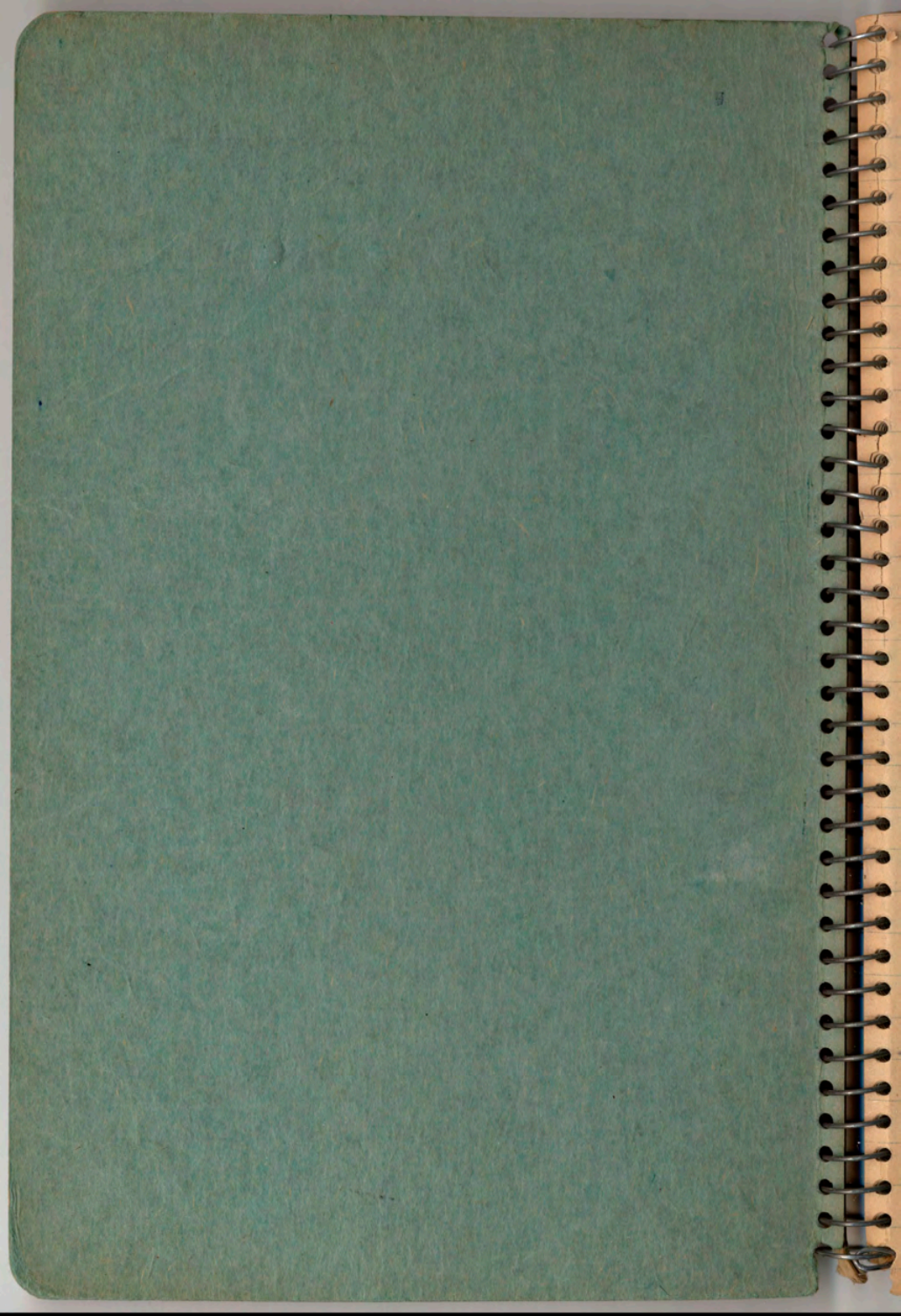




Winter Quarter 1949

1

Explanation of a nuclear chart

Stable + radioactive nuclei

β^- β^+ α radioactivities

Half life and mean life

$$\frac{dn}{dt} = -\frac{n}{\tau} \quad n = n_0 e^{-t/\tau}$$

$$t_{1/2} = \tau \times \ln 2 = .693 \tau$$

Principle of the mass-spectrograph

$$\frac{mv^2}{2} = ZeV$$

$$mv^2 = ZeV$$

$$\frac{mv^2}{r} = \frac{Ze}{c} vH$$

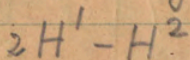
$$mv = \frac{Ze}{c} rH$$

$$v = \frac{2cV}{rH}$$

$$m = \frac{Ze r^2 H^2}{2Vc^2}$$

Problem: Draw a chart of the stable isotopes

Problem: Design a mass spectrograph for measuring the mass difference



2

Nuclear masses

Unit of atomic weight:

$$\frac{1}{N} = 1.6599 \times 10^{-24} \text{ gr}$$

$$(N = 6.0245 \times 10^{23} \text{ at/mole})$$

Energy equivalent

$$1.6599 \times 10^{-24} (2.9976 \times 10^{10})^2 =$$

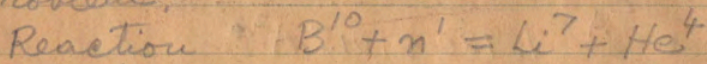
$$= 1.4915 \times 10^{-3} \text{ ergs} = 931.0 \text{ MeV}$$

$$\frac{e\hbar}{2M_{\text{proton}}c} = 5.049 \times 10^{-24}$$

Nucleus	At. wt.	Spin	Magn. Mom.	Quadrupole moment
H ¹	1.00813	1/2	2.7896	
n ¹	1.00898	1/2	-1.9103	
H ²	2.01471	1	.85647	$2.73 \times 10^{-27} = \frac{3e^2 - e_z^2}{4}$
He ³	3.01700	1/2	-2.12506	
H ³	3.01702	1/2	2.9756	
He ⁴	4.00390	0		
Li ⁶	6.01697	1	.8214	
He ⁶	6.02094			
Be ⁷	7.019			
Li ⁷	7.01822	3/2	3.2527	
Be ⁹	9.01503	3/2	-1.176	
B ¹⁰	10.01618	3	1.7987	
B ¹¹	11.01284	3/2	2.6857	
C ¹²	12.00382	0		

Magn. moment
of proton = 1.4102×10^{-23}
#2

Problem:



Find: Velocity of reaction products in slow neutron reaction.

For inverse reaction:

Threshold of α -particles on Li^7 target.

Mass formula (See P.R. 94 398 1954 for newer data)

$$M = M_H Z + M_n (A-Z) - a_1 A + a_2 A^{2/3} + a_3 \frac{\left(\frac{A}{2} - Z\right)^2}{A} + \frac{3}{5} \frac{Z^2 e^2}{r_0 A^{1/3}} \frac{1}{1.66 \times 10^{-24} \text{C}^2}$$

$$r_0 = 1.48 \times 10^{-13}$$

Electrostatic energy $.000627 Z^2 / A^{1/3}$

$$M = 1.00898 A - .00085 Z - a_1 A + a_2 A^{2/3} + a_3 \frac{\left(\frac{A}{2} - Z\right)^2}{A} + .000627 \frac{Z^2}{A^{1/3}}$$

Stable $Z = Z_A$ from

$$\frac{\partial M}{\partial Z_A} = 0 \quad Z_A = \frac{a_3 + .00085}{2a_3 + .001254 A^{2/3}} A$$

Fit with $a_3 = .083$

4

$$Z_A = \frac{A}{1.980 + .015 A^{2/3}}$$

Coefficients a_1, a_2 from fitting of packing fraction curve

$$M_A = (1 + f) A$$

A	$f \times 10^4$	A	$f \times 10^4$	A	$f \times 10^4$
20	-1.5	80	-6.6	180	+1.5
30	-4.6	90	-6.2	200	+2.0
40	-7.0	100	-5.7	220	+4.2
50	-7.2	120	-4.6	240	+5.8
60	-7.1	140	-3.3		
70	-6.9	160	-1.3		

One finds

$$a_1 = .01507$$

$$a_2 = .014$$

Suggested formula

$$M(A, Z) = .99391 A - .00085 Z + .014 A^{2/3} + .083 \frac{\left(\frac{A}{2} - Z\right)^2}{A} + .000627 \frac{Z^2}{A^{1/3}} + \delta$$

$$\delta = 0 \text{ for } A \text{ odd}$$

$$\delta = \pm .036 / A^{3/4} \text{ for } A \text{ even, } Z \begin{cases} \text{even} \\ \text{odd} \end{cases}$$

Comments:

Results to be expected from a degenerate gas model

$$\text{Max. momentum} \begin{cases} P_p = (3\pi^2)^{1/3} \hbar (Z/V)^{1/3} \\ P_n = (3\pi^2)^{1/3} \hbar (Z/V)^{1/3} \end{cases}$$

$$V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} r_0^3 A = 1.36 \times 10^{-38} A$$

Total ^{kin.} energy of protons & neutrons

$$\frac{3^{7/3} \pi^{2/3} \hbar^2}{5 \times 2^{7/3} M r_0^2} \frac{Z^{5/3} + (A-Z)^{5/3}}{A^{2/3}}$$

One finds a_3 on assumption that kin. energy only counts

$$a_3(\text{kin}) = \frac{3^{1/3} \pi^{2/3}}{2} \left(\frac{\hbar}{M r_0 c} \right)^2 = .0315$$

Too small by factor about 2.6

Also one finds

$$a_1(\text{kin}) = - \frac{3^{7/3} \pi^{2/3}}{40} \left(\frac{\hbar}{M r_0 c} \right)^2 = -.0142$$

Which indicates that a_1 (potential)

6
must be about

$a_1(\text{pot}) = .015 + .014 = .029$
corresponding to about 27 MeV
potential depth.

Estimate of order of magnitude
of capillary coefficient a_2

Assuming that nucleons within
 r_0 of surface have only $1/2$ the
bulk binding energy one finds

$$\frac{a_2 A^{2/3}}{a_1 A} = \frac{1}{2} \frac{4\pi R^2 r_0}{4\pi R^3} = \frac{3}{2 A^{1/3}}$$

hence

$$\frac{a_2}{a_1} = \frac{3}{2}$$

instead of experimental .9

Surface tension:

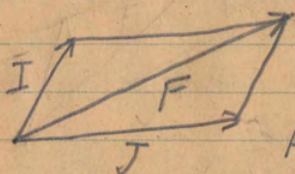
$$= \frac{a_2 A^{2/3} \times Mc^2}{4\pi (r_0 A^{1/3})^2} = \frac{a_2 Mc^2}{4\pi r_0^2} = 7.59 \times 10^{19} \text{ ergs/cm}^2$$

Problem: Calculate with the mass formula
the binding energy of an α -particle to
 U^{235} and the energies released when a proton
or a neutron is attached to U^{235}

Nuclear spins

Methods:

a) From hyperfine structures



$$A = \frac{\mu H}{IJ}$$

H = electron mag. field at nucleus

$$\frac{1}{2} A [F(F+1) - J(J+1) - I(I+1)]$$

Interaction energy due to magnetic field of electron orbit acting on nuclear magnetic moment.

Interaction energy;

$$\mathcal{H} = - e \alpha \cdot A$$

e = abs. value of electron charge

$$A = \frac{\mu_o \times r}{r^3}$$

μ_o = nuclear magnetic moment

$$\mathcal{H} = - e \frac{\alpha \cdot \mu_o \times r}{r^3} = e \mu_o \cdot \frac{\alpha \times r}{r^3}$$

Averaging over electron orbit gives

$$\mathcal{H} = \mu_o \cdot e \int \frac{\tilde{\Psi} \times \Psi \times r}{r^3} d^3 r$$

8

Non relativistic approximation
for one electron gives

s-terms $a = \frac{16\pi}{3I} \mu \mu_0 \psi^2(0)$

μ = Bohr magneton

μ_0 = nuclear magn. moment

Other terms

$$a = \frac{2l(l+1)}{Ij(j+1)} \mu \mu_0 \frac{1}{r^3}$$

For further details

E. Fermi, Zs.f. Phys. 60 320 1930

Fermi & Segre, Zs.f. Phys. 82 729 1933

b) From band spectra

Alternating intensities ratios

$$\frac{I+1}{I}$$

c) From atomic beams

$$\text{Deflection} \approx \frac{L \mu_z}{AMv^2} \frac{\partial H}{\partial z} \quad (\text{angular deflection})$$

In particular zero moments method

d) Magnetic resonance

$$\nu = \frac{\mu H}{2\pi I \hbar}$$

$$\frac{\text{nuclear magneton}}{2\pi \hbar} = 762.5$$

Method is used with atomic or molecular beams or with microwave induction or absorption

e) Nuclear scattering

f) Polarization of resonance radiation

For results see table p.2

Regularities:

All even even nuclei have zero spin

$$\text{Proton} + 2.7896 \quad 1 + 1.7896$$

$$\text{Neutron} - 1.9103 \quad 0 - 1.9103$$

Connection with Dirac-type theory and with attempts at explaining with meson theory.

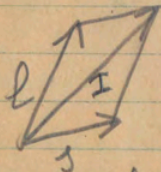
$$\text{Deuterium: } .84647$$

$$(\text{Proton} + \text{Neutron}) = .8793$$

in fair agreement with 3S state of deuteron.

(Mention correction due to quadrupole moment)

Heavy nuclei with odd neutron or odd proton



Landé type formula

$$\mu_I = \mu_s \cos \hat{sI} + \mu_l \cos \hat{lI}$$

$$g_I = g_s \frac{s(s+1) + I(I+1) - l(l+1)}{2I(I+1)} + \\ + g_l \frac{l(l+1) + I(I+1) - s(s+1)}{2I(I+1)}$$

For odd neutron

$$j = \frac{1}{2} \quad g_s = -2 \times 1.91 \quad g_l = 0$$

$$l = I + \frac{1}{2}$$

$$\mu_I = 1.91 \frac{I}{I+1}$$

$$l = I - \frac{1}{2} \quad \mu_I = -1.91$$

For odd proton

$$j = \frac{1}{2} \quad g_s = 2 \times 2.79 \quad g_l = 1$$

$$l = I + \frac{1}{2} \quad \mu_I = \frac{I^2 - 1.29 I}{I+1}$$

$$l = I - \frac{1}{2} \quad \mu_I = \frac{I^2 + 3.29 I + 2.29}{I+1}$$

Agreement is indifferent but indicates that model may have some correct feature.

(See Goldsmith & Inglis - Brookhaven)
BNL-1-5 1948

Quadrupole moments

Definition:

$$Q = \frac{1}{e} \int \rho (3z^2 - r^2) d^3r$$

(z is the nuclear spin axis)

Note that when $I < 1$ Quadrupole moment vanishes

(Proof: for $I = \frac{1}{2}$

$$z \text{ prop to } \frac{r \cdot I}{I} = x I_x + y I_y + z I_z$$

$$z^2 \text{ prop to } x^2 I_x^2 + y^2 I_y^2 + z^2 I_z^2 + \\ + xy(I_x I_y + I_y I_x) + \dots = \frac{1}{4} r^2$$

$$\text{use } I_x I_y = -I_y I_x, I_x^2 = \frac{1}{4}$$

Quad. moments are observed usually from the interaction with the quadrupole part of the potential at the nucleus in atoms or molecules

Mostly atomic fine structures in a few cases molecular beam and magnetic resonance techniques

Deuteron has

$$Q = .00273 \times 10^{-24} \text{ cm}^2$$

which indicates admixture of d -eigenfunction.

For various nuclei Q ranges from

to $-.6 \times 10^{-24}$ for I_{53}^{127}

$+ 7 \times 10^{-24}$ for Lu_{71}^{176}

Negative Q means a flattened shape

Positive Q means an elongated shape

A rotation ellipsoid with semiaxis a, b and uniformly distributed charge has

$$Q = \frac{2Z}{5} (a^2 - b^2)$$

$$\approx \frac{4}{5} \times 10^{-24} A^{2/3} Z \left(\frac{a}{b} - 1 \right) \quad \text{for } \left| \frac{a}{b} - 1 \right| \ll 1$$

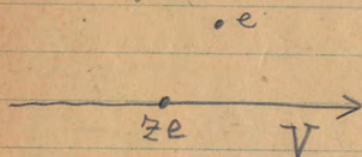
$$\approx .0175 \times 10^{-24} A^{2/3} Z \left(\frac{a}{b} - 1 \right)$$

For $\text{Lu}_{71}^{176} \quad \frac{a}{b} = 1.18$

For $\text{I}_{53}^{127} \quad \frac{a}{b} = .98$

Passage of radiations through matter

Charged particles
Ionization loss:



$$(2) \frac{ze^2}{b^2} \frac{b}{V} \approx \text{momentum transmitted}$$

$$\left(\int E_{\perp} dx \right) \times 2\pi b = 4\pi ze \quad (\text{also relativistic})$$

$$\int E_{\perp} dx = \frac{2ze}{b} \quad dx = V dt$$

$$p_{\perp} = \frac{2ze^2}{bV}$$

$$\frac{p^2}{2m} = \frac{2z^2 e^4}{mV^2 b^2}$$

$$\text{Loss per unit path} = \frac{2z^2 e^4}{mV^2 b^2} 2\pi b n db$$

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4 n}{mV^2} \log \frac{b_{\max}}{b_{\min}}$$

Upper limit of b : $b_{\max}$

From need of adequate frequency

Classical:

$$\frac{V}{b} \gg v$$

Relativistic:

$$\frac{V}{b \sqrt{1 - B^2}} \gg v \quad B = \frac{V}{c}$$

also, for very hi velocities, the polarization effect:

(Fermi, Phys. Rev. 57 485 1948)

gives in the limit $B \rightarrow 1$

$$b_{\max} \approx \sqrt{\frac{mc^2}{3.17 \pi n e^2}} \quad (3.17 = e^2 \times \text{Euler constant})$$

Lower limit of b

Classical limit:

As order of magnitude energy transmitted to electron cannot exceed mV^2 (non relativistic)

Hence

$$b_{\min} \gtrsim \frac{ze^2}{mV^2}$$

16.

Also quantum limit: $b_{\min} \approx$
of wave length of apparent electron
wave packets

$$b_{\min} \approx \frac{\hbar \sqrt{1-B^2}}{mV}$$

For a wide energy range

$$\frac{b_{\max}}{b_{\min}} \approx \frac{mV^2}{\hbar \sqrt{1-B^2}}$$

For non relativistic velocities

Bethe recommends: (Hand. d. Phys. XXIV p. 522)

$$\log \frac{b_{\max}}{b_{\min}} = \begin{cases} \log \frac{2mV^2}{E} & \text{for heavy particles} \\ \log \left(\frac{mV^2}{2E} \sqrt{\frac{e}{z}} \right) & \text{for electrons} \end{cases}$$

$\bar{E}$ For various substances

	$\bar{E}$
H ₂	17.6 eV
O ₂	37
A	59
He	28

For relativistic velocities

Electrons

$$-\frac{dE}{dx} = \frac{2\pi ne^4}{mV^2} \times$$

$$\times \left\{ \log \frac{mV^2 E'}{2\bar{E}^2(1-B^2)} - (2\sqrt{1-B^2} - 1 + B^2) \log 2 + 1 - B^2 \right\}$$

(E' = energy of electron
exclusive of rest
energy)

Heavy particles

$$-\frac{dE}{dx} = \frac{4\pi ne^4 z^2}{mV^2} \left(\log \frac{2mV^2}{\bar{E}(1-B^2)} - B^2 \right)$$

Using the approximate formula

$$-\frac{dE}{dx} = \frac{4\pi z^2 ne^4}{mV^2} \log \frac{mV^2}{h\nu(1-B^2)}$$

one finds

Approximate rules:

- 1) Particles of equal velocity lose energy per unit path proportional to z^2
- 2) The ionization loss has a flat minimum at the relativistic boundary beyond which it increases extremely slowly with energy (logarithmically)
- 3) By equal n heavy elements are slightly less effective than light elements in producing ionization losses.

Energy loss in MeV/gr/cm²

Electron	air	19.5	3.67	1.69	1.95	2.47	2.99	3.48
	Pb	10	2.2	1.10	1.35	1.75	2.17	2.56

Proton	air	300	47	7.6	2.31	2.31
	Pb	150	27.5	5.0	1.56	1.63

Log (MeV
of primary) 4 5 6 7 8 9 10

(No density effect included)

Including density effect the asymptotic formula for very large energies is (case of non electrons)

$$-\frac{dE}{dx} = \frac{2\pi ne^4}{mc^2} \left[\log \frac{m^3 c^4}{\pi ne^2 h^2 (1-B^2)} - 1 \right]$$

$$= nmc^2 \times .499 \times 10^{-24} \log \frac{7.06 \times 10^{30}}{n(1-B^2)}$$

α - particles in air (15°C 760 cm Hg)

Range	α particles Energy (MeV)
0 cm	0
.1	.10
.2	.24
.5	.85
1	1.92
2	3.40
3	4.52
5	6.31
7	7.76
10	9.68
12	10.80
15	12.31
18	13.70
20	14.54
22 cm	15.32

For intermediate ranges $R_{cm} = .32 (\text{MeV})^{3/2}$

Protons in air (15° 760 mm Hg)

Range (cm) (Protons)	Energy (MeV)	Deuterons	
0 cm	0 MeV	0 cm	0 MeV
.2	.145	.4	.29
.5	.375	1	.75
1	.56	2	1.12
2	.91	4	1.82
3	1.18	6	2.36
4	1.42	8	2.84
5	1.625	10	3.25
7	1.99	14	3.98
10	2.45	20	4.90
15	3.11	30	6.22
20	3.68	40	7.36
25	4.20	50	8.40
30	4.66	60	9.32
50	6.24	100	12.48
70	7.56	140	15.12
100	9.25	200	18.50
150	11.62	300	23.24
200	13.65	400	27.30

(Data from Bethe, Rev of Mod Phys)

C July 1937

Ranges for high energies

Protons

Protons	Air range	Al (gr/cm ²)	Pb (gr/cm ²)
.232 MeV	2.58×10^{-4} gr/cm ²		
.500	8.59×10^{-4}		
2	8.57×10^{-3}	10.8×10^{-3}	.029
10	.141	.1667	.36
15	.292	.339	
25	.725	.837	
50	2.52	2.878	
100	8.70	9.854	16.6
200	29.22	32.84	
300	57.88	64.80	
400	92.49	103.3	
500	131.55	146.7	
700	219.21	243.8	
1000	365.60	405.5	624
2000	900.37	994.1	
5000	2518.2	2763	
10000	5036.4	5501	8020
34556	15489		
93959	38306		60500

Minimum ionization loss in MeV/gr/cm²

Protons in air 1.825 at 2500 MeV energy

Protons in Al 1.664 at 2000 MeV

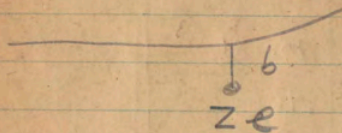
Protons in Pb 1.118 at 2000

(Data from E.P. Gross, Princeton)

22

Scattering (non nuclear)

Side momentum



$$\frac{zeZe^2}{bv}$$

$$\theta \approx \frac{zeZe^2}{bv^2}$$

$$2\pi b db = \frac{8\pi ze^2 Z^2 e^4}{v^2 p^2} \frac{d\theta}{\theta^3}$$

Exact (non relativistic) formulae

$$\tan \frac{\theta}{2} = \frac{zeZe^2}{mbv^2}$$

$$d\sigma = \frac{\pi e^4 Z^2}{m^2 v^4} \frac{d\theta}{\tan^3 \frac{\theta}{2} \cos^2 \frac{\theta}{2}}$$

$$\frac{d\sigma}{d\omega} = \frac{1}{4} \left(\frac{e^2 Z z}{m v^2} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}}$$

Comments on limits of validity
 Non relativistic formula is correct
 for Coulomb forces except when particle
 enters the nucleus. Also effect of
 exchange in collision of like particles

Multiple scattering

$$\overline{\theta^2} = \frac{8\pi z^2 Z e^4 N}{v^2 p^2} \log \frac{D_{\max}}{D_{\min}}$$

Let $(N = \text{atoms/cm}^2 \text{ encountered})$
 $D_{\max} \approx 1$

$D_{\min}$ limited by screening at

$$b \approx \frac{a}{Z^{1/3}} \quad a = \text{Bohr radius}$$

$$D_{\min} \approx \frac{2\pi Z e^2}{a v p} = \frac{4 \text{ Ryd}}{v p} \approx Z^{4/3}$$

Example: electrons in Pb.
 (non relativistic range)

$$\overline{\theta^2} \approx \frac{3 \times 10^7 \times \delta x}{(\text{keV})^2} \times (1/20) \approx \frac{6 \times 10^8 \delta x}{(\text{keV})^2}$$

Hence very rapid loss of direction
 Electrons in air at N.P.T.

$$\overline{\theta^2} \approx \frac{7000 \delta x}{(\text{keV})^2}$$

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Multiple scattering is important in determining apparent absorption coeff. of β -rays.

Practical formula
end point of β spectrum in MeV

$$= 8.70 T_{1/2}^{.677}$$

$T_{1/2}$ = half thickness in g/cm^2 Al

Absorption of γ -rays

a) Photoeffect (Bethe p. 477)

Absorption cross-section for one $1s$ electron

$$\sigma = \frac{128\pi}{3} \frac{e^2}{mc} \frac{\nu_1^3}{\nu^4} \frac{e^{-4n' \arctan n'}}{1 - e^{-2\pi n'}}$$

with

$$\nu_1 = \frac{\text{Rydberg} \times Z^2}{h}$$

$$n' = \sqrt{\frac{\nu_1}{\nu - \nu_1}}$$

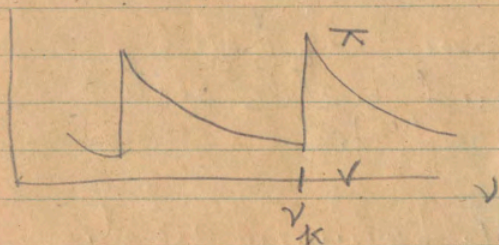
Near absorption edge

$$\sigma \approx \frac{6.31 \times 10^{-18}}{Z^2} \left(\frac{\nu_1}{\nu} \right)^{8/3}$$

Mass-absorption coeff due to K-shell

$$\left\{ \begin{array}{ll} \frac{7.6 \times 10^6}{AZ^2} \left(\frac{\nu_1}{\nu} \right)^{8/3} & \nu - \nu_1 \ll \nu_1 \\ \frac{6.5 \times 10^7}{AZ^2} \left(\frac{\nu_1}{\nu} \right)^{7/2} & \nu - \nu_1 \gg \nu_1 \end{array} \right.$$

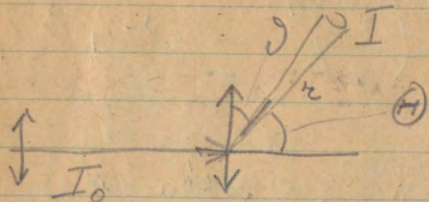
L-absorption at K-absorption edge is about 1/5 of K absorption and varies approximately like ν^{-3} .



$$\frac{\tau(\nu_K + \epsilon)}{\tau(\nu_K - \epsilon)} \approx 5.35$$

Thomson cross-section

$$\frac{\frac{2}{3} \frac{e^2}{c^3} \left(\frac{eE}{m} \right)^2}{c \frac{E^2}{4\pi}} = \frac{8\pi \left(\frac{e^2}{mc^2} \right)^2}{3} = 6.65 \times 10^{-24}$$

Compton effect and
Klein - Nishina formula

θ = angle between directions of primary & scattered radiation

ϑ = angle between scattered radiation and electric vector of primary radiation

I_0 = intensity of primary rad.

I = " " scattered " at distance r from electron

$$a = \frac{h\nu}{mc^2}$$

$$\frac{I}{I_0} = \frac{e^4}{m^2 c^4 r^2} \frac{\sin^2 \theta}{[1 + \alpha(1 - \cos \theta)]^3} \times$$

$$\times \left(1 + \alpha^2 \frac{(1 - \cos \theta)^2}{2 \sin^2 \theta [1 + \alpha(1 - \cos \theta)]} \right)$$

Total cross section is.

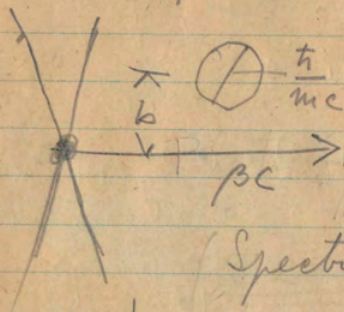
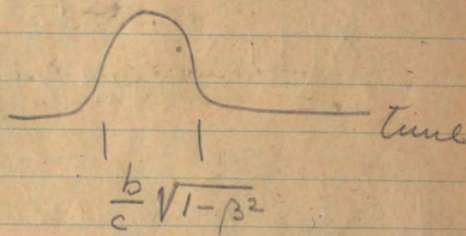
$$\sigma_{KN} = \frac{2\pi e^4}{m^2 c^4} \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha} \log(1+2\alpha) \right] + \right. \\ \left. + \frac{1}{2\alpha} \log(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}$$

Scattered wave length is

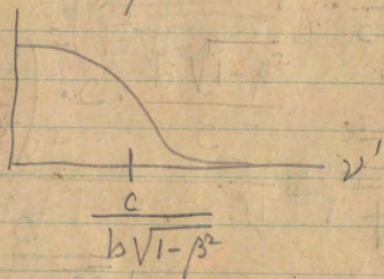
$$\lambda_{\text{scatt}} - \lambda_{\text{primary}} = \frac{h}{mc} (1 - \cos \theta)$$

Radiative loss of fast electrons
Field of nucleus passing by

$$\frac{Ze}{b^2} \frac{1}{\sqrt{1-\beta^2}} = \text{field}$$



Spectrum



No of photons per second

$$\int \frac{Z^2 e^2}{b^4 (1-\beta^2)} \cdot \frac{b}{c} \sqrt{1-\beta^2} \cdot c \cdot \frac{d\nu'}{\frac{c}{b\sqrt{1-\beta^2}}} \cdot \frac{1}{h\nu'} N b db$$

$$= \frac{Z^2 e^2 N}{c h} \frac{d\nu'}{\nu'} \log \frac{b_{\max}}{b_{\min}}$$

Use ^{later} relativity transformation

$$\nu = \frac{\nu' (1 + \beta \cos \theta')}{\sqrt{1 - \beta^2}} \approx \frac{\nu'}{\sqrt{1 - \beta^2}}$$

$$b_{\max} = \text{smallest of } \frac{c}{\nu' \sqrt{1 - \beta^2}} \text{ and } \frac{a_0}{Z^{1/3}} = \frac{\hbar^2}{Z^{1/3} m e^2}$$

at very large β

$$b_{\max} = \frac{\hbar^2}{m e^2 Z^{1/3}}$$

$$b_{\min} = \frac{\hbar}{m c} ; \quad \frac{b_{\max}}{b_{\min}} = \frac{137}{Z^{1/3}}$$

Photons are scattered (Thomson) at rate

$$\approx \frac{Z^2 e^2 N}{c \hbar} \left(\frac{e^2}{m c^2} \right)^2 \log \frac{137}{Z^{1/3}} \frac{d\nu'}{\nu'}$$

Relativity transformation gives

$$\frac{d\nu'}{\nu'} \approx \frac{d\nu}{\nu}$$

Final formula approximately

$$\sigma_{d\nu} = 4 \frac{Z^2}{137} \left(\frac{e^2}{m c^2} \right)^2 \log \frac{137}{Z^{1/3}} \frac{d\nu}{\nu}$$

Mean Radiative loss per unit path

$$-\left(\frac{dE}{dx}\right)_{\text{rad}} = E \frac{4Z^2}{137} \left(\frac{e^2}{mc^2}\right)^2 \log \frac{137}{Z^{1/3}}$$

Discuss the fluctuations

Radiation unit (where in the average energy is lost)

$$\frac{1}{l} = \frac{4Z^2 N}{137} \left(\frac{e^2}{mc^2}\right)^2 \log \frac{137}{Z^{1/3}}$$

Air $l \approx 360 \text{ m}$

Al $l \approx 11 \text{ cm}$

Pb $l \approx .58 \text{ cm}$

Pair formation cross-section

$$\frac{h\nu}{mc^2} \quad \sigma_{\text{pair}} / \left[\frac{Z^2}{137} \left(\frac{e^2}{mc^2}\right)^2 \right]$$

2 .00

3 .12

4 .36

5 .64

6 .91

10 2.0

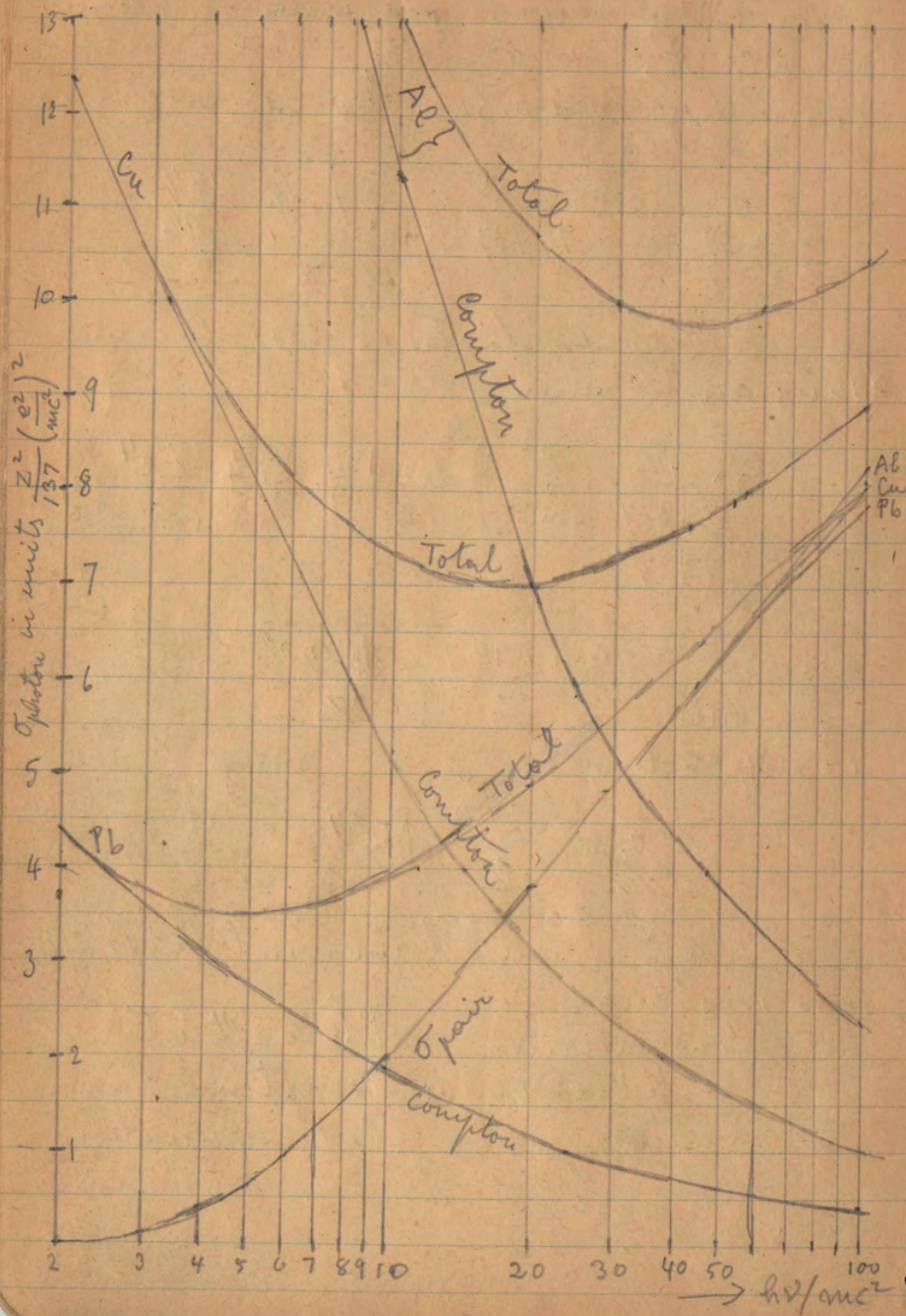
20 3.8

40 5.7

60 6.8

Bethe + Heitler
P.R.S. 146 83 1934

82



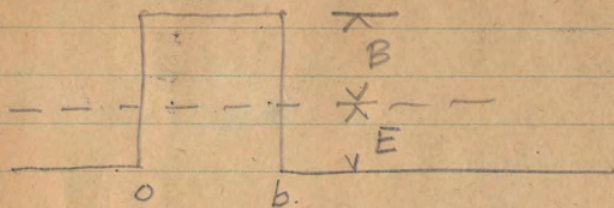
Lowest absorption coefficient

in Pb = $1/(25 \text{ gr/cm}^2)$ at ~ 2 to 3 MeV

in Cu = $1/(31 \text{ gr/cm}^2)$ at $\sim 20 \text{ MeV}$

in Al = $1/(47 \text{ gr/cm}^2)$ at $\sim 35 \text{ MeV}$

A brief discussion of the cascade
process (qualitative)

Emission of α -particlesTransparency of a barrier

$$\psi'' + \frac{2m}{\hbar^2} \begin{cases} E \\ -B \end{cases} \psi = 0$$

$$p^2 = \frac{2mE}{\hbar^2}$$

$$Q^2 = \frac{2mB}{\hbar^2}$$

Left of 0

$$\alpha e^{ipx} + \beta e^{-ipx}$$

 $0 < x < b$

$$A e^{Qx} + B e^{-Qx}$$

 $x > b$

$$\gamma e^{ipx}$$

$$\alpha + \beta = A + B$$

$$ip(\alpha - \beta) = Q(A - B) \quad \left| \begin{array}{l} A e^{Qb} + B e^{-Qb} = \gamma e^{ipb} \\ Q(A e^{Qb} - B e^{-Qb}) = \\ = ip\gamma e^{ipb} \end{array} \right.$$

one finds

$$\alpha = \frac{\gamma e^{iPb}}{4} \left\{ \left(1 + \frac{Q}{iP}\right) \left(1 + \frac{iP}{Q}\right) e^{Qb} + \left(1 - \frac{Q}{iP}\right) \left(1 - \frac{iP}{Q}\right) e^{-Qb} \right\}$$

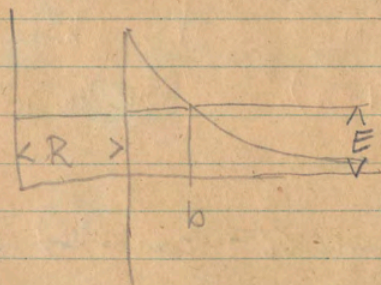
Approximately

$$\frac{|\alpha|^2}{|\gamma|^2} = e^{2Qb}$$

$$\text{Transparency} \approx e^{-2\sqrt{\frac{2m}{\hbar^2}} B b}$$

Simple Gamow Theory

$$\frac{1}{\tau} \approx \frac{V_{\text{inside}}}{R} e^{-2\sqrt{\frac{2m}{\hbar^2}} \int_R^b \sqrt{\frac{2Ze^2}{r} - E} dr}$$



$$\frac{2Ze^2}{b} = E$$

$$\int_R^b \sqrt{\frac{1}{r} - \frac{1}{b}} dr = \sqrt{b} \left\{ \arccos \sqrt{\frac{R}{b}} - \sqrt{\frac{R}{b} - \frac{R^2}{b^2}} \right\}$$

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$$\tau = \frac{R \sim 10^{-21}}{v_{\text{inside}}} e^G$$

$$G = \sqrt{\frac{8\pi Z e^2 b}{\hbar^2}} \left\{ \arccos \sqrt{\frac{R}{b}} - \sqrt{\frac{R}{b} - \frac{R^2}{b^2}} \right\}$$

$$\frac{Z e^2}{b} = \frac{1}{2} m v^2$$

$$G = \frac{4\pi Z e^2}{\hbar v} \left\{ \arccos \sqrt{\frac{R}{b}} - \sqrt{\frac{R}{b} - \frac{R^2}{b^2}} \right\}$$

$$x = \arccos \sqrt{x} - \sqrt{x - x^2}$$

0	$\frac{\pi}{2} = 1.571$	}	$\frac{\pi}{2} - 2\sqrt{x}$	(hi barrier)
.01	1.371			
.05	1.129	}		
.1	.950			
.2	.755	}		
.3	.537			
.4	.396	}		
.6	.192			
.8	.063	}	$\frac{2}{3}(1-x)^{3/2}$	(low barrier)
1.0	0			

$$G \approx \frac{2\pi Z e^2}{\hbar v}$$

$$- \sqrt{\frac{32 R Z e^2 m c^2}{\hbar^2}}$$

Geiger - Nuttall Law (R = range)

$$\frac{1}{\tau} = \text{const } R^{60}$$

$$\text{U family } \text{const} = 10^{-42.3}$$

$$\text{Th } \text{const} = 10^{-44.2}$$

$$\text{Ac } \text{const} = 10^{-46.3}$$

Potential barrier for α -particles

$$\frac{2Ze^2}{R} \approx \frac{2Ze^2}{1.48 \times 10^{-13} A^{1/3}} = 3.11 \times 10^{-6} \frac{Z}{A^{1/3}} \text{ ergs}$$

$$= 1.94 \frac{Z}{A^{1/3}} \text{ MeV}$$

For ${}_{92}^{238}\text{U}$

$$B = 28.8 \text{ MeV}$$

For ${}_{88}^{226}\text{Ra}$

$$B = 27.7$$

For ${}_{82}^{207}\text{Pb}$

$$B = 26.9$$

For ${}_{47}^{107}\text{Ag}$

$$B = 19.2$$

For ${}_{26}^{56}\text{Fe}$

$$B = 13.2$$

For ${}_{13}^{27}\text{Al}$

$$B = 8.4$$

$$b = \frac{2.875 \times 10^{-13}}{\text{MeV}}$$

Examples (Bethe B p 166)

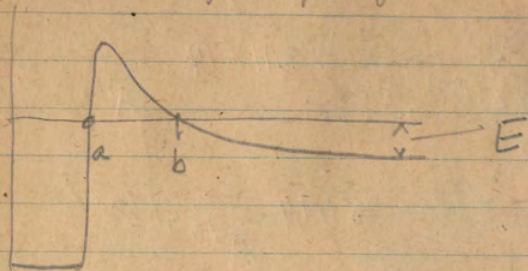
(Old radius)

			$\lambda = 1/c$	$R(\text{old})$
Th	$88+2$	4.34 MeV	1.2×10^{-18}	8.7
U^{238}	$90+2$	4.15	5×10^{-18}	9.8
Th C	$82+2$	6.20	6.7×10^{-5}	7.0
Ra C'	$82+2$	7.83	7×10^4	9.4
	$(Z-2)+2$			
Sm	$60+2$	2.55	1.7×10^{-19}	6.7

Discuss limitations of Gamow theory

Ex. Take $A=100, 150, 200, 250, 300$
 $Z=44, 62, 80, 97, 113$

nuclei. Calculate their life time against α -decay.

Better proof of Gamow formula

$$g = \frac{2m}{\hbar^2} (E - U)$$

$$r_1 \approx 1 \quad r_2 \approx 1$$

$$\begin{aligned} \left\{ \begin{array}{l} C_- e^{-\int_a^x \sqrt{-g} dx} \\ C_+ e^{\int_a^x \sqrt{-g} dx} \end{array} \right. & \longleftrightarrow C_- r_1 e^{-\int_a^b \sqrt{-g} dx} \sin \int_a^b \sqrt{g} dx \\ & \longleftrightarrow C_+ r_2 e^{\int_a^b \sqrt{-g} dx} \cos \int_a^b \sqrt{g} dx \end{aligned}$$

$$C_+ + C_- = \sin \sqrt{g} a$$

$$G = 2 \int_a^b \sqrt{-g} dx$$

$$C_+ - C_- = \frac{\sqrt{g}}{\sqrt{-g_a}} \cos \sqrt{g} a$$

$$(\text{amp outside})^2 = C_+^2 r_2^2 e^G + C_-^2 r_1^2 e^{-G}$$

Virtual level where

$$C_+ = 0 \quad \text{or} \quad \frac{\sqrt{g_i}}{\sqrt{-g_a}} \cot \sqrt{g_i} a = -1$$

Near virtual level (distance ϵ) has order of magnitude

$$C_+ \approx \frac{\epsilon}{\delta E}$$

$$C_- \approx 1$$

$\delta E =$ interval between virtual levels
 $\epsilon =$

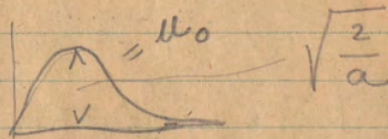
$$\frac{(\text{amp outside})^2}{(\text{amp inside})^2} \approx \frac{\epsilon^2}{\delta \epsilon^2} e^G + e^{-G}$$

(Comments on meaning of this)

Normalized e.f.

$$u = \frac{\sqrt{2/L}}{\sqrt{\frac{\epsilon^2}{\delta \epsilon^2} e^G + e^{-G}}} \left(\sqrt{\frac{a}{2}} u_0(x) \right) \sin \sqrt{g} x \leftrightarrow \leftrightarrow$$

Initial wave funst
(not stationary state)



$$\psi(0) \approx u_0(x)$$

Coeff of development of $\psi(0)$;

$$\psi(0) \approx \sum_n \frac{a}{L} \frac{u_0(x)}{\frac{\epsilon_n^2}{\delta \epsilon^2} e^G + e^{-G}} \quad (\text{inside})$$

$$\psi(t) = u_0(x) \sum_n \frac{a}{L} \frac{e^{-\frac{i t}{\hbar} \epsilon_n}}{\frac{\epsilon_n^2}{\delta \epsilon^2} e^G + e^{-G}}$$

$$\epsilon_{n+1} - \epsilon_n = v(p_{n+1} - p_n) = v \frac{\pi \hbar}{L} \quad (v = \text{velocity outside})$$

$$\sum_n = \int \frac{L d\epsilon}{\pi \hbar v}$$

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is amp of probability to be inside

$$\frac{a}{L} \frac{L}{\pi \hbar \nu} \int_{-\infty}^{\infty} \frac{e^{-\frac{it}{\hbar} E} dE}{\frac{E^2}{8E^2} e^G + e^{-G}}$$

Using

$$\int_{-\infty}^{\infty} \frac{e^{-i\alpha x} dx}{1+x^2} = \pi e^{-\alpha}$$

find amp. proportional to

$$e^{-\frac{L}{\hbar} \frac{\delta E}{2}} = e^{-\frac{t}{2T}}$$

which yields mean life

$$T = \frac{\hbar}{2\delta E} e^G$$

Comments on width of levels
(from p.39) half width at half
maximum for amplitude is

$$\epsilon_{1/2} = \delta E \times e^{-G}$$

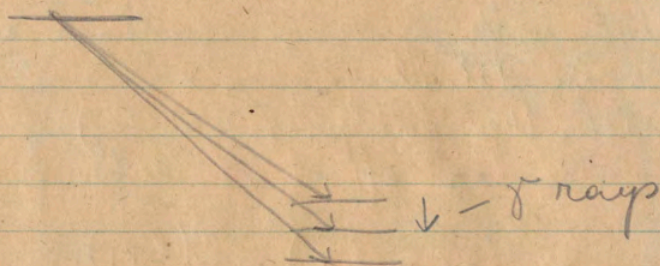
$$\epsilon_{1/2} T = \frac{\hbar}{2} \quad \left(\text{Heisenberg} \right. \\ \left. \text{indeterminacy} \right)$$

Fine structure of α -particles

Examples Ra - Two lines with disintegration energies 5.612 and 5.550.

		Dis. energy	Intensity
Th C	α_1	6.200	27%
	α_2	6.160	70%
	α_3	5.873	1.8
	α_4	5.728	.2
	α_5	5.709	1.1

Theory



Using for Gamow factor approximate formula (p. 36)

$$G = \frac{2\pi zZe^2}{\hbar v} - \sqrt{\frac{32 R z Z m e^2}{\hbar^2}}$$

$$\delta G = \frac{2\pi zZe^2}{\hbar v^2} \delta v = \frac{2\pi zZe^2}{2\hbar v E} \delta E = \frac{\pi zZe^2 \sqrt{m} \delta E}{\sqrt{2} \hbar E^{3/2}}$$

$$\left(\text{for } Z=86\right) \approx \frac{170 \delta E}{E^{3/2}} \quad (\text{energies in MeV})$$

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Hence energy should drop like

$e^{-170 \frac{\delta E}{E^{3/2}}}$ (E in MeV)
Drop by factor 100 for

$$\frac{\delta E}{.01} \approx \frac{4.6}{170} E^{3/2} = .027 E^{3/2}$$

for $E = 6$ MeV

$$\delta E_{.01} = .027 \times 6^{3/2} = .4 \text{ MeV}$$

Comparison with experimental observations

Agreement with general features and failure in detail

Reasons for the failures

In particular effect of centrifugal force

$$g = \frac{2m}{\hbar^2} \left(\frac{ZZe^2}{r} - E \right) + \frac{l(l+1)}{r^2}$$

$$\frac{l(l+1)}{r^2}$$

$$\frac{\frac{2mZZe^2}{\hbar^2 r}}{\frac{l(l+1)}{r^2}} \approx 2 \times 10^{-3} l(l+1)$$

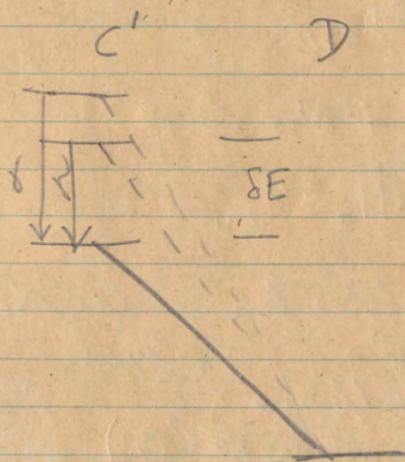
about factor 10 for $l \approx 5$ hence rather small

Long range α -particles

Observed only on very short living products Ra C', Th C'

$$(\tau = 10^{-5} \quad \tau = 3.2 \times 10^{-7} \text{ sec})$$

2.2×10



Order of magnitude of relative intensity

$$\approx \frac{\frac{1}{\tau} e^{170 \delta E / E^{3/2}}}{\frac{1}{\tau_\delta}} \quad (E \text{ in MeV})$$

Example Th C' Two long range lines
 $\delta E = .73$ and 1.80 $E = 8.95$ $\tau = 3 \times 10^{-7}$
 intensity 3×10^{-5} and 19×10^{-5}

Formula gives $\tau_\delta \approx \begin{matrix} 10^{-14} \\ 16 \times 10^{-17} \end{matrix}$ } reasonable order of magnitude

Emission of β^- particles

Generalities on continuous spectrum
 The neutrino hypothesis
 Consequences of energy conservation

$$M_Z c^2 > M_{Z+1} c^2$$

In this case the normal β^- -emission with

$$E_{\max} = M_Z - M_{Z+1}$$

(Total energy of electron binding amounts to

$-1.54 \text{ Rydbergs} \times Z^{7/3}$
 but does not change result.)

If

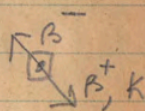
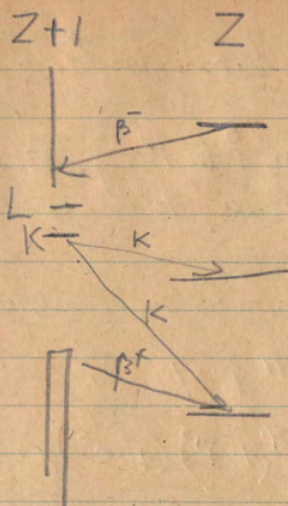
$$M_Z < M_{Z+1} - \begin{cases} E_K \\ E_L \\ E_M \end{cases}$$

one might have K, L, M capture

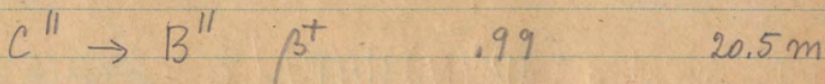
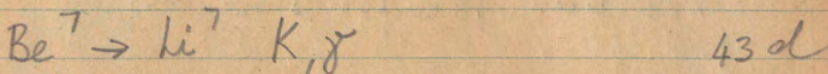
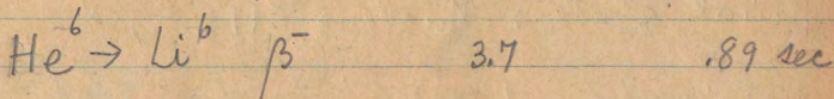
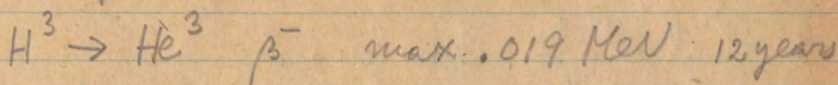
If

$$M_Z < M_{Z+1} - 2mc^2$$

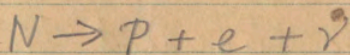
positron emission (β^+)



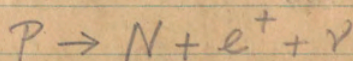
Examples



β^- emission as neutron $\rightarrow$ proton transformation

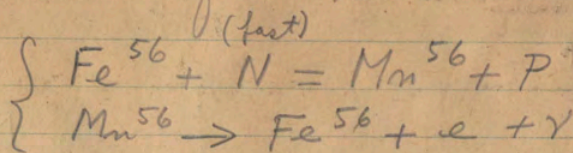


β^+ as

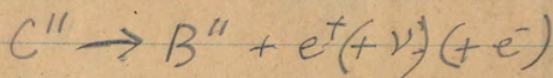
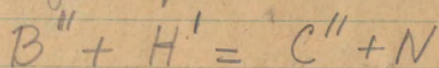


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Conversion of a neutron into a proton



Conversion of a proton into a neutron



Simplified outline of β theory

Initial state: Nucleus Z

Final state: Nucleus $Z+1$, e , γ

Matrix element: Various forms of theory differ.

One form gives:

$$(F) \quad \psi_p \psi_n g \int \psi^*(N) \psi(P) d\tau \quad (\text{for allowed})$$

Other theories give

$$(GT) \quad \psi_p \psi_n g \int \psi^*(N) \sigma \psi(P) d\tau \quad (\text{for allowed})$$

Assuming for a moment that the integral is M_0 and Z is small so that electron waves are plane
Also neglect spin

$$\mathcal{W} = \frac{g}{\Omega} d\Omega$$

Prob. of emission of electron of given $\vec{p}$

$$\frac{2\pi}{\hbar} \frac{g^2 d\Omega^2}{\Omega^2} \frac{4\pi p_v^2 dp_v d\Omega}{8\pi^3 \hbar^3 c dp_v} = \frac{g^2 W_v^2 d\Omega^2}{\pi \hbar^4 c^3 \sqrt{2}}$$

Probability of emission of electron p, dp

$$\frac{g^2 d\Omega^2}{2\pi^3 \hbar^7 c^3} \left(\sqrt{m^2 c^4 + c^2 p_o^2} - \sqrt{m^2 c^4 + c^2 p^2} \right)^2 p^2 dp$$

$$\frac{1}{c} = \frac{g^2 d\Omega^2 m^5 c^4}{2\pi^3 \hbar^7} F(\eta_o)$$

$$F(\eta_o) = \int_0^{\eta_o} \left(\sqrt{1+\eta_o^2} - \sqrt{1+\eta^2} \right)^2 \eta^2 d\eta$$

$$= -\frac{1}{4} \eta_o - \frac{1}{12} \eta_o^3 + \frac{1}{30} \eta_o^5 + \frac{1}{4} \sqrt{1+\eta_o^2} \log(\eta_o + \sqrt{1+\eta_o^2})$$

η_o	$F(\eta_o)$	η_o	$F(\eta_o)$
0	$\frac{2}{105} \eta_o^7$	3	6.23
.5	.00012	4	29.5
1	.0115	∞	$\frac{1}{30} \eta_o^5$
2	.557		

When Z is not negligible

$$f(Z, w_0) = \int_1^{w_0} w dw \sqrt{w^2 - 1} (w_0 - w)^2 F(iZ, w)$$

with

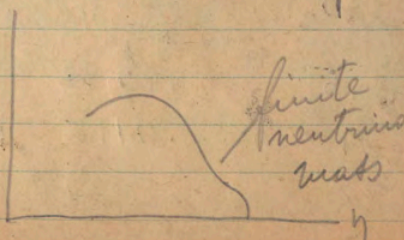
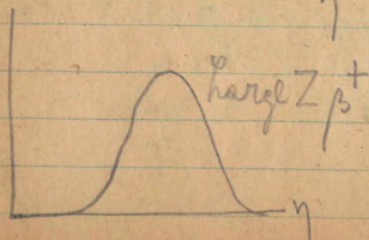
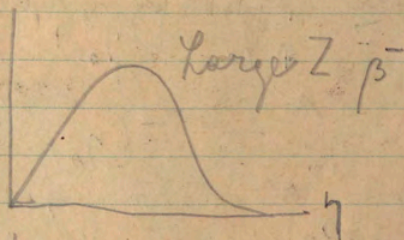
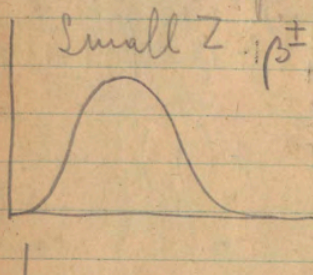
$$w = \sqrt{1 + \eta^2}$$

$$F(Z, w) = 4(2\eta R)^{2s-2} e^{\pi \alpha Z w / \eta} \times$$

$$\times \frac{|\Gamma(s + i \alpha Z w / \eta)|^2}{\{\Gamma(2s + 1)\}^2} \frac{1+s}{2}$$

$$\alpha = 1/137 \quad s = \sqrt{1 - \alpha^2 Z^2}$$

Various shapes of β -spectra



Allowed & forbidden transitions

Assuming matrix elements on p. 46

For form (F)

Allowed $I = I'$ no parity change

For form (GT)

Allowed: $I = I' \pm 1, I'$ "no"
(excluded $0 \rightarrow 0$)

When transition is permitted intensity is determined by M^2 . Maximum value of $M \approx 1$ for good overlap

As a rule expect M to decrease from light to heavy elements.

Wigner elements

Active isotope	Particles	Max energy	$T_{1/2}$	$f T_{1/2}$
H^1	β^-	.79 MeV	—	—
H^3	β^-	.019	12 yrs	800
C^{11}	β^+	.99	20.5 m	4300
N^{13}	β^+	1.24	10.1 m	5400
O^{15}	β^+	1.7	126 sec	4000
F^{17}	β^+	2.1	70 sec	5400
Ne^{19}	β^+	2.2	20.3 sec	1930

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Other light elements

Isotope	Radiat	Energy	$\tau_{1/2}$	$f \tau_{1/2}$
He ⁶	β^-	3.7 MeV	.87 sec	1180
C ¹⁰	β^+	2.2	19.1 sec	2100
O ¹⁴	β^+	1.8	76.5 sec	3450
F ¹⁸	β^+	.7	112 m	5400
B ¹²	β^-	12	.027 sec	13200
Li ⁸	β^-	12	.89 sec	4.2×10^5
C ¹⁴	β^-	.154	6400 yrs	10^9
Be ¹⁰	β^-	.56	2.5×10^6 yrs	4×10^{13}

For medium & heavy elements

$$f \tau \approx 10^4 \text{ to } 10^5$$

For extensive $f \tau$ - table

E. J. Konopinski - Rev. Mod. Phys.

15, p. 209, 1943Value of g for G.T. interaction is about

$$g \approx 3 \times 10^{-49} \text{ cm}^5 \text{ gr sec}^{-2}$$

However great uncertainty on precise form of interaction.

K-capture

Usually observed by X-ray emission

Prob of transition (See p 47)

$$\frac{1}{\tau} = \frac{g^2 W_\gamma^2 M^2}{\pi \hbar^4 c^3} \psi_e^2(0)$$

Put

$$\psi^2(0) = \frac{Z^3}{\pi a^3 n^3}$$

$$a = \frac{\hbar^2}{m e^2}$$

$$\frac{1}{\tau} = \frac{g^2 M^2 W_\gamma^2 Z^3}{\pi^2 \hbar^4 c^3 a^3 n^3}$$

$$= 7 \times 10^{-10} M^2 \left(\frac{W_\gamma}{m c^2} \right)^2 \frac{Z^3}{n^3}$$

For example Be^7

$$W_\gamma \approx 2 m c^2 \quad Z=4 \quad n=1$$

$$\frac{1}{\tau} = 1.8 \times 10^{-8} M^2$$

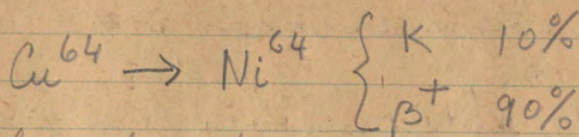
One finds $\tau = 5.4 \times 10^6$
Hence

$$M \approx .3$$

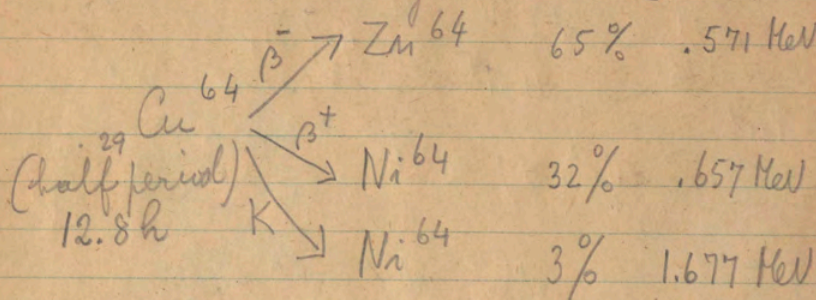
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K-capture + β^+ emission are often observed at same time.

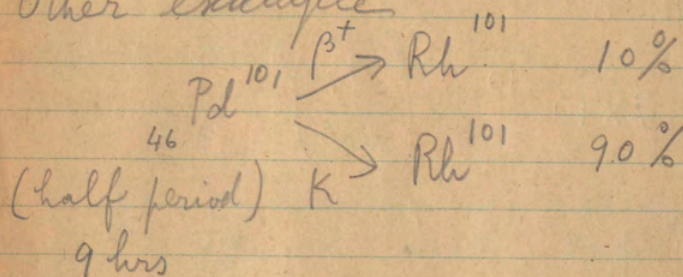
E.g.



Cu^{64} also disintegrates to Zn^{64} .



Other example



For very heavy elements K-capture usually predominates on β^+ emission due to nuclear charge effect.

Various forms of β interaction

Most current types are

$$g \int_{\alpha} [\bar{\psi}(N) A_{\alpha} \psi(P)] [\bar{\psi}_r A_{\alpha} \psi_e] d\tau$$

+ complex conjugate

where A_{α} may be

a scalar

$$A = \gamma_4$$

a 4-vector

$$A_{\nu} = \gamma_4 \gamma_{\nu}$$

a pseudoscalar

$$A = \gamma_1 \gamma_2 \gamma_3$$

a pseudovector

$$\gamma_2 \gamma_3, \gamma_3 \gamma_1, \gamma_1 \gamma_2, -\gamma_1 \gamma_2 \gamma_3 \gamma_4$$

a tensor (sex-vector)

$$A_{\mu\nu} = \gamma_4 \gamma_{\mu} \gamma_{\nu}$$

(See Quantum mechanics p. 29)

Notations

$$\gamma_1 = \frac{\beta \alpha_x}{i}$$

$$\gamma_2 = \frac{\beta \alpha_y}{i}$$

$$\gamma_3 = \frac{\beta \alpha_z}{i}$$

$$\gamma_4 = -\beta$$

Allowed transitions are obtained by neglecting relativistic effects for the heavy particles and disregarding the variations of the light particle contribution over the nuclear volume

Light particle contribution to the square of matrix element involves factor

$$\left(\int \tilde{\Psi}_\nu A \Psi_e \right) \left(\int \tilde{\Psi}_e' A^\dagger \Psi_\nu' \right) =$$

$$= \iint \tilde{\Psi}_e' A \Psi_\nu' \tilde{\Psi}_\nu A \Psi_e$$

Using (Quantum Mechanics p. 34) (for $m_\nu = 0$)

$$\Omega \Psi_\nu' \tilde{\Psi}_\nu = \frac{1}{2} \left(1 - c \frac{p_\nu}{|p_\nu|} \cdot \alpha \right) e^{\frac{i}{\hbar} p(r'-r)}$$

Neglecting phase factor for allowed transitions and averaging over all directions of neutrino moment

$$\Psi_\nu' \tilde{\Psi}_\nu = \frac{1}{2}$$

Using also $A_\alpha^2 = 1$ one finds light particle factor (Neglect difference between Ψ_ν' and $\frac{1}{2} \tilde{\Psi}_\nu$)

In allowed approx square of matrix element is

$$\frac{g^2}{2} (\tilde{\psi}_e \psi_e) \sum_{\alpha} (N | A_{\alpha} | P)^2 = |{}^0\mathcal{H}|^2$$

For vector interaction dominant term is $\alpha=4$ and gives

$$|{}^0\mathcal{H}|^2 = \frac{g^2}{2\Omega} (\tilde{\psi}_e \psi_e) \int \tilde{\psi}(N) \psi(P)$$

leading to F-selection rules
Allowed $J'=J$, "no"

For pseudovector or tensor interaction dominant terms are the space components which in non relativistic approximation are identical to the spin. They give

$$|{}^0\mathcal{H}|^2 = \frac{g^2}{2} (\tilde{\psi}_e \psi_e) \sum_i \int \tilde{\psi}(N) \sigma_i \psi(P)$$

leading to G.T.-selection rules

Allowed $J'=J \pm 1$ } "no"
or $J'=J$

with exception of $0 \rightarrow 0$ which is forbidden

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There are several more types of interaction e.g.

$$(\bar{\psi}(N) A_2 \psi_e)(\bar{\psi}_\nu A_2 \psi(P))$$

or any scalar built by a linear combination of products of

$$\sum_{l,m,n,r} c_{lmnr} \psi_e^*(N) \psi_m(P) \psi_\nu^* \psi_r^e$$

Also possible description of neutrinos a la Majorana
(See 104 p 18)

Outline of Yukawa mechanism for β interaction

$$N \rightarrow P + \pi$$

$$\pi \rightarrow e + \nu$$

Discussion:

$$\pi \rightarrow e + \nu$$

disintegration is not observed and is probably to slow by at least a factor 10 to 100.

Properties of neutrinos

Recoil of emitted neutrino

Example in case of the Be^7

K-capture. Neutrino energy $\approx 2mc^2$

Neutrino momentum $= 2mc$

$$\text{Be}^7 \text{ recoil energy } \frac{(2mc)^2}{2M_{\text{Be}}} = 2mc^2 \frac{m}{M_{\text{Be}}}$$

$$= \frac{2 \times 510000}{1840 \times 7} \text{ eV} = 79 \text{ eV}$$

When electron & neutrino are emitted usually there is an angular correlation between the two directions that depends on details of theory

Hamilton, P.R. 71 p 456, 1947

Attempts to detect neutrinos give an upper limit to the cross-section of about 10^{-31}

Meaning of the distinction between neutrinos and antineutrinos

Double β -transitions (Fireman,

62

Total prob. of transition for dipole radiation:

$$\text{factor} = \underbrace{4\pi}_{\text{solid angle}} \underbrace{(2)}_{\text{polarisation}} \underbrace{\left(\frac{1}{3}\right)}_{\text{direction average}} = \frac{8\pi}{3}$$

$$\begin{aligned} \frac{1}{\tau_f}(\text{dipole}) &= \frac{4\omega}{3\hbar c^3} \left| \left(\sum_s \frac{e_s}{m_s} \vec{p}_s \right)_{nn'} \right|^2 \\ &= \frac{4\omega^3}{3\hbar c^3} \left| \left(\sum_s e_s x_s \right)_{nn'} \right|^2 \end{aligned}$$

$\frac{3 \cancel{3} \cancel{2} \cancel{2} \cancel{x}^2}{4 \cancel{m}^2 \cancel{c}^4} = \frac{3}{4}$

Case of a single particle

Use sum rule

$$\sum_m (E_m - E_n) |x_{nm}|^2 = \frac{\hbar^2}{2m}$$

Put

$$|x_{nm}|^2 = \frac{\hbar}{2m\omega} f_{nm}$$

f = oscillator number

would give

$$\frac{1}{\tau_f}(\text{dipole}) = \frac{2}{3} \frac{e^2 \omega^2}{m c^3} f$$

Dipole, quadrupole, hipo γ rays

For a proton:

$$\frac{1}{\tau_p} = \frac{2}{3} \frac{e^2}{\hbar c} \frac{m_e}{m_p} \frac{m_e c^2}{\hbar} \left(\frac{\hbar \omega}{m_e c^2} \right)^2 \times f$$

$$= 2.06 \times 10^{15} \left(\frac{\hbar \omega}{m_e c^2} \right)^2 \times f$$

Dipole transitions are usually very weak

Argument: no dipole transitions if e/m were the same for all parts of nucleus

Quadrupole radiation

Order of magnitude (from end of p. 113)
($M = \text{nucleon mass}$)

$$\frac{1}{\tau} = 4\pi \frac{\omega}{2\pi \hbar c^3} \left| \frac{e\omega}{Mc} \left(\xi \eta \right)_{nn'} \right|^2$$

$$\approx \frac{2\omega}{\hbar c^3} \frac{e^2 \omega^4}{c^2} \left| \left(\xi \eta \right)_{nn'} \right|^2 =$$

$$= \frac{2e^2}{\hbar c^5} \omega^5 \left| \left(\xi \eta \right)_{nn'} \right|^2$$

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with capillary forces only

Liquid drop model gives frequencies

$$\omega^2 = \left(\frac{G}{\rho S} \right) \frac{l(l+2)(l-1)}{R^3}$$

$G =$ ^{total} surface energy of nucleus

$\rho =$ density of nucleus

$R, S =$ radius & surface of nucleus

From p 4

$$G = .014 A^{2/3} \times 1.49 \times 10^{-3} = 2.09 \times 10^{-5} A^{2/3}$$

$$R = 1.48 \times 10^{-13} A^{1/3}$$

$$S = 4\pi R^2 = 2.75 \times 10^{-25} A^{2/3}$$

$$\rho = \frac{1.66 \times 10^{-24} A}{\frac{4\pi}{3} R^3} = 1.22 \times 10^{14}$$

$$\omega = \frac{1.39 \times 10^{22}}{\sqrt{A}} \sqrt{l(l-1)(l+2)}$$

$$\hbar\omega = 9.1 \text{ MeV} \times \sqrt{\frac{l(l-1)(l+2)}{A}}$$

Frequency of ellipsoidal ($l=2$) vibration including electrostatic energy is (See NG 15)

$$\omega = 2\pi\gamma = 3.9 \times 10^{22} \sqrt{\frac{1}{A} - .0224 \frac{Z^2}{A^2}}$$

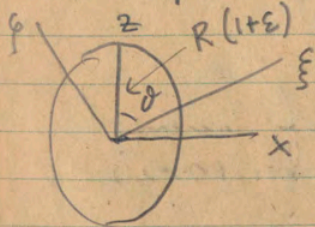
$$= 25.7 \sqrt{\frac{1}{A} - .0224 \frac{Z^2}{A^2}} \text{ MeV}$$

For $A=100$	$Z=44$		$\omega \text{ (MeV)}$
			1.94 MeV

For $A=238$	$Z=92$		$\omega \text{ (MeV)}$
			.75 MeV

Classical calculation gives for the emitted energy (p. 60)

$$\frac{d\omega}{d\omega} = \frac{\omega^2}{8\pi c} \left| \int J_{0\theta} \frac{\omega}{c} \xi d\tau \right|^2 \quad (\text{per sterad})$$



$$J_{0x} = -\frac{\rho}{c} x \frac{\dot{\epsilon}_0}{2}$$

$$J_{0z} = \frac{\rho}{c} z \dot{\epsilon}_0$$

$$\frac{d\omega}{d\omega} = \frac{\omega^2}{8\pi c} \frac{\rho^2}{c^2} \dot{\epsilon}_0^2 \frac{\omega^2}{c^2} \left(\int (z \sin \theta + \frac{x}{2} \cos \theta) (z \cos \theta + x \sin \theta) d\tau \right)^2$$

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$$\int = \frac{1}{2} \left(\int_0^R r^2 4\pi r^2 dr \right) \sin \theta \cos \theta$$

$$= \frac{2\pi}{5} R^5 \sin \theta \cos \theta$$

Integrating on all directions

$$W = \frac{3}{500} \frac{e^2 Z^2 R^4 \omega^4}{c^5} \epsilon_0^2$$

Energy of mode is on the other hand (NG 18)

$$\frac{3}{20} M A R^2 \epsilon_0^2$$

Hence classical time in which energy is lost

$$\frac{1}{\tau} = \frac{1}{25} \frac{\omega^4}{c^5} \frac{e^2 R^2}{M} \frac{Z^2}{A}$$

This is equal to prob. of transition when one quantum only is excited.

$$\frac{1}{\tau} = \frac{1}{25} \frac{e^2 R^2}{M c^5} \frac{Z^2}{A} (3.9 \times 10^{22})^4 \left(\frac{1}{A} - .0224 \frac{Z^2}{A} \right)^2$$

$$= \frac{1}{25} \frac{e^2 \epsilon_0^2}{M c^5} (3.9 \times 10^{22})^4 \frac{Z^2}{A^{7/3}} \left(1 - .0224 \frac{Z^2}{A} \right)^2$$

or

$$\frac{1}{\tau} = 1.17 \times 10^{16} \frac{Z^2}{A^{7/3}} \left(1 - .0224 \frac{Z^2}{A}\right)^2$$

For U^{238}

$$\frac{1}{\tau} = 1.16 \times 10^{13}; \quad \frac{1}{\tau} = .0076 \text{ eV}$$

From p. 63 follows that a well permitted quadrupole radiation may have

$$\frac{1}{\tau} \approx \frac{e^2 \omega^5}{\hbar c^5} R^4 \approx 10^{13} \times (\text{MeV})^5$$

For dipole radiation one adds at each step a factor of order of magnitude

$$\left(\frac{\omega R}{c}\right)^2$$

giving for 2^l -pole

$$\frac{1}{\tau} \approx \frac{\omega}{137} \left(\frac{\omega R}{c}\right)^{2l}$$

For large l and small ω , τ can be long (example $l=4$)

$$\tau = 600 \text{ sec}$$

$$\begin{aligned} \hbar\omega &= 10^5 \text{ eV} \\ R &= 5 \times 10^{-13} \end{aligned}$$

This is the qualitative explanation of metastable states, however more on this later.

Magnetic dipole radiation

Qualitative discussion

Analogy with p. 62 gives

$$\frac{1}{\tau}(\text{mag. dipole}) = \frac{4\omega^3}{3\hbar c^3} |\vec{\mu}_{nm}|^2$$

As order of magnitude

$$\mu_{nm} \approx \frac{e\hbar}{2Mc}$$

$$\frac{1}{\tau}(\text{mag. dipole}) \approx \frac{e^2 \hbar \omega^3}{3 c^5 M^2} \approx 4 \times 10^{12} (\text{MeV})^3$$

At $\hbar\omega \approx 1 \text{ MeV}$ quadrupole and magnetic dipole have comparable intensities

Magnetic pole radiation of order 2^l has

$$\frac{1}{\tau}(\text{mag. } 2^l\text{-pole}) \approx \frac{e^2 \hbar \omega^3}{3 M^2 c^5} \left(\frac{\omega R}{c}\right)^{2l-2}$$

Internal conversion

Non relativistic theory (Electric dipole)

Initial state

$$\psi_0 = s(r)$$

Final state

$$\psi_1 = \frac{1}{\sqrt{L}} p(r) \cos \theta$$

Perturbation

$M \cos \omega t$ = electric moment

$$\begin{aligned} & \frac{eM}{r^2} \cos \theta \cos \omega t \\ \mathcal{H}_b &= \frac{eM}{\sqrt{L}} \int \frac{\cos \theta}{r^2} s(r) p(r) \cos \theta \cdot 2\pi \sin \theta d\theta r^2 dr \\ &= \frac{4\pi}{3} M Q e / \sqrt{L} \frac{e^{\pm i\omega t}}{2} \end{aligned}$$

with

$$Q = \int s(r) p(r) dr$$

$$\frac{1}{\tau} (\text{conversion}) = \frac{2\pi}{\hbar} |\mathcal{H}_b|^2 (\text{density factor})$$

$$= \frac{2\pi}{\hbar} \cdot \frac{4\pi^2 e^2 M^2 Q^2}{9 L} \frac{L}{v\pi\hbar} = \frac{8\pi^2 e^2 M^2 Q^2}{9 \hbar^2 v}$$

Rate of emission of photons is

$$\frac{1}{\tau} (\text{photons}) = \frac{e^2 M^2 \omega^3}{3 \hbar c^3}$$

Internal conversion coefficient is defined

$$\alpha_k = \frac{2 \Gamma_{\text{conv}}}{2 \Gamma_{\text{conv}} + \Gamma_{\text{photon}}}$$

When $\alpha_k \ll 1$ non relativistic result is

$$\alpha_k = \pi \left(\frac{4 \hbar c}{e^2} \right)^3 \left(\frac{\text{Ryd}}{\hbar \omega} \right)^4 Z^4 \frac{e^{-4n \arctan \frac{1}{m}}}{e^{2\pi n} - 1}$$

with

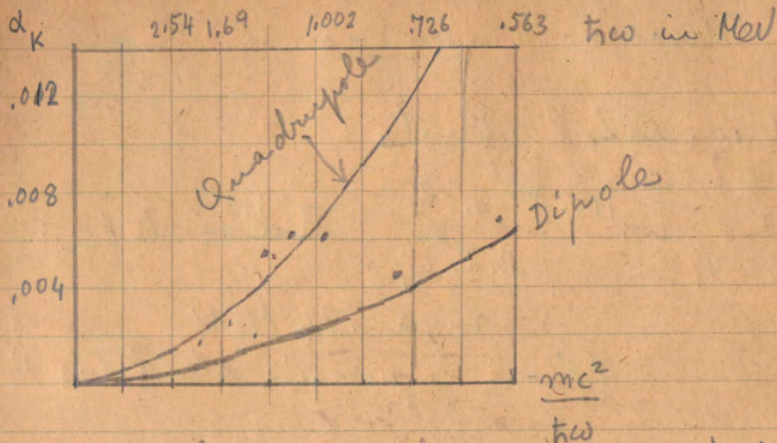
$$m = \frac{Z^2 \text{Ryd}}{\hbar \omega - Z^2 \text{Ryd}} \quad \alpha_k \approx \frac{\sqrt{2}}{137^4} \frac{Z^3}{(\hbar \omega / m c^2)^{3/2}}$$

Relativistic calculations are important for heavy elements
Also calculations needed for
0-pole dipole quadrupole
magnetic dipole etc...

(Taylor, Mott, Hulme, F. Oppenheimer)

Rose - P.R. 83 79 (1951)

Blatt + Weisskopf p. 619



Internal conversion coefficients for heavy (Naturally radioactive) atoms for dipole & quadrupole radiation

Also

$$\alpha_K : \alpha_L : \alpha_M \approx 100 : 15 : 5$$

Noteworthy case: the 1.414 line of RaC' is totally converted. Its life time is $\tau(\text{conversion}) = 8 \times 10^{-12}$ sec about 100 times longer than γ -life times in that region.

For detailed discussion see
Bethe-B p. 229 or
Rasetti - p. 142

Also internal conversion by pair formation with probability $\approx 10^{-3}$ for 2.6 MeV γ -rays (Jaeger + Hulme)

For magnetic 2^l -pole

$$\begin{cases} |I - I'| \leq l \\ I + I' \geq l \end{cases} \quad \begin{array}{l} \text{parity change} \\ (-1)^{l+1} \end{array}$$

Internal conversion permits all transitions including

$0 \rightarrow 0$ "no" (See 105 p 75)

$0 \rightarrow 0$ "yes"

with varying probabilities

Report on Nuclear Isomerism
(Segre & Helmholz-

Rev. of Mod. Phys. 21, 271, April 1949)

$$\frac{1}{\tau} = \frac{8\pi}{9} \underbrace{\frac{e^6 m^2}{a^3 \hbar^5}}_{1.47 \times 10^{50}} \frac{Z^4}{1 - e^{-\frac{2\pi Z e^2}{\hbar v}}} \left| (R^2)_{12} \right|^2$$

Natural Radioactivity

Generalities on natural & artificial radioactivity

Radioactive chains (Problems)

Statistical laws of radioactive decay

Observational methods

Counters (G.M. - Proportional -
scintillation - Pulse
chambers)

Ionization chambers

Other methods

Natural radioactive families

Uranium family $\frac{1}{2}$ -life

${}_{92}\text{U}^{238}$ (UI) $4.498 \times 10^9 \text{ y}$ α 4.21 MeV

$\downarrow \alpha$

${}_{90}\text{Th}^{234}$ (UX₁) 24.1 d β^- 0.20 γ 0.092

$\downarrow$

${}_{91}\text{Th}^{234}$ (UX₂) 1.14 m β^- , γ , (IT)

$\downarrow$

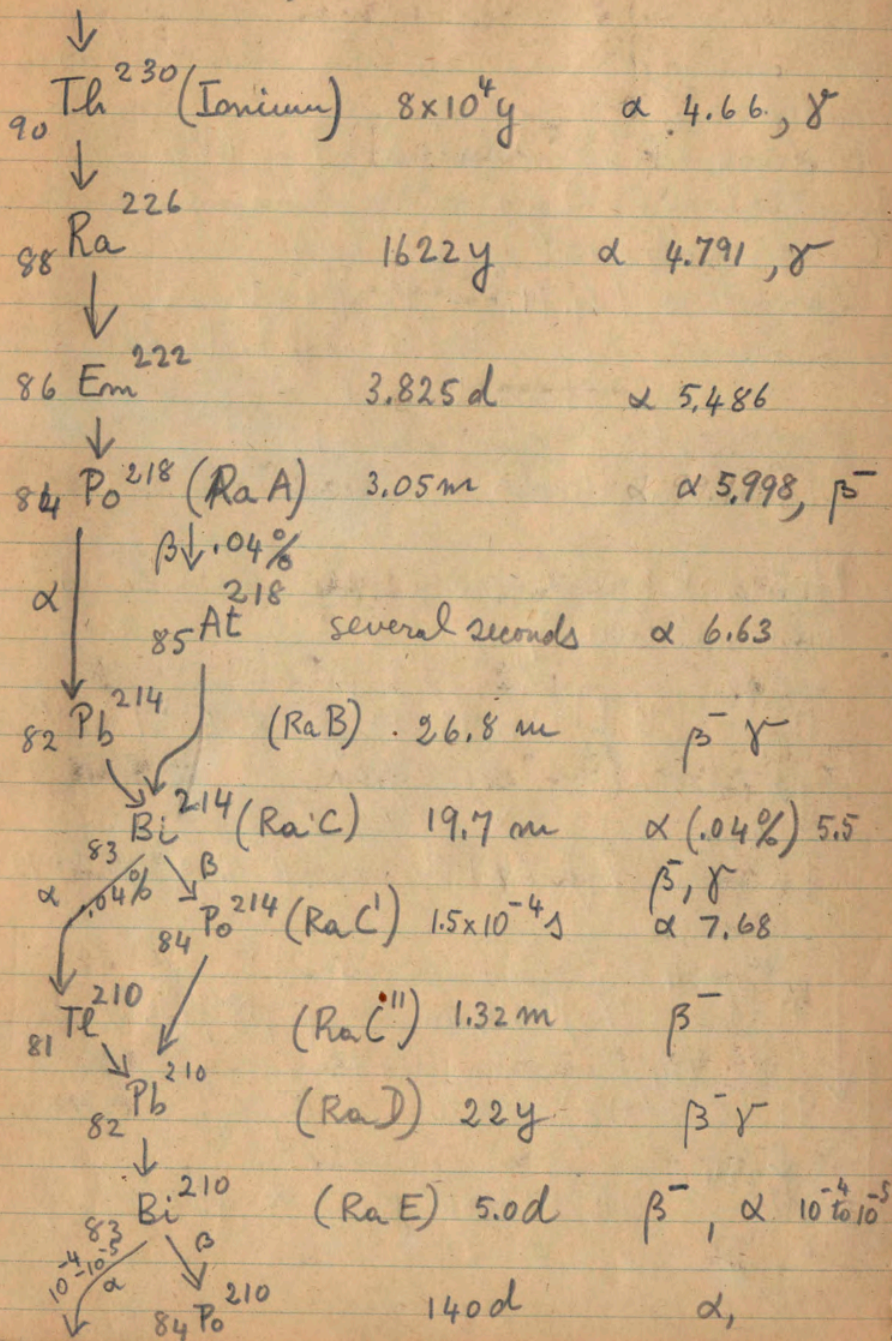
${}_{91}\text{Th}^{234}$ (UZ) 6.7 h β^- , γ

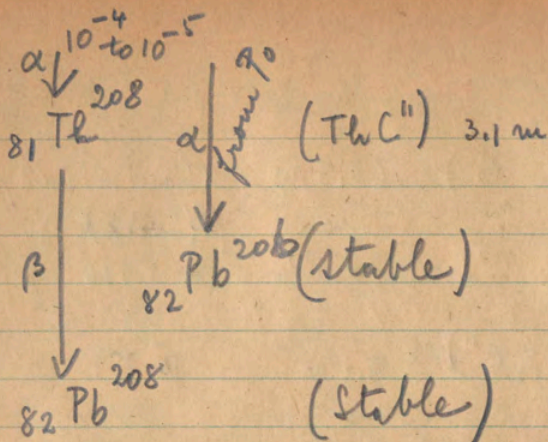
$\downarrow$

${}_{92}\text{U}^{234}$ (VII) $1.62 \times 10^5 \text{ y}$ α 4.825

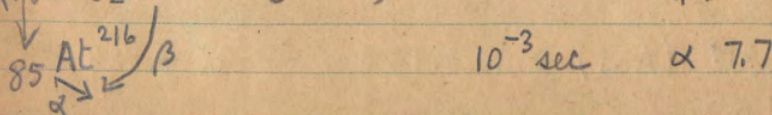
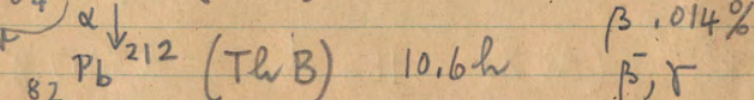
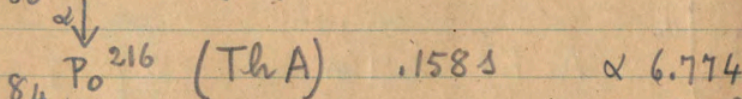
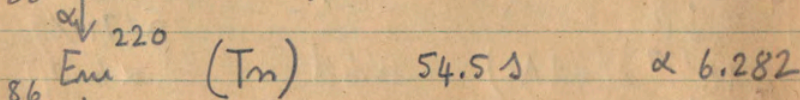
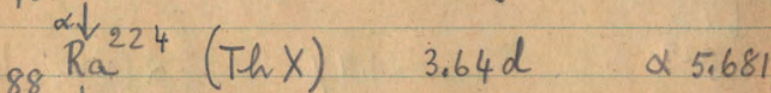
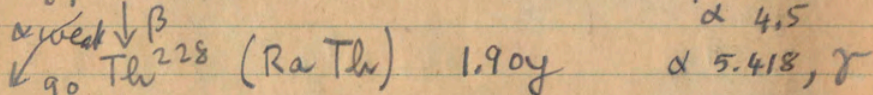
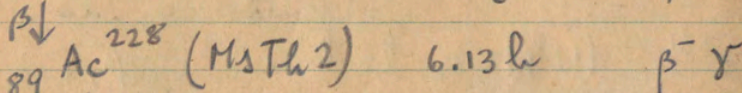
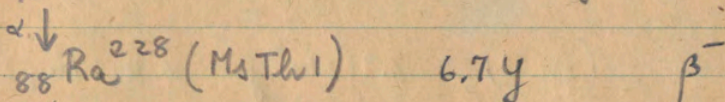
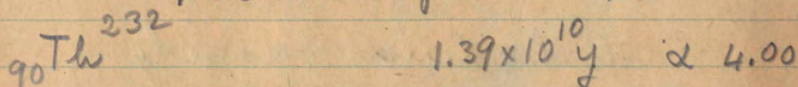
$\downarrow$

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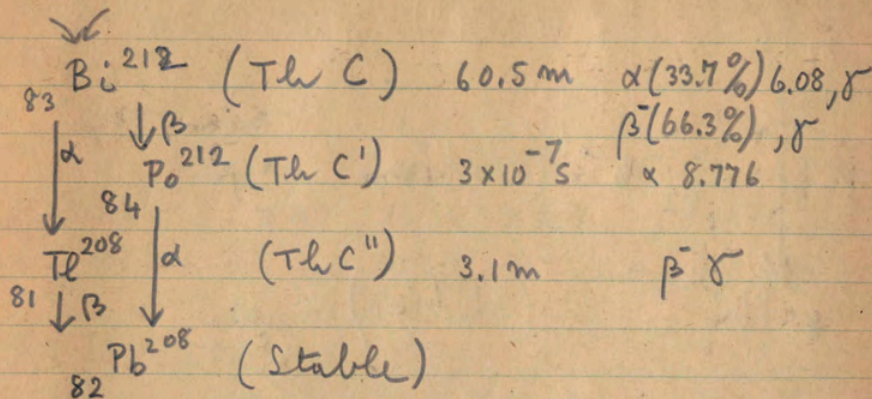




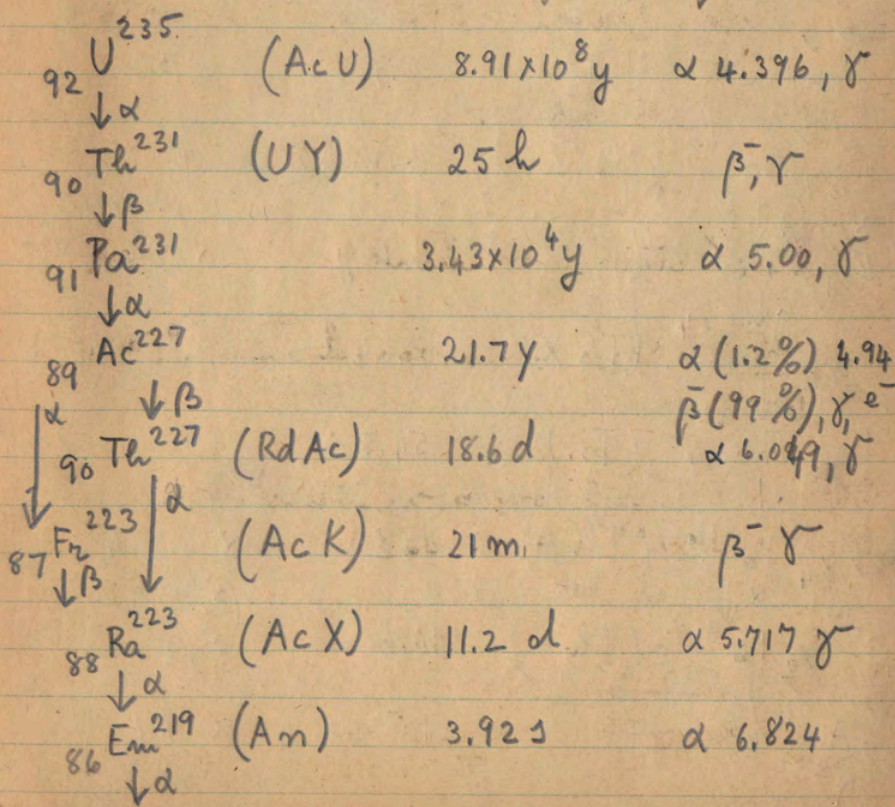
Thorium families

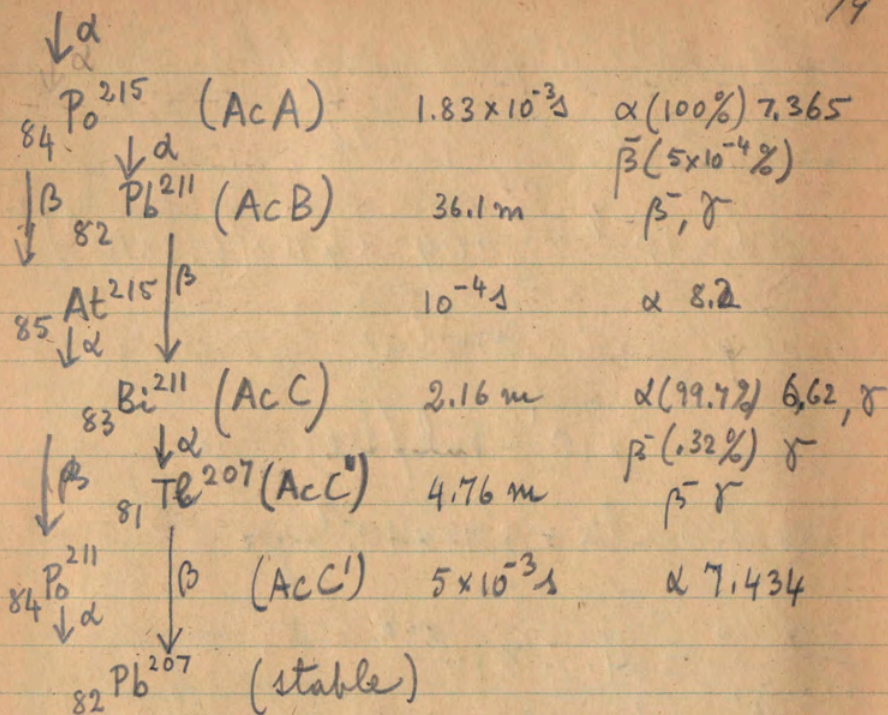


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Actinium family





Ratio of disintegrations

$$\frac{\text{Act family}}{\text{U family}} = .0071 \times \frac{4.498 \times 10^9}{8.91 \times 10^8} = .036$$

Other natural radioactivities

$^{75}_{75}\text{Re}^{187}$	$3 \times 10^{12} \text{ y}$	$\bar{\beta} .043$
$^{71}_{71}\text{Lu}^{176}$	$2.4 \times 10^{10} \text{ y}$	$\bar{\beta}, \text{K}, \gamma$
$^{62}_{62}\text{Sm}^{148}$	$1.4 \times 10^{10} \text{ y}$	$\alpha 2.0$
$^{37}_{37}\text{Rb}^{87}$	$6.3 \times 10^{10} \text{ y}$	$\bar{\beta}, \gamma$
$^{19}_{19}\text{K}^{40}$	$2.4 \times 10^8 \text{ y}$	$\bar{\beta}, \bar{\beta}^+, \text{K}$

Radioactivity & Geophysics

Heat lost by Earth

$$\underbrace{3 \times 10^{-4}}_{\text{Temp grad}} \times \underbrace{.004}_{\text{conduct.}} \times 4\pi \times (6.3 \times 10^8)^2$$

Temp grad conduct.

$$= 6 \times 10^{12} \text{ cal/sec}$$

$$\text{Mass of earth} = 5.98 \times 10^{27} \text{ gr}$$

$$\frac{\text{ergs}}{\text{sec gr}} = 4.2 \times 10^{-8}$$

$$1 \text{ gr } \bar{U} = \frac{1}{238} \times 6 \times 10^{23} \times \frac{1}{2 \times 10^{17}} \times 47 \times 1.6 \times 10^{-6}$$

(47 MeV per disintegration)

= .95 ergs/sec

1 gr Th produces .27 ergs/sec

(41 MeV/dis)

$$U \sim 5 \text{ ppm} \quad .95 \times 5 \times 10^{-6}$$

$$Th \sim 10 \text{ ppm} \quad .27 \times 10 \times 10^{-6}$$

$$7.5 \times 10^{-6} \text{ ergs/sec gr}$$

or about 100 times need if
uniformly spread

However thin surface layer only

Potassium also important esp.
in early geological ages

$$\frac{Pb}{U} = e^{\frac{t}{2 \times 10^{17}} - 1}$$

$$t = 2 \times 10^{17} \ln \left(1 + \frac{Pb^{208}}{U^{238}} \right)$$

Up to about 2×10^9 years

at large r

$$\psi \rightarrow \sum_{l=0}^{\infty} \frac{P_l^{(0)}(\theta)}{r} \left\{ e^{\frac{i p r}{\hbar}} \left[-a_l - \frac{i \hbar (2l+1)}{2p} \right] + e^{-\frac{i p r}{\hbar}} (-1)^l \frac{i \hbar (2l+1)}{2p} \right\}$$

One has exactly

$$\psi = \sum_l \frac{P_l(\theta)}{r} c_l v_l(r)$$

where $v_l(r)$ are solutions of radial Schrödinger equations

$$v_l''(r) + \left\{ \frac{2mE}{\hbar^2} - \frac{2mV(r)}{\hbar^2} - \frac{l(l+1)}{r^2} \right\} v_l = 0$$

normalized as follows

$$r^{l+1} \longleftrightarrow v_l(r) \longleftrightarrow A_l \cos\left(\frac{pr}{\hbar} - \frac{l+1}{2}\pi + \beta_l\right)$$

One finds then

$$-a_l = \frac{\hbar(2l+1)}{p} e^{i\beta_l} \sin \beta_l$$

$$c_l = (-1)^l e^{i(\beta_l - \frac{l\pi}{2})} \frac{\hbar(2l+1)}{p A_l}$$

Follows: cross-section

$$\sigma = \sum_{l=0}^{\infty} \frac{4\pi \hbar^2}{p^2} (2l+1) \sin^2 \beta_l$$

Also density $|\psi(0)|^2$ at origin

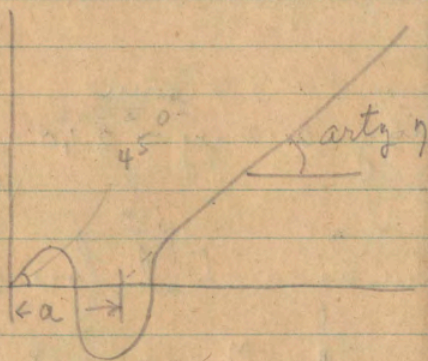
$$|\psi(0)|^2 = \frac{\hbar^2}{p^2 A_0^2}$$

In limiting case of large wave length only s -scattering counts.

For $E \approx 0$

$$v''(r) - \frac{2mV}{\hbar^2} v(r) = 0$$

Solution



$$v(r) \rightarrow \eta(r-a) \quad \text{for } E=0$$

$$v(r) \rightarrow \frac{\eta \hbar}{p} \sin \frac{p(r-a)}{\hbar} \quad \text{for } E = \frac{p^2}{2m} \text{ small}$$

Hence

$$\beta_0 = -\frac{pa}{\hbar}, \quad A_0 = \frac{\eta \hbar}{p}$$

$$\frac{1}{k} \tan k r_0 = -\frac{1}{\alpha}$$

$$\frac{1}{k r_0} \tan k r_0 = -\frac{1}{\alpha r_0}$$

Assume (independent evidence)

$$r_0 = \frac{e^2}{mc^2} = 2.82 \times 10^{-13}$$

$$\frac{1}{k r_0} \tan k r_0 = -1.53$$

$$k r_0 = 1.902$$

$$k = \frac{1.902}{2.82 \times 10^{-13}} = 6.74 \times 10^{12}$$

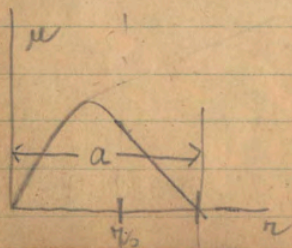
$$V - W_0 = \frac{\hbar^2 k^2}{M} = 30.12 \times 10^{-6}$$

$$V_3 = 33.7 \times 10^{-6} \text{ ergs} = 21.0 \text{ MeV}$$

Slow neutron-proton scattering.

(Instead Salpeter 8260)

Calculation of a



$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_0 k^2$$

$$u = \sin kr$$

$$r < r_0$$

$$k = \frac{\sqrt{MV}}{\hbar}$$

a from

$$\sin kr_0 + (a - r_0) k \cos kr_0 = 0$$

$$a = r_0 - \frac{1}{k} \tan kr_0 = r_0 \left(1 - \frac{1}{kr_0} \tan kr_0\right)$$

Disregarding spin one would find

$$\sigma = 4\pi r_0^2 = 4\pi r_0^2 \left(1 - \frac{1}{kr_0} \tan kr_0\right)^2$$

From deuteron $3S$ -State

$$k_3 = \frac{\sqrt{M \times 33.7 \times 10^{-6}}}{\hbar} = 7.13 \times 10^{12}$$

$$k_3 r_0 = 2.01$$

$$\frac{1}{k_3 r_0} \tan k_3 r_0 = -1.09$$

$$a_3 = .589 \times 10^{-12}$$

$$\sigma_3 = 4\pi a_3^2 = 4.40 \times 10^{-24}$$

Experimentally

$$\sigma_{sc} = 20.3 \times 10^{-24} = \frac{3}{4} \sigma_3 + \frac{1}{4} \sigma_1$$

$$= 3.3 \sigma_3 + \frac{1}{4} \sigma_1$$

Giving

$$\sigma_1 = 68 \times 10^{-24}$$

Confluent Hypergeometric functions

$$F(a, b, z) = 1 + \frac{a}{b} z + \frac{a(a+1)}{b(b+1)2!} z^2 + \dots$$

is a solution of

$$z F''(z) + (b-z) F'(z) - a F(z) = 0$$

Solution also

$$G(z) = F(z) \int \frac{e^{\xi} d\xi}{\xi^b F^2(\xi)}$$

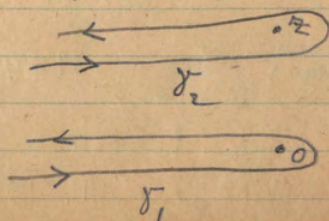
(See Yost, Wheeler & Breit, P.R. 49, 174, 1936)

Asymptotic expansion for

b positive integer and z pure imaginary

$$F(z) \rightarrow W_1 + W_2(z)$$

$$W_1(z) = \frac{(b-1)!}{2\pi i} \int_{\gamma_1} \frac{e^t dt}{t^b} \left(1 - \frac{z}{t}\right)^{-a}$$



asymptotic expansions

$$W_1 \rightarrow \frac{\Gamma(b)}{\Gamma(b-a)} (-z)^{-a}$$

$$W_2 \rightarrow \frac{\Gamma(b)}{\Gamma(a)} z^{a-b} e^z$$

$$\Gamma(n) = (n-1)!$$

Solution irregular at origin is

$$i(W_1 - W_2)$$

Radial hydrogenlike Schrödinger equation (repulsive force)

$$\frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + \left(k^2 - \frac{2}{r} - \frac{l(l+1)}{r^2} \right) R = 0$$

$$\left(\text{unit of length} = \frac{\hbar^2}{m e^2 Z Z'} \right)$$

Solve by

$$R = r^l e^{ikr} F$$

gives

$$r \frac{d^2 F}{dr^2} + (2l+2+2ikr) \frac{dF}{dr} - F(2-ik(2l+2)) = 0$$

Change variable to

$$z = -2ikr, \quad \left(\alpha = \frac{1}{k} = \frac{mZZ^2}{k^2} \right)$$

$$z F''(z) + (2l+2-z)F'(z) - (l+1+i\alpha)F = 0$$

$$F(z) = F(l+1+i\alpha, 2l+2, -2ikr)$$

Giving asymptotic behavior

$$R(r) = r^l + \dots \quad \text{for } r \text{ small}$$

$$R(r) = \frac{e^{\frac{\pi}{2}\alpha}}{(2k)^2} \frac{(2l+1)!}{|\Gamma(l+1+i\alpha)|} \frac{1}{kr} \times$$

$$\times \sin \left\{ kr - \alpha \ln(2kr) - \frac{l\pi}{2} - \arg \Gamma \right\}$$

(Use in evaluating

$$\Gamma(1+z)\Gamma(1-z) = \frac{\pi z}{\sin \pi z}$$

$$|\Gamma(1+i\alpha)|^2 = \frac{2\pi\alpha}{e^{\pi\alpha} - e^{-\pi\alpha}}$$

$$\Gamma(n) = (n-1)!$$

Scattering by Coulomb center

$$\nabla^2 \psi + \left(k^2 - \frac{2}{r} \psi\right) = 0$$

Put

$$\psi = e^{ikz} F(\xi)$$

$$\xi = r - z$$

One finds

$$\xi \frac{d^2 F}{d\xi^2} + (1 - ik\xi)F - F = 0$$

Hypergeometric solution is

$$F\left(-\frac{i}{k}, 1, ik\xi\right) = F$$

Normalized ψ to $|\psi|^2 \rightarrow 1$

$$\psi = e^{-\frac{\pi}{2k}} \Gamma\left(1 + \frac{i}{k}\right) e^{ikz} F\left(-\frac{i}{k}, 1, ik(r-z)\right)$$

For $z=0$

$$\psi = e^{-\frac{\pi}{2k}} \Gamma\left(1 + \frac{i}{k}\right) \quad |\psi(0)|^2 = \frac{2\pi/k}{e^{\frac{2\pi}{k}} - 1}$$

In normal units

(v = rel. velocity)

$$|\psi(0)|^2 = \frac{2\pi\alpha}{e^{2\pi\alpha} - 1}$$

$$\alpha = \frac{e^2}{\hbar v} Z Z'$$

98.

For r large:

$$\psi_c \rightarrow I_c + S_c f_c(\theta) \quad \left(\begin{array}{l} \text{index } c \\ = \text{"Coulomb"} \end{array} \right)$$

 I_c = incident wave

$$= \left\{ 1 - \frac{\alpha^2}{ik(r-z)} \right\} e^{ikz + i\alpha \ln k(r-z)}$$

 $S_c f_c(\theta)$ = scattered wave

$$S_c = \frac{1}{k} e^{ikr - 2i\alpha \ln(kr)} \quad ?$$

$$f_c(\theta) = \frac{1}{2k^2} \frac{1}{\sin^2 \frac{\theta}{2}} e^{-i\alpha \ln(1-\cos\theta) + i\pi + 2i\eta_0}$$

where

$$\eta_0 = \arg \Gamma\left(1 + \frac{i}{k}\right)$$

Coulomb center with short range
scattered force.

First expand solution of previous sections in spherical harmonics.
One finds (See Schiff, Q.M., p 118)
(index ψ in ψ_c means "Coulomb")

$$\psi_e = e^{-\frac{\pi}{2k}} \sum_0^{\infty} \frac{\Gamma(l+1 + \frac{i}{k})}{(2l)!} (2ikr)^l e^{ikr} \times \quad 99$$

$$\times F\left(l+1 + \frac{i}{k}, 2l+2, -2ikr\right) P_l(\cos \theta)$$

If F is written as on p. 94

$$F = W_1 + W_2$$

one sees that the term with W_2 represents incoming wave and the term with W_1 represents outgoing wave

An additional short range force will exert its effect as a change of the ratio of the coefficients of W_1 and W_2 (mixing in of irregular solution).

This ratio will no longer be 1 but $e^{2i\delta}$ (modulus one because no absorption).

Modified ψ will then be:

$$\psi = e^{-\frac{\pi}{2k}} \sum_0^{\infty} \frac{\Gamma(l+1 + \frac{i}{k})}{(2l)!} (2ikr)^l e^{ikr} \times$$

$$\times \left\{ W_2\left(l+1 + \frac{i}{k}, 2l+2, -2ikr\right) + e^{2i\delta} W_1\left(l+1 + \frac{i}{k}, 2l+2, -2ikr\right) \right\} P_l(\cos \theta)$$

Follows

$$\psi = \psi_c + e^{-\frac{\pi}{2k}} \sum_0^{\infty} \frac{\Gamma(l+1+\frac{i}{k})}{(2l)!} (2ikr)^l e^{ikr} x$$

$$\times (e^{2i\delta_l} - 1) W_l(l+1+\frac{i}{k}, 2l+2, -2ikr)$$

Developing ψ_c as on p. 98
one finds that

$$\psi = I + S f(\theta)$$

$$I = I_c \quad S = S_c$$

$$f(\theta) = f_c(\theta) + \sum_0^{\infty} \frac{2l+1}{k} e^{i(2\eta_l + \delta_l)} \sin \delta_l P_l(\cos \theta)$$

$$\eta_l = \arg \Gamma(l+1+\frac{i}{k})$$

In particular when only δ_0 is $\neq 0$
one finds

$$\frac{\sigma(\theta)}{\sigma_c(\theta)} = \left| 1 - 2k \sin^2 \frac{\theta}{2} \sin \delta_0 e^{i\delta_0 - \frac{i}{k} \ln(1-\cos \theta)} \right|^2$$

Proton - proton scattering
Previous discussion changed by spin
& statistics.

Wave function spatially symmetric

$$\psi(r) = \frac{\psi(r) + \psi(-r)}{\sqrt{2}}$$

gives scattering cross-section

$$\vartheta \leq \frac{\pi}{2}$$

$$d\sigma = \frac{dw}{2\pi} |f(\vartheta) + f(\pi - \vartheta)|^2 \quad (v = \text{speed})$$

(Anomalous contribution from even
l's only because $P_l(\cos \vartheta) + P_l(-\cos \vartheta) = 0$
for l odd)

For spatially antisymmetric wave fts

$$d\sigma = \frac{dw}{2\pi} |f(\vartheta) - f(\pi - \vartheta)|^2$$

For 0 proton - proton collisions

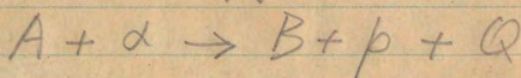
$$\begin{aligned} d\sigma &= \frac{dw}{2\pi} \left\{ \frac{3}{4} |f(\vartheta) - f(\pi - \vartheta)|^2 + \frac{1}{4} |f(\vartheta) + f(\pi - \vartheta)|^2 \right\} \\ &= \frac{dw}{2\pi} \left\{ |f(\vartheta)|^2 + |f(\pi - \vartheta)|^2 - \text{Real}(f(\vartheta)f^*(\pi - \vartheta)) \right\} \end{aligned}$$

Nuclear Reactions

Various types of reactions

α, p	d, n	α, γ
p, d	p, n	p, γ
d, d	d, n	d, p
n, α	n, p	n, γ
γ, d	γ, p	γ, n
m, f	α, f	p, f
		d, f
		γ, f

Reaction energy Q



$$Q = (m_A + m_\alpha - m_B - m_p) c^2$$

Various Q values possible when B is excited.

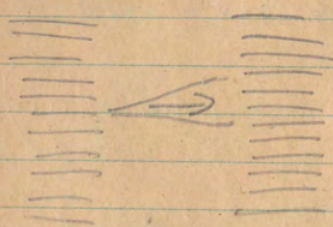
Relationship between Q and the threshold

$$\text{Threshold} = \frac{m_\alpha}{m_\alpha + m_A} Q = \frac{m_A + m_\alpha}{m_A} Q$$

Born approximation

$A + \alpha_1$

$B + \alpha_2$



(neglect spin)

$$\frac{\sigma_1 \Omega_1}{\Omega} = \frac{2\pi}{\hbar} |\mathcal{H}|^2 \frac{4\pi p_2^2 dp_2 \Omega_1}{8\pi^3 \hbar^3 \Omega_2 dp_2}$$

$$\sigma_1 = \frac{1}{\pi \hbar^4} \left| \Omega \mathcal{H} \right|^2 \frac{p_2^2}{\Omega_1 \Omega_2}$$

Crude guess

$$\Omega \mathcal{H} \approx V U e^{-\frac{1}{2}(G_1 + G_2)}$$

Slightly better guess (see p. 97)

V = nuclear volume; U ^{interaction} energy

$$|\Omega \mathcal{H}|^2 = V^2 U^2 \frac{2\pi \alpha_1}{e^{-1} - 1} \frac{2\pi \alpha_2}{e^{-1} - 1}$$

However should be completed by effect of finite nuclear radii.

Such descriptions do not include resonance (See later).

From σ -formula follows

$$\frac{\sigma(1 \rightarrow 2)}{\sigma(2 \rightarrow 1)} = \frac{p_2^2}{p_1^2} \quad (\text{no spin})$$

Alternate proof by detailed balancing

$$p_1^2 dp_1 v_1 \sigma_1 = p_2^2 dp_2 v_2 \sigma_2$$

But

$$v_1 dp_1 = dE = v_2 dp_2$$

Hence

$$p_1^2 \sigma_1 = p_2^2 \sigma_2$$

If spins are involved

$$(2I_A + 1)(2I_B + 1) p_1^2 \sigma_{1 \rightarrow 2} = (2I_2 + 1)(2I_B + 1) p_2^2 \sigma_{2 \rightarrow 1}$$

Problem: $\text{Be}^9 + \text{H}^1 \rightarrow \text{Li}^6 + \text{He}^4$

(Data Bethe C p 310)

Plan an experiment for observation of the inverse reaction

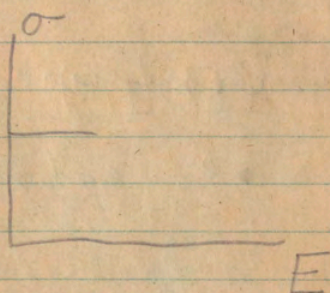
Shapes of the thresholds
are determined by statistical
factors

$$p_2^2 / \sigma_1 \sigma_2$$

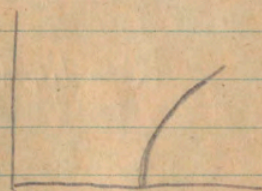
and when it is the case by the
gamma factors

Examples

(n, n) - elastic

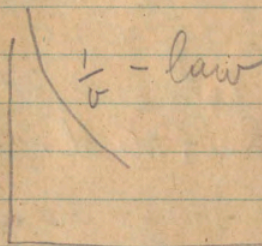


(n, n) - anelastic



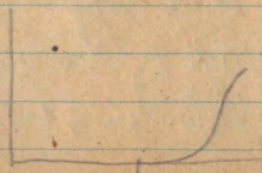
(n, γ) (n, α) (n, p) (n, f)

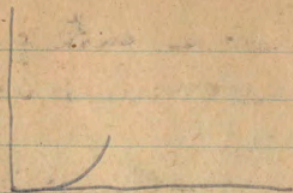
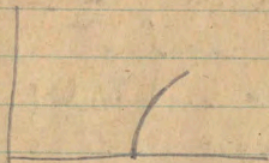
$$Q > 0$$



(n, α) (n, p) (n, f)

$$Q < 0$$



$(p, n) (p, \alpha) (p, r)$ $(\alpha, n) (\alpha, r) (\alpha, p)$ $Q > 0$  $(p, n) (\alpha, n)$ $Q < 0$ 

Resonance & Compound Nucleus

General features of resonance

Very pronounced in (n, r) processes

^{117}In	$R = 1.44 \text{ eV}$ (half width = $.042 R_{\text{eV}}$)	$\sigma = 27,000 \times 10^{-24}$
^{107}Ag	$R = 5.2 \text{ eV}$ (.063 eV)	$\sigma_R = 24,000 \times 10^{-24}$

More data Goldsmith & others

Rev. Mod. Phys. 19, p 259, 1947

In other types of reactions see

Horiya & Lauritsen, R. Mod. Phys. 20, 191, 1948

Fowler, Lauritsen & Lauritsen, R. H. P. 20, 236, 1948

Qualitative interpretation in terms of compound nucleus.

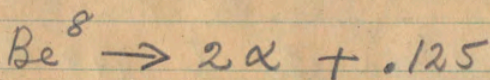
Various machineries for quasi-stability of compound nucleus

(Gamow factors, Relatively slow γ -emission, Partition of energy among many degrees of freedom)

Expect in general:

Width of resonance $\approx \hbar \times$ inverse life time

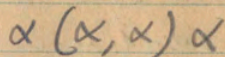
Example - Compound nucleus Be^8
Ground state unstable



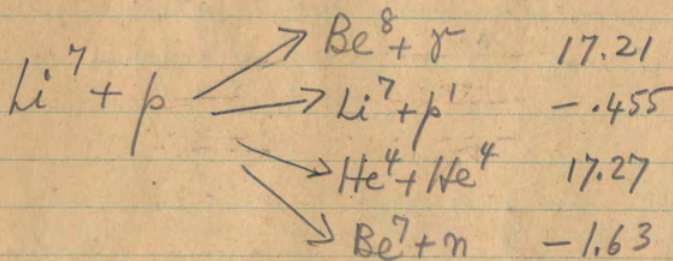
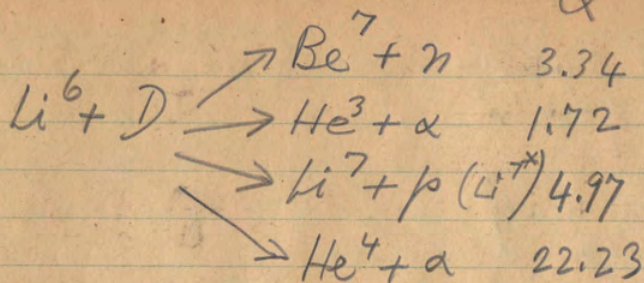
Life time estimated $\sim 10^{-14 \pm 1}$ sec

(Millar + Cameron, Chicago Meeting Nov 1950)

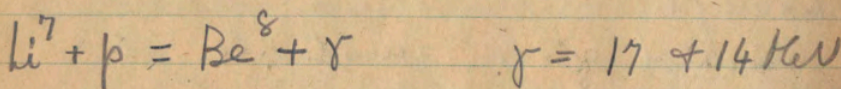
Is formed in give $(5.3 \pm 1.1) \times 10^{-14}$ sec



Observed broad resonance at ~ 3.4 MeV (There should be also a resonance at .1 MeV too small for observation)



Comments



Sharp resonance at .440 MeV

Li^7 has a level at .453 (known from $\text{Be}^7 \rightarrow \text{Li}^7 + \gamma$ disintegration) observed as ~~the~~ threshold in inelastic proton scattering

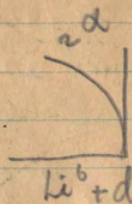
Discussion of the parity of level at 17.57 (odd)

Hi even levels are so broad that no resonances are observed for them.

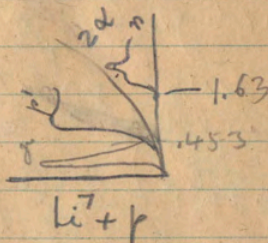
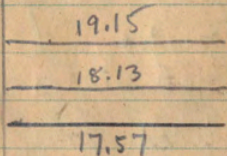
MeV-scale

///

25



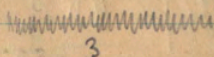
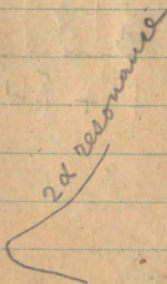
20



15

10

5



0

$2d - .085$

0

-2

$.112$

Be^8

Breit-Wigner formulaDiscuss for example of (n, γ) reactions R = energy of compound state v = neutron speed ; $v_R = \sqrt{\frac{2R}{M}}$

normalize to one neutron/cc

 a = amplitude compound state b_s = amplitude of neutron states $(\tau, \tau_n, \tau_\gamma)$ life times for all processes,
neutron emission, γ emission

$$\Gamma = \hbar/(2\tau) \quad \Gamma_n = \hbar/(2\tau_n) \quad \Gamma_\gamma = \hbar/2\tau_\gamma$$

$$\Gamma = \Gamma_n + \Gamma_\gamma$$

$$\dot{a} = -\frac{\Gamma}{\hbar} a + \frac{i}{\hbar} \psi_0 e^{\frac{i}{\hbar}(R-W)t}$$

$$a = \frac{i \psi_0 e^{\frac{i}{\hbar} \delta W t}}{\Gamma + i \delta W} \quad |a|^2 = \frac{|\psi_0|^2}{\Gamma^2 + \delta W^2}$$

From (neglect spin)

$$\begin{aligned} \frac{1}{\tau_n} &= \frac{2\pi}{\hbar} |\psi_0|^2 \frac{4\pi p_m^2 dp_m}{8\pi^3 \hbar^3 v_m dp_m} \\ &= \frac{|\psi_0|^2}{\pi \hbar^4} M^2 v_m = \frac{2\Gamma_n}{\hbar} \end{aligned}$$

one gets

$$\Gamma_n = \frac{1}{2\pi \hbar^3} |\psi_0|^2 \frac{p_m^2}{v_m}$$

$$|\gamma_b|^2 = \frac{2\pi\hbar^3 \Gamma_n}{M^2 v_n}$$

$$|a|^2 = \frac{2\pi\hbar^3}{M^2 v} \frac{\Gamma_n(W)}{\Gamma^2 + (\delta W)^2}$$

$$v \sigma_{abs} = \frac{1}{\tau_r} |a|^2 =$$

$$v \sigma_{scatt} = \frac{1}{\tau_n} |a|^2$$

$$\sigma_{abs} = 4\pi\lambda^2 \frac{\Gamma_r \Gamma_n}{\Gamma^2 + \delta W^2}$$

$$\sigma_{scatt} = 4\pi\lambda^2 \frac{\Gamma_n^2}{\Gamma^2 + \delta W^2}$$

Pact

$$\Gamma_n = \frac{v_r}{v_R} \Gamma_n(R)$$

$$\sigma_{abs} = 4\pi\lambda_R^2 \frac{\Gamma_n(R) \Gamma_r}{\Gamma^2 + \delta W^2}$$

(note $\frac{1}{v}$ -law
at low
energy)

$$\sigma_{scatt} = 4\pi\lambda_R^2 \frac{\Gamma_n^2(R)}{\Gamma^2 + \delta W^2}$$

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Introducing spin I of original nucleus and $I \pm \frac{1}{2}$ of compound nucleus.

Probability with $l=0$ for neutron
Compound nucleus Probability p.p.t.o

$I + \frac{1}{2}$

$$2(I + \frac{1}{2}) + 1 = 2(I+1)$$

$I - \frac{1}{2}$

$$2(I - \frac{1}{2}) + 1 = 2I$$

Hence add to B.W. formula factor

$$\frac{1}{2} \left(1 + \frac{1}{2I+1} \right)$$

For examples

Goldsmith, R. Mod. Phys 19 259 1947

Problem: Give one example each for the following reactions

(α, p) (α, n) (p, γ) (n, α) (n, p) (n, γ)
collect the available data on Q and on resonances

Problem: Outline a possible experimental technique for the determination of the resonance level in the (p, p') reaction on Li^7

Statistical theory of compound nucleus.

Bohr's qualitative arguments on nucleus as system with many degrees of freedom in order to explain.

- rather long life of compound nucleus
- large frequency of levels for its excitation.

Introduction of "nuclear temperature" and qualitative discussion of nuclear evaporation.

For example degenerate gas model of nucleus

V = nuclear volume $A = Z + N$

T = temperature in eV

Max momentum

$$P_p = (3\pi^2)^{1/3} \hbar (Z/V)^{1/3} \quad (\text{for protons})$$

$$P_n = (3\pi^2)^{1/3} \hbar (N/V)^{1/3} \quad (\text{for neutrons})$$

Relationship between energy & T

$$W_p = \frac{V M P_p}{6 \hbar^3} T^2 = \left(\frac{\pi^2}{72} \right)^{1/3} \frac{V M}{\hbar^2} \left(\frac{Z}{V} \right)^{1/3} T^2$$

Assuming radius $1.48 \times 10^{-13} A^{1/3}$
 fluid $V = 1.36 \times 10^{-38} A$

$$W = 4.4 \times 10^{-4} A^{2/3} \{ Z^{1/3} + (A-Z)^{1/3} \} T^2$$

For example $A=100$ $Z=44$
 gives

$$W = 7 \times 10^6 T^2$$

$$\text{For } W = 10 \text{ MeV} = 1.6 \times 10^{-5}$$

$$T = 1.5 \times 10^{-6} \approx 1 \text{ MeV}$$

Entropy: if $W = a T^2$

$$S = \int \frac{dW}{T} = 2aT = 2\sqrt{aW}$$

e^S can be interpreted as proportional to the level density.

In previous example level density increases from $W=0$ to $W=8 \text{ MeV}$ by factor

$$e^{2\sqrt{7 \times 10^6 \times 8 \times 1.6 \times 10^{-6}}} = e^{19} \approx 10^8$$

This perhaps is even too much!

Fission

Deformation of a drop-nucleus
axes

$$a = R(1+\epsilon) \quad b = R/\sqrt{1+\epsilon}$$

(to keep constant volume)

$$\text{Surface} \approx 4\pi R^2 \left(1 + \frac{2}{5} \epsilon^2\right)$$

$$\text{Electrostatic energy} \approx \frac{3}{5} \frac{Z^2 e^2}{R} \left(1 - \frac{\epsilon^2}{5}\right)$$

$$\text{Capillary energy} = .014 A^{2/3} \left(1 + \frac{2}{5} \epsilon^2\right)$$

mass units

$$\text{Electrost. energy} = .000627 \frac{Z^2}{A^{1/3}} \left(1 - \frac{1}{5} \epsilon^2\right)$$

Coeff. of ϵ^2

$$\begin{aligned} & \frac{2}{5} \cdot .014 A^{2/3} - .000627 \frac{1}{5} \frac{Z^2}{A^{1/3}} \\ & = .0056 A^{2/3} - .0001254 \frac{Z^2}{A^{1/3}} \end{aligned}$$

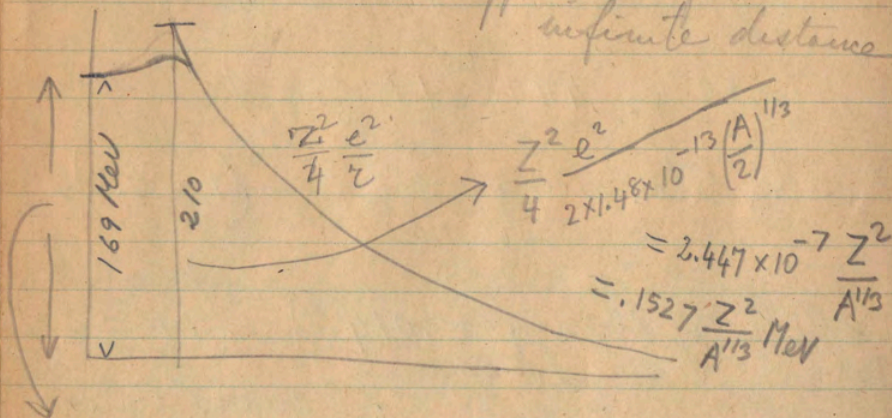
Condition for stability

$$\frac{Z^2}{A} < 44.6$$

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For	Z^2/A
Fe	12
Ag	21
Pb	32
U^{235}	36

approach from
infinite distance



$$M(A, Z) - 2M\left(\frac{A}{2}, \frac{Z}{2}\right) = \quad (\text{from p 4})$$

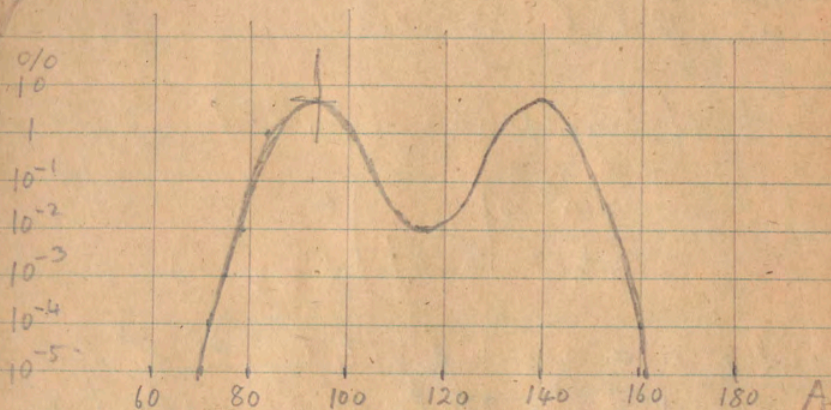
$$= .014 A^{2/3} \left(1 - 2\left(\frac{1}{2}\right)^{2/3}\right) +$$

- .26

$$+ .000627 \frac{Z^2}{A^{1/3}} \left(1 - 2\frac{(1/2)^2}{(1/2)^{1/3}}\right) = \left\{ \begin{array}{l} .000232 \frac{Z^2}{A^{1/3}} \\ - .00364 A^{2/3} \end{array} \right.$$

For U^{235} calculated available energy
is = 169 Mev

Actual distribution of fission yields



Maximum at $A = 95$ and 140
 Max yield $\sim 6\%$

Explain also

Radioactivity of fission products

Emission of neutrons

Emission of α -particles

Delayed neutrons

$1/2$ life Delayed/Prompt

55.6 sec

.00021

22.0

.00139

4.51

.00179

1.53

.00201

.42

.00071

.006

Some slow neutron capture reactions.
 Neutron capture by protons
 by magnetic dipole

Prob. of transition (p. 68)

$$\frac{4\omega^3}{3\hbar c^3} |\mu_{ij}|^2$$

Index 0 = ground state 3S of deuteron

Index 1 = 1S state of low positive energy w/ (this is only important state)

$$\psi_1 = \psi(^1S) = \frac{v(r)}{r} \frac{(+ -) - (- +)}{\sqrt{2}}$$

first spin proton
 second spin neutron

$$v''(r) + \frac{M}{\hbar^2} \{w - U_1(r)\} v = 0$$

Final states

($M =$ neutron mass)

$$\frac{1}{r} u(r) (++)$$

$$\frac{u(r)}{r} \frac{(+ -) + (- +)}{\sqrt{2}}$$

$$\frac{u(r)}{r} (--)$$

$$4\pi \int u^2 dr = 1$$

$$u''(r) + \frac{M}{\hbar^2} (w_0 - U_3(r)) u = 0$$

$$\mu_z = \mu_p \sigma_{z1} + \mu_n \sigma_{z2}$$

$$|\mu_{z01}|^2 = (\mu_p - \mu_n)^2 \left(4\pi \int u v dr \right)^2$$

$$|\mu_{01}|^2 = 3 \times (4\pi)^2 (\mu_p - \mu_n)^2 \left(\int u v dr \right)^2$$

Prob of transition $1S \rightarrow 3S$

$$\frac{64\pi^2 \omega^3}{\hbar c^3} (\mu_p - \mu_n)^2 \left(\int u v dr \right)^2$$

u from p. 87

v from p. 90

If v is normalized so that

$$v \rightarrow a+r \rightarrow \frac{\sin k(a+r)}{k}$$

corresponds to 1 $1S$ per unit volume or 4 neutrons/cc

Hence prob of transition = $4v\sigma$

$$v\sigma = \frac{16\pi^2 \omega^3}{\hbar c^3} (\mu_p - \mu_n)^2 \left(\int_0^\infty u v dr \right)^2$$

122

In first approx. put

$$u = \frac{1}{\sqrt{2\pi b}} e^{-r/b}$$

$$b = 4.31 \times 10^{-13}$$

$$v = a + r$$

$$a = 2.32 \times 10^{-12}$$

$$\int_0^{\infty} u v dr = \frac{1}{\sqrt{2\pi b}} (ab + b^2)$$

$$\sigma = \frac{8\pi\omega^3}{\hbar c^3 v} (\mu_p - \mu_n)^2 (a+b)^2 b$$

Put

$$\hbar\omega = 2.23 \text{ MeV} = 3.58 \times 10^{-6}$$

$$\mu_p = 2.4896 \times \frac{e\hbar}{2Mc}$$

$$\mu_n = -1.9103 \times \frac{e\hbar}{2Mc}$$

$$v = 2.2 \times 10^5$$

Find

$$\sigma_{2.2 \times 10^5} = 2.9 \times 10^{-25}$$

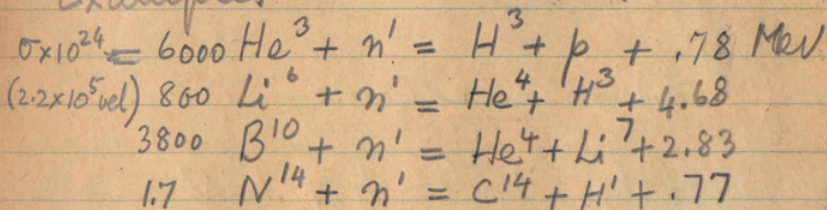
Better calculation gives $.31 \times 10^{-24}$ Expt $.30 \times 10^{-24}$

(n, α) and (n, p) reactions prominent in light elements when energetically possible.

For slow neutrons follow $1/v$ law because resonances very broad.

Examples

Barrier



From p. 105

$$\sigma \approx \frac{V^2 U^2 e^{-G_1 - G_2}}{\pi \hbar^4} \frac{m_2^2 \sigma_2}{v_1}$$

Actually gives order of magnitude of 1000×10^{-24} when no Gamow factor

Slow neutron (n, γ) reactions have usually very small $\sigma \approx .01 \times 10^{-24}$ to .001

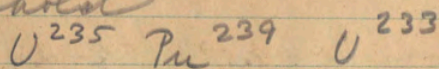
Estimate

$$v \sigma \approx \frac{V}{U}$$

$$\sigma \approx \frac{10^{-37}}{10^{-15} \times 2 \times 10^5} = 5 \times 10^{-3}$$

For heavier nuclei resonance becomes dominant

(n, f) reaction for low neutron energies follows the $1/v$ law when above the threshold



all containing odd number of protons (Bohr argument)

(γ, n) - reactions

Thresholds Be^9 1.71 MeV
 H^2 2.22

Other thresholds much higher ranging around 6 to 15 MeV
Max C^{12} 17.7 MeV

Max cross sections few MeV above threshold usually are $\approx 10^{-27}$

Theory of H^2 - photoeffect (Bethe-Peierls approximation)

Two processes are important:

a) Magnetic dipole ${}^3\text{S} \rightarrow {}^1\text{S}$ obtainable by det. balancing from p. 122 completed to include energy variations as follows

$$\sigma(\text{mag. dipole neutron capture}) = \frac{8\pi\omega^3}{hc^3\nu} (\mu_p - \mu_n)^2 \times \frac{b(a+b)^2}{(1+b^2k^2)(1+a^2k^2)}$$

where

$$k = \frac{Mv}{\hbar}$$

1st. balancing (p.106) gives

$$\begin{aligned}\sigma_{\gamma, n} &= \frac{2 \times 2 \times \rho_1^2}{3 \times 2 \times \rho_2^2} \sigma_{n\gamma} = \frac{2}{3} \frac{M^2 v^2}{4 \left(\frac{\hbar^2 \omega^2}{c^2} \right)} \sigma_{n\gamma} \\ &= \frac{1}{6} \frac{M^2 v^2 c^2}{\hbar^2 \omega^2} \frac{8\pi \omega^3}{\hbar v c^3} (\mu_p - \mu_n)^2 \frac{b(a+b)^2}{(1+b^2 k^2)(1+a^2 k^2)} \\ &= \frac{4\pi}{3} \frac{M^2 v \omega}{\hbar^3 c} (\mu_p - \mu_n)^2 \frac{b(a+b)^2}{(1+b^2 k^2)(1+a^2 k^2)}\end{aligned}$$

Use (ω_0 = threshold)

$$\hbar(\omega - \omega_0) = \frac{M}{4} v^2 = \frac{k^2 \hbar^2}{M}$$

Find

$$\begin{aligned}\sigma_{\gamma, n}(\text{mag. dipole}) &= \frac{8\pi}{3} \frac{M^{3/2} (\mu_p - \mu_n)^2}{\hbar^{5/2} c} b(a+b)^2 \times \\ &\quad \times \frac{\omega \sqrt{\omega - \omega_0}}{\left\{ 1 + \frac{b^2 M}{\hbar} (\omega - \omega_0) \right\} \left\{ 1 + \frac{a^2 M}{\hbar} (\omega - \omega_0) \right\}} \\ &= 1.82 \times 10^{-27} \frac{\omega_M \sqrt{\omega_M - \omega_{M0}}}{[1 + 4.49(\omega_M - \omega_{M0})][1 + 13.0(\omega_M - \omega_{M0})]} \\ &\quad (\omega_M \text{ in MeV})\end{aligned}$$

Nuclear models

- a) Nuclear matter & saturation
- b) Drop model - Mass formula p. 3
- c) ~~Statistical~~ ^{Quasiparticle} gas model pp 115, 131
- d) Single particle model - shell model p. 130

a) Nuclear matter & saturation

Quite empirical facts $R \approx 1.4 \times 10^{-13} A^{1/3}$

$$\rho \approx 1.45 \times 10^{14}$$

Binding energy $\approx 8 \text{ MeV per nucleon}$

Two particle attractive potential (range r_0)
 N nucleons in volume V $\frac{4\pi}{3} r_0^3 = V_0$

$$\text{Kin energy} \approx \frac{\hbar^2}{2M} \left(\frac{N}{V} \right)^{2/3}$$

$$\text{Pot energy} \approx -N \times V_0 \times \frac{V_0 N}{V} = - \frac{V_0 V_0 N^2}{V}$$

Pot energy dominant leads to $V \propto V_0$

No saturation

Two particle - exchange potential



Problem: even within range only overlapping part of wave function contributes. Therefore

No gain beyond making wave packets of range size

Probably comments on repulsive cores and many body forces

that strong spin orbit interaction
lowers $1g_{7/2}$, raises $1g_{9/2}$

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Heisenberg's assumes Hamiltonian

$$\frac{p^2}{2M} + V(r) - 15 \frac{\hbar}{2Mc^2} \sigma \cdot \nabla V \times \sigma$$

$$\nabla V = \frac{V'(r)}{r} r$$

$$-15 \frac{\hbar^2}{2Mc^2} \frac{V'(r)}{r} l \cdot \sigma = - \frac{15 \hbar^2}{2Mc^2} \frac{V'(r)}{r} \left\{ \begin{matrix} l \\ -l-1 \end{matrix} \right.$$

For oscillator potential

$$V(r) = \frac{M}{2} \omega^2 r^2$$

$$\frac{V'(r)}{r} = M\omega^2 \quad \text{Splitting} = \frac{15 (\hbar\omega)^2}{2 Mc^2} (2l+1)$$

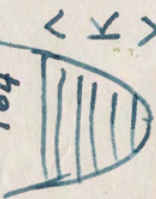
$$(\hbar\omega)^2 \approx \frac{30 \times 10^{-10}}{A^{2/3}} \quad \text{Splitting} \approx \frac{10 \text{ MeV}}{A^{2/3}} (2l+1)$$

Order of magnitude of

h orbit splitting

$$\frac{(n+1)(n+2)}{2} = \text{no. of states } n\hbar\omega$$

$$\frac{10}{164^{2/3}} (2 \times 5 + 1) = 3.7 \text{ MeV}$$



$$Z = \frac{A}{4} = \frac{n^3}{8}$$

$$K \approx n\hbar\omega$$

$$\hbar\omega = \frac{\hbar^2}{(5A)^{1/3}} \quad \hbar\omega = \frac{2^{1/3} \hbar^2}{5^{1/3} A^{1/3}}$$

Section 1

Statistical model (See p 115)

$$\text{Add: } \frac{Z}{V} \approx \frac{N}{V} \approx \frac{A}{2V} \approx \frac{10^{38}}{2 \times 1.36} = 3.7 \times 10^{37}$$

$$P_p \approx P_m \approx (2\pi^2)^{1/3} \hbar \left(\frac{Z}{V} \right)^{1/3} = 9.5 \times 10^{-15}$$

corresponding to max energy = $27 \times 10^{-6} = 17 \text{ MeV}$

$$\text{splitting} = \frac{15^5 K}{2^{\frac{1}{3}} 3^{\frac{2}{3}} M^2} \frac{K}{M^2} \frac{2l+1}{A^{\frac{2}{3}}} \approx \left(\frac{5.5}{2} \text{ MeV} \right) \frac{2l+1}{A^{\frac{2}{3}}}$$

for $K=30 \text{ MeV}$

$$\hbar\omega = \frac{26}{A^{\frac{1}{3}}} \text{ MeV}$$

Examples

1/93 f

2

A

$\hbar\omega / \text{MeV}$

~~splitting~~ splitting / MeV

f z y 1

d 1 s 1

1	12	11.4	3.2
2	28	8.6	3.0
3	56	6.8	2.6
4	100	5.6	2.3
5	164	4.76	2.0
6	252	4.12	1.8

Order of

8

that strong spin orbit interaction
lowers $1g_{7/2}$, raises $1g_{9/2}$

$1h_{11/2}$

$1h_{9/2}$

$1i_{13/2}$

$1i_{11/2}$

giving shells	population	total
1s	2	2
1p	6	8
1d 2s	12	20
1f 2p $1g_{9/2}$	30	50
$1g_{7/2}$ 2d 3s $1h_{11/2}$	32	82
$1h_{9/2}$ 2f 3p $1i_{13/2}$	44	126
$\frac{3^5}{2^{12}}$ $\frac{3^1}{2^2}$		

Arguments against single orbit
and their discussion

Other models

Drop model (See p. 3)

also extensive discussion in Ref B
Section 9

Statistical model (See p 115)

$$\text{Add: } \frac{Z}{V} \approx \frac{N}{V} \approx \frac{A}{2V} \approx \frac{10^{38}}{2 \times 1.36} = 3.7 \times 10^{37}$$

$$P_p \approx P_n \approx (2\pi^2)^{1/3} k \left(\frac{Z}{V} \right)^{1/3} = 9.5 \times 10^{-15}$$

corresponding to max energy $= 27 \times 10^{-6} = 17 \text{ MeV}$

Mesons and nuclear forces (Levi Martin)

Generalities on π and μ mesons

$$m_{\pi^0} = 265 m_e$$

$$m_{\pi} = 276 m_e = 148 \text{ MeV} \quad \begin{array}{l} \text{New values} \\ 276 \pm 6 \end{array}$$

$$m_{\mu} = 210 m_e = 107.5 \text{ MeV} \quad \begin{array}{l} 210 \pm 4 \end{array}$$

(Dec 1949)

Powell's experiments

Lattes & Gardner experiments

Reactions

$$\pi \rightarrow \mu (+ \gamma) \quad 2\gamma$$

(probably neutrino. Life time $\approx 10^{-8} \text{ sec}$)

$$\mu \rightarrow e^+ + (2\gamma) \quad \tau \approx 2.15 \times 10^{-6}$$

Evidence of strong interaction between π -meson & nucleon and weak interaction between μ -meson & nucleon (Covversi Paurini Piccioni experiment)

Yukawa theory of nuclear forces

Assumes meson field (π -meson)

in which quantum has mass

Hence momentum energy relationship

$$w^2 = \mu^2 c^4 + c^2 p^2$$

This corresponds to wave of type

$$e^{\frac{i}{\hbar}(p \cdot r - wt)} = \varphi$$

obeying diff equation

$$\nabla^2 \varphi = -\frac{p^2}{\hbar^2} \varphi \quad \frac{\partial^2 \varphi}{\partial t^2} = -\frac{w^2}{\hbar^2} \varphi$$

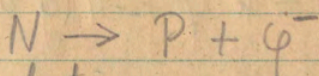
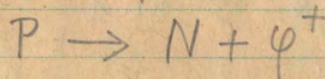
$$\nabla^2 \varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} - \left(\frac{\mu c}{\hbar}\right)^2 \varphi = 0$$

put

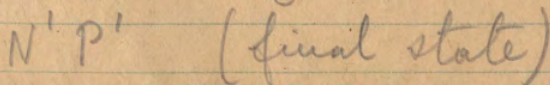
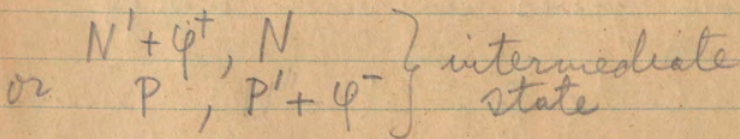
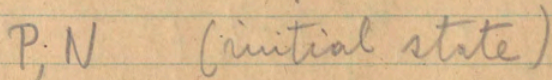
$$\frac{\mu c}{\hbar} = k$$

$$\nabla^2 \varphi - \frac{1}{c^2} \ddot{\varphi} - k^2 \varphi = 0$$

Coupling with nucleons corresponds to processes of type



Forces between nucleons appear as second order processes with pattern



This pattern therefore gives primarily exchange forces and only in higher approximation potential forces

A neutral meson interacting according to pattern

$$\begin{matrix} P \\ N \end{matrix} \} \rightarrow \begin{matrix} P \\ N \end{matrix} \} + \mu^0$$

gives potential forces.

One gets an approximate understanding by a classical description as follows:

View nucleus as sources of meson field with charge g and equation

$$\nabla^2 \varphi - \frac{1}{c^2} \ddot{\varphi} - k^2 \varphi = -4\pi g \delta(\mathbf{r} - \mathbf{r}')$$

Static solution

$$\varphi = \frac{g}{r} e^{-kr}$$

Action of this "potential" on other nucleon gives mutual energy

$$U = \frac{g^2}{r} e^{-kr} \quad (\text{with neutral mesons})$$

or

$$U = \frac{g^2}{r} e^{-kr} P_{\text{exc}} \quad (\text{with charged mesons})$$

This particular theory (scalar meson) gives forces that are repulsive and

Nuclear forces according to

M.M. Lévy, P.R. 88 725 1952

$$V = V_2(r) + V_4(r) + V_{\text{repulsive}}$$

$$V_2 = \frac{g^2}{4\pi} \left(\frac{\mu}{2M} \right)^2 \frac{\tau_1 \cdot \tau_2}{3} \times$$

$$\times \left\{ \sigma_1 \cdot \sigma_2 + \left[1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right] S_{12} \right\} \frac{e^{-\mu r}}{r}$$

$$\mu = \frac{c}{\hbar} \times \text{pion mass}$$

$$V_4 = -3 \left(\frac{g^2}{4\pi} \right)^2 \left(\frac{\mu}{2M} \right)^2 \frac{1}{\mu r^2} \times$$

$$\times \left\{ \frac{2}{\pi} K_1(2\mu r) + \left[\frac{2}{\pi} K_0(\mu r) \right]^2 \right\}$$

$V_{\text{repulsive}}$ = infinite repulsive core

Lévy takes $\frac{g^2}{4\pi} = 9.7 \pm 1.3$

$$r_0 \text{ of repulsive core} = (0.38 \pm 0.03) \frac{\hbar}{\mu c} = 1.53 \times 10^{-13} \text{ cm}$$

In non relativistic approximation for nucleons

$$S_m = \bar{\psi} \sigma_m \psi \quad S_4 = 0 \quad q = 0$$

$\sigma_1, \sigma_2, \sigma_3$ = Pauli operators

This pattern therefore gives primarily exchange forces and only in higher

nucleon gives mutual energy

$$U = \frac{g^2}{\hbar} e - kr \quad (\text{with neutral mesons})$$

or

$$U = \frac{g^2}{\hbar} e - kr \quad P_{\text{exc}} \quad (\text{with charged mesons})$$

This particular theory (scalar meson) gives forces that are repulsive and

Simplified Yukawa theory

$$\nabla^2 \varphi - \frac{1}{c^2} \ddot{\varphi} - k^2 \varphi = 0 \quad \text{at } k = mc/\hbar$$

$$W = \frac{1}{2} \int \left(\frac{1}{c^2} \dot{\varphi}^2 + (\nabla \varphi)^2 + k^2 \varphi^2 \right) d\Omega$$

$$\varphi = A e^{\frac{i}{\hbar} (p \cdot r - wt)} + \text{comp. conj.}$$

Find

$$W = \sqrt{c^2 p^2 + \mu^2 c^4}$$

and

$$\frac{\varphi^2}{\varphi} = 2 |A|^2$$

$$\frac{1}{\sqrt{2}}$$

$$W = 2 |A|^2 \left(k^2 + \frac{p^2}{\hbar^2} + \frac{w^2}{\hbar^2 c^2} \right) \Omega$$

$$= \frac{2 w^2}{\hbar^2 c^2} |A|^2 \Omega = w$$

gives

$$|A| = \frac{\hbar c}{\sqrt{2 \Omega w}}$$

Matrix element

$$\sum_{(N)} \frac{\hbar c e_z}{\sqrt{2 \Omega w}} e^{\frac{i}{\hbar} p \cdot r_N}$$

In non relativistic approximation for nucleons

$$S_n = \tilde{\psi} \sigma_n \psi \quad S_4 = 0 \quad q = 0$$

$\sigma_1, \sigma_2, \sigma_3 =$ Pauli operators

This pattern therefore gives primarily exchange forces and only in higher

Neutral meson - $N = \text{nucleon}$

Matrix $\frac{\partial \psi}{\partial t} \rightarrow N + N \rightarrow N + N + \psi$

$$g_0 = \frac{e_2 \hbar c}{\sqrt{2\Omega\omega}} \left(1 + e^{\frac{i}{\hbar} \mathbf{p} \cdot \mathbf{r}} \right)$$

$$\Delta W = - \frac{|g_0|^2}{\omega} = - \frac{e_2^2 \hbar^2 c^2}{2\Omega} \left[\frac{2 + 2 \cos \frac{\mathbf{p} \cdot \mathbf{r}}{\hbar}}{\omega^2} \right]$$

$$= - \frac{e_2^2 \hbar^2 c^2}{2\Omega} \int \frac{\cos \frac{\mathbf{p} \cdot \mathbf{r}}{\hbar}}{\mu^2 c^4 + c^2 p^2} \frac{d^3 p}{8\pi^3 \hbar^3} + \text{const}$$

$$\frac{e^{-\mu c r / \hbar}}{4\pi c^2 \hbar^2 r}$$

$$U = - \frac{e_2^2}{4\pi} \frac{e^{-\mu c r / \hbar}}{r}$$

nucleon gives mutual energy

$$U = \frac{g^2}{r} e^{-kr} \quad (\text{with neutral mesons})$$

or

$$U = \frac{g^2}{r} e^{-kr} p_{\text{exc}} \quad (\text{with charged mesons})$$

This particular theory (scalar meson) gives forces that are repulsive and spin independent.

Exchange forces

$$\frac{e^2 \hbar c}{\sqrt{2} \Omega w} e^{\frac{i}{\hbar} \mathbf{p} \cdot \mathbf{r}} \quad \text{or} \quad \begin{array}{l} P \rightarrow N + \pi^+ \\ N \rightarrow P + \pi^- \end{array}$$

$$\begin{array}{c} \text{States} \\ \hline \begin{array}{cc} \pi^0 & \pi^+ \pi^- \\ P & N \end{array} \end{array} \quad \begin{array}{c} (PN) \\ (PN) \end{array}$$

and

$$\begin{array}{cc} N & P \\ \hline & \end{array} \quad \begin{array}{c} (NP) \\ (NP) \end{array}$$

Combination states

$$\frac{1}{\sqrt{2}} \{ (PN) \pm (NP) \}$$

Perturbing states

$$\begin{array}{c} (NN) + \pi^+ \\ \hline \frac{1}{\sqrt{2}} \frac{e^2 \hbar c}{\sqrt{2} \Omega w} \left(1 \pm e^{\frac{i}{\hbar} \mathbf{p} \cdot \mathbf{r}} \right) \end{array} \quad \begin{array}{c} (PP) + \pi^- \\ \text{same} \end{array}$$

Except for constant:

$$\begin{aligned} \Delta E &= -2 \times \frac{1}{2} \frac{e^2 \hbar^2 c^2}{2 \Omega} \frac{(\pm p \cos \frac{\mathbf{p} \cdot \mathbf{r}}{\hbar})}{\mu^2 c^4 + p^2} \frac{d^3 p}{8 \pi^3 \hbar^3} \\ &= \mp \frac{e^2 \hbar^2 c^2}{4 \pi} \frac{\exp(-2 \mu r / \hbar)}{r} \end{aligned}$$

In non relativistic approximation for nucleons

$$S_m = \bar{\psi} \sigma_m \psi \quad S_4 = 0 \quad q = 0$$

$\sigma_1, \sigma_2, \sigma_3$ = Pauli operators

This pattern therefore gives primarily exchange forces and only in higher

$$\Delta E = \mp U(r)$$

$$-U(r) P_x (PN) = -U(r) (NP)$$

$$-U(r) P_x (NP) = -U(r) (PN)$$

$$-U(r) P_x \frac{(PN) \pm (NP)}{\sqrt{2}} = \mp U(r) \frac{(PN) \pm (NP)}{\sqrt{2}}$$

Hence neutral energy

$$-U(r) P_x$$

$$-(\pi + (qq)) + (\pi + (pN))$$

$$U = -\frac{g^2}{4\pi} \left(\frac{q^2}{k^2 \pm 1} \right) \frac{2\pi^2}{(2\pi)^3} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}$$

Transform of type

$$\frac{q^2}{k^2 \pm 1} \frac{1}{\pi} \frac{2\pi^2}{(2\pi)^3} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} = \Delta E$$

nucleon gives nuclear energy

$$U = \frac{g^2}{k} e^{-kr} \quad (\text{with neutral mesons})$$

or

$$U = \frac{g^2}{k} e^{-kr} P_{ex} \quad (\text{with charged mesons})$$

This particular theory (scalar meson) gives forces that are repulsive and spin independent.

Therefore attempts have been made in different directions.

Pseudoscalar meson field

a) Neutral field

Lagrangian density $x_4 = ict$
latin 1, 2, 3; greek 1, 2, 3, 4

$$\mathcal{L} = -\frac{\mu^2}{2} \varphi^2 - \frac{1}{2} \left(\frac{\partial \varphi}{\partial x_\nu} \right)^2 - g \frac{\partial \varphi}{\partial x_\nu} s_\nu - f \varphi q$$

($\mu = \frac{c}{\hbar} \times \text{mass}$)

$$s_\nu = \tilde{\psi} \frac{1}{i} \left\{ \begin{array}{c} \sigma_2 \sigma_3 \\ \sigma_3 \sigma_1 \\ \sigma_1 \sigma_2 \\ -\sigma_1 \sigma_2 \sigma_3 \sigma_4 \end{array} \right\} \psi \quad \text{is a pseudovector}$$

$$q = \tilde{\psi} i \sigma_1 \sigma_2 \sigma_3 \psi \quad \text{is a pseudoscalar}$$

ψ is nucleon wave function
 for notations see p. 53

In non relativistic approximation for nucleons

$$s_m = \tilde{\psi} \sigma_m \psi \quad s_4 = 0 \quad q = 0$$

$\sigma_1, \sigma_2, \sigma_3 = \text{Pauli operators}$

Canonical conjugate to φ

$$\chi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \frac{1}{ic} \frac{\partial \mathcal{L}}{\partial \dot{x}_4} =$$

$$= -\frac{1}{ic} \frac{\partial \varphi}{\partial x_4} - \frac{1}{ic} s_4$$

Hamiltonian density

$$\mathcal{H} = \chi \frac{\partial \varphi}{\partial t} - \mathcal{L}$$

$$= \frac{\mu^2}{2} \varphi^2 + \frac{1}{2} \nabla \varphi^2 + \frac{1}{2c^2} \dot{\varphi}^2 + g \frac{\partial \varphi}{\partial x_n} s_n + f \varphi g$$

Field equations (from variation: $\delta \int \mathcal{L} dt d\tau = 0$)

$$\square^2 \varphi - \mu^2 \varphi = -g \frac{\partial s_n}{\partial x_n} + f g$$

Interaction energy $\pm$ volume integral of

$$g \frac{\partial \varphi}{\partial x_n} s_n + f \varphi g$$

In static case and (of course) non relativistic approximation.

Field equation

$$\nabla^2 \varphi - \mu^2 \varphi = -g \nabla \cdot \vec{s} \quad ; \quad \vec{s} \equiv \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

φ produced by first nucleon (index a)

$$\begin{aligned}\varphi_a &= \frac{g}{4\pi} \int (\nabla \cdot \mathbf{s}_a) \frac{e^{-\mu r}}{r} d\tau_a \\ &= \frac{g}{4\pi} \int \mathbf{s}_a \cdot \nabla \frac{e^{-\mu r}}{r} d\tau_a\end{aligned}$$

(sign (+) because ∇ is at end point of r and not at point a)

If nucleon (a) is a small wave packet

$$\int \mathbf{s}_a d\tau_a = \sigma_a \quad \vec{r} = \vec{P} - \vec{P}_a$$

$$\varphi_a = \frac{g}{4\pi} (\sigma_a \cdot \vec{r}) \frac{1}{r} \frac{d}{dr} \left(\frac{e^{-\mu r}}{r} \right)$$

Interaction energy with nucleon at b is

$$\vec{r} = \vec{P}_b - \vec{P}_a$$

$$U = g \sigma_b \cdot \nabla \varphi_a$$

$$= \frac{g^2}{4\pi} \sigma_b \cdot \nabla \left[(\sigma_a \cdot \vec{r}) \frac{1}{r} \frac{d}{dr} \frac{e^{-\mu r}}{r} \right]$$

$$= \frac{g^2}{4\pi} \left\{ (\sigma_b \cdot \sigma_a) \frac{1}{r} \frac{d}{dr} \frac{e^{-\mu r}}{r} + \right.$$

$$\left. + (\sigma_b \cdot \vec{r})(\sigma_a \cdot \vec{r}) \frac{1}{r} \frac{d}{dr} \left(\frac{1}{r} \frac{d}{dr} \frac{e^{-\mu r}}{r} \right) \right\}$$

$$= \frac{g^2}{4\pi} \left\{ -(\sigma_a \cdot \sigma_b) \left(\frac{1}{r^3} + \frac{\mu}{r^2} \right) e^{-\mu r} + \right.$$

$$\left. + (\sigma_a \cdot \vec{r})(\sigma_b \cdot \vec{r}) \left(\frac{3}{r^5} + \frac{3\mu}{r^4} + \frac{\mu^2}{r^2} \right) e^{-\mu r} \right\}$$

$$U = \frac{g^2}{4\pi} \left[\left\{ \frac{3(\sigma_a \cdot r)(\sigma_b \cdot r)}{r^2} - (\sigma_a \cdot \sigma_b) \right\} \left(\frac{1}{r^3} + \frac{\mu}{r^2} + \frac{\mu^2}{3r} \right) e^{-\mu r} + (\sigma_a \cdot \sigma_b) \frac{\mu^2}{3r} e^{-\mu r} \right] = \frac{g^2}{4\pi} \mathcal{O}$$

b) Charged pseudoscalar field

Instead of complex field φ introduce real fields φ_1, φ_2 with

$$\varphi = \varphi_1 + i\varphi_2 \quad \varphi^* = \varphi_1 - i\varphi_2$$

In $\mathcal{L}$ and $\mathcal{H}$ meson part is merely the sum of two terms like previously, one for φ , one for φ_2 .
In interaction terms substitute

$$\varphi \text{ by } \varphi Q + \varphi^* Q^* = \quad Q = N \rightarrow P$$

$$= \varphi_1 (Q + Q^*) + \varphi_2 \frac{Q^* - Q}{i} = \varphi_1 \tau_1 + \varphi_2 \tau_2$$

$$Q = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} \text{Neutron} \\ \text{Proton} \end{vmatrix}$$

$$Q^* = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$\tau_1 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \quad \tau_2 = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} \quad \text{also } \tau_3 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

τ_1, τ_2, τ_3 also called isotopic spin

energy

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Interaction between two nucleons becomes with O from previous page

$$\frac{g^2}{4\pi} \left\{ \tau_1(a) \tau_1(b) + \tau_2(a) \tau_2(b) \right\} \times O = \frac{g^2}{2\pi} (Q_a Q_b^* + Q_a^* Q_b) O$$

If both nucleons are protons or if both are neutrons no interaction.

If one is a proton and one is a neutron.

$Q_a Q_b^* + Q_a^* Q_b =$ Heisenberg exchange operator $= H_{ex}$

$$\text{Interaction} = 2 H_{ex} U$$

Various combinations of charged and neutral mesons are possible.

Divergence at $r=0$ is sometimes eliminated by cut-off.

(Bethe. P.R. 54 pp 260, 390, 1940)

Vector meson fieldVector $\varphi_1, \varphi_2, \varphi_3, \varphi_4 = i\varphi_0$

Put

$$f_{\lambda\nu} = \frac{\partial \varphi_\nu}{\partial x_\lambda} - \frac{\partial \varphi_\lambda}{\partial x_\nu}$$

a) Pure meson field (neutral)

$$\mathcal{L} = -\frac{1}{4} f_{\lambda\nu}^2 - \frac{\mu^2}{2} \varphi_\lambda^2$$

Field equations

$$\frac{\partial f_{\nu\lambda}}{\partial x_\nu} = \mu^2 \varphi_\lambda$$

Variables conjugate to φ_ν

$$\pi_\nu = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_\nu} = \frac{1}{ic} \frac{\partial \mathcal{L}}{\partial \frac{\partial \varphi_\nu}{\partial x_4}} = \frac{1}{ic} f_{\nu 4}$$

$$\pi_m = \frac{1}{ic} f_{m4} \quad \pi_4 = 0$$

$$\mathcal{H} = \pi_\nu \dot{\varphi}_\nu - \mathcal{L} + \text{divergence}$$

$$= f_{\nu 4} \frac{\partial \varphi_\nu}{\partial x_4} + \frac{1}{4} f_{\lambda\nu}^2 + \frac{\mu^2}{2} \varphi_\lambda^2 + \text{divergence}$$

$$\begin{aligned}
 \mathcal{H} &= f_{n4} \left(\frac{\partial \varphi_m}{\partial x_4} - \frac{\partial \varphi_4}{\partial x_m} \right) + f_{n4} \frac{\partial \varphi_4}{\partial x_m} + \\
 &+ \frac{1}{4} f_{ln}^2 + \frac{1}{2} f_{m4}^2 + \frac{\mu^2}{2} \varphi_m^2 + \frac{\mu^2}{2} \varphi_4^2 + \text{divergence} \\
 &= -\frac{1}{2} f_{m4}^2 - \varphi_4 \frac{\partial f_{m4}}{\partial x_m} + \frac{\mu^2}{2} \varphi_4^2 + \frac{\mu^2}{2} \varphi_m^2 + \frac{1}{4} f_{ln}^2
 \end{aligned}$$

Use field equations to eliminate

$$\varphi_4 = \frac{1}{\mu^2} \frac{\partial f_{m4}}{\partial x_m} = \frac{ic}{\mu^2} \frac{\partial \pi_m}{\partial x_m}$$

Use vector notation

$$\vec{\pi} = (\pi_1, \pi_2, \pi_3) \quad \pi_4 = 0$$

$$\vec{\varphi} = (\varphi_1, \varphi_2, \varphi_3) \quad \varphi_4 = \frac{ic}{\mu^2} \nabla \cdot \vec{\pi}$$

Find

$$\mathcal{H} = \frac{c^2}{2} \pi^2 + \frac{1}{2} (\nabla \times \varphi)^2 + \frac{\mu^2}{2} \varphi^2 + \frac{c^2}{2\mu^2} (\nabla \cdot \pi)^2$$

b) Neutral meson and nucleon field

Nucleon field = ψ

$$M = \frac{c}{\hbar} \times \text{nucleon mass}$$

$$\mathcal{L} = -\hbar c \tilde{\psi} \gamma_4 \left(\gamma_\nu \frac{\partial}{\partial x_\nu} + M \right) \psi - \frac{1}{4} f_{\lambda\nu}^2 - \frac{\mu^2}{2} \varphi_\nu^2 + \\ + i a \varphi_\nu \tilde{\psi} \gamma_4 \gamma_\nu \psi + \frac{i b}{2\mu} f_{\lambda\nu} \tilde{\psi} \gamma_4 \gamma_\lambda \gamma_\nu \psi$$

a, b are interaction constants

Field equations are

$$\frac{\partial f_{\nu\lambda}}{\partial x_\nu} - \mu^2 \varphi_\lambda = -i a \tilde{\psi} \gamma_4 \gamma_\lambda \psi + \frac{i b}{\mu} \sum_{\nu \neq \lambda} \frac{\partial}{\partial x_\nu} (\tilde{\psi} \gamma_4 \gamma_\nu \gamma_\lambda \psi)$$

$$\left[-\hbar c \gamma_4 \left(\gamma_\nu \frac{\partial}{\partial x_\nu} + M \right) + i a \varphi_\nu \gamma_4 \gamma_\nu + \frac{i b}{2\mu} \gamma_4 \gamma_\nu \gamma_\lambda f_{\nu\lambda} \right] \psi = 0$$

Hamiltonian: (Use $\varphi_4 = \frac{1}{\mu^2} \left(i c \frac{\partial \pi_m}{\partial x_m} + i a \tilde{\psi} \gamma_4 \psi \right)$)

$$\pi_m = \frac{1}{i c} f_{m4} + \frac{b}{\mu c} \tilde{\psi} \gamma_m \psi \quad \pi_4 = 0$$

In three dimensional vector notation

$$\vec{\varphi} \equiv (\varphi_1, \varphi_2, \varphi_3) \quad \pi = (\pi_1, \pi_2, \pi_3) \quad \vec{\gamma} = (\gamma_1, \gamma_2, \gamma_3)$$

$$\pi = \frac{1}{i c} \nabla \varphi_4 + \frac{1}{c^2} \dot{\vec{\varphi}} + \frac{b}{\mu c} \tilde{\psi} \vec{\gamma} \psi$$

also from field equation for φ_4

$$\varphi_4 = \frac{i c}{\mu^2} \nabla \cdot \pi + \frac{i a}{\mu^2} \tilde{\psi} \psi$$

Notice also (See Quantum Mechs p. 23)

conjugate to ψ is $-\frac{\hbar}{i}\tilde{\psi}$

Find Hamiltonian density, adding a divergence similar to p 140 §

$$\begin{aligned} \mathcal{H} = & c\hbar\tilde{\psi}\gamma_4\left(\gamma_m\frac{\partial}{\partial x_m} + M\right)\psi + \\ & + \frac{c^2}{2}\pi^2 + \frac{1}{2}(\nabla\times\varphi)^2 + \frac{\mu^2}{2}\varphi^2 + \frac{c^2}{2\mu^2}(\nabla\cdot\pi)^2 - \\ & - i a \varphi \cdot \tilde{\psi} \gamma_4 \gamma \psi + \frac{c a}{\mu^2} (\nabla\cdot\pi) \tilde{\psi} \psi - \\ & - \frac{i b}{2\mu} (\nabla\times\varphi) \cdot \tilde{\psi} \gamma_4 (\gamma\times\gamma) \psi + \frac{c b}{\mu} \pi \cdot \tilde{\psi} \gamma \psi + \\ & + \frac{a^2}{2\mu^2} (\tilde{\psi}\psi)^2 + \frac{b^2}{2\mu^2} (\tilde{\psi}\gamma\psi)^2 \end{aligned}$$

Forces between nucleons at rest

Field equations reduce to

$$\nabla_4^2 \varphi - \mu^2 \varphi = -i a \tilde{\psi} \psi$$

$$\frac{\partial f_{mm}}{\partial x_m} - \mu^2 \varphi_m = \frac{b}{2\mu} \left\{ \nabla \times (\tilde{\psi} \sigma \gamma) \right\}_m$$

or

$$\begin{cases} \nabla^2 \varphi_4 - \mu^2 \varphi_4 = -ia \bar{\psi} \psi \\ \nabla^2 \varphi - \nabla(\nabla \cdot \varphi) - \mu^2 \varphi = \frac{b}{2\mu} \nabla \times \bar{\psi} \sigma \psi \end{cases}$$

In static approx, interaction energy is

$$\frac{a}{i\mu^2} (\nabla^2 \varphi_4) \bar{\psi} \psi + \frac{b}{\mu} (\nabla \times \varphi) \cdot \bar{\psi} \sigma \psi$$

Field of one nucleon

$$\varphi_4 = \frac{ia}{4\pi} \frac{e^{-\mu r}}{r}$$

$$\nabla^2 \varphi_4 = \mu^2 \varphi_4 = \mu \frac{ia}{4\pi} \frac{e^{-\mu r}}{r}$$

Interaction potential due to "a" is

$$V_a = \frac{a^2}{4\pi} \frac{e^{-\mu r}}{r}$$

also:

$$\varphi = \frac{b}{4\pi\mu} \sigma \times \nabla \frac{e^{-\mu r}}{r}$$

Interaction due to "b"

$$\begin{aligned}
 V_b &= \frac{b^2}{8\pi\mu^2} \nabla \times (\sigma_1 \times \nabla \frac{e^{-\mu r}}{r}) \cdot \sigma_2 \\
 &= -\frac{b^2}{8\pi\mu^2} \left\{ 2 \left(\frac{e^{-\mu r}}{r^2} + \mu \frac{e^{-\mu r}}{r} \right) \sigma_1 \cdot \sigma_2 + \right. \\
 &\quad \left. + \frac{1}{r} \left[\frac{d}{dr} \left(\frac{e^{-\mu r}}{r^2} + \mu \frac{e^{-\mu r}}{r} \right) \right] (\sigma_1 \times r) \cdot (\sigma_2 \times r) \right\}
 \end{aligned}$$

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3 - Neutral pions

(Stanford Meeting Dec 1949

York & others - P.R. 77, 213, 1950

For 345 MeV protons energy of
observed γ -rays becomes direction
independent and equal 75 MeV
in system with $\beta = .32$

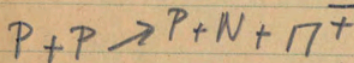
Cross-section per Carbon nucleus
 $\approx 1 \times 10^{-27}$
(probably per photon)

345 MeV protons on C-target

Production of positive & negative
pions

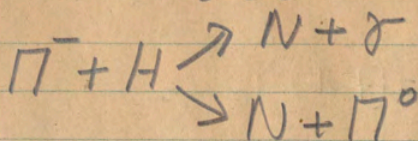
Comments π^+/π^- ratios

Garnow factor
Mc Millan - Teller

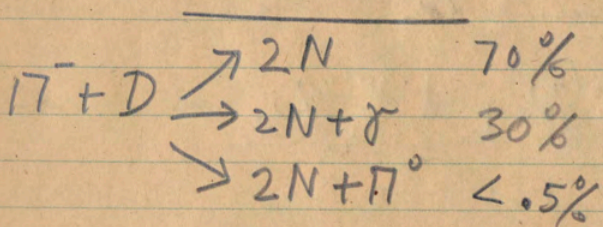


Nucleon binding and
deuteron formation

Reaction at rest



$$\mu_{\pi^0} = \mu_{\pi^-} - 11 m_{el}$$



Insert on p. 92

Let $s = (++) Y_0^0$

$$d = \frac{1}{\sqrt{10}} \left\{ (++) Y_2^0 - \sqrt{3} \frac{(+ -) + (- +)}{\sqrt{2}} Y_2^1 + \sqrt{6} (- -) Y_2^2 \right\}$$

Solution will be

$$\psi = s \frac{u(r)}{r} + d \frac{v(r)}{r}$$

Potential

$$U(r) + V(r) S \quad S = S_{12}$$

$U(r)$ = central force for triplet state

Find

$$\begin{cases} S s = \sqrt{8} d \\ S d = -2d + \sqrt{8} s \end{cases}$$

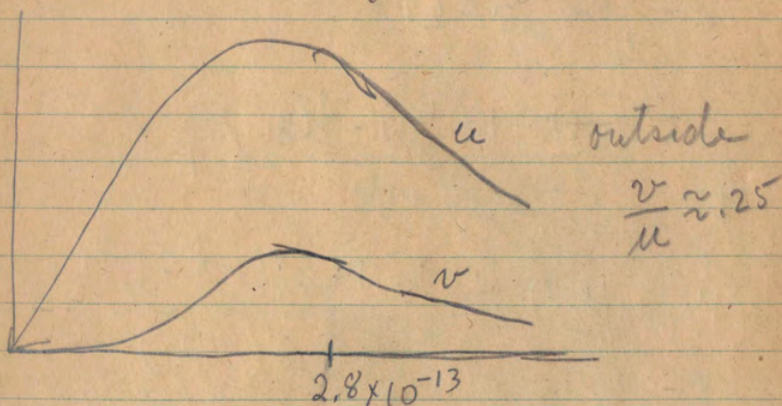
$$\text{Hamiltonian} = -\frac{\hbar^2}{M} \nabla^2 - U - V S$$

yields coupled equations

$$\frac{\hbar^2}{M} u'' + (E+U)u + \sqrt{8} V \cdot v = 0$$

$$\frac{\hbar^2}{M} \left(v'' - \frac{6}{r^2} v \right) + (E+U-2V)v + \sqrt{8} V u = 0$$

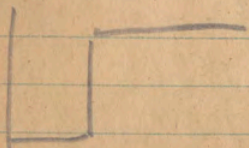
Rarita & Schwinger solution



$$Q = \frac{\sqrt{2}}{10} \int_0^{\infty} r^2 (uv - v^2/\sqrt{8}) dr \leftarrow = 2.73 \times 10^{-27}$$

$$Y_2^0 = \sqrt{\frac{5}{16\pi}} (3z^2 - r^2)/r^2$$

$$\frac{1}{4} (3z^2 - r^2) = \sqrt{\frac{8}{5}} r^2 Y_2^0 =$$



	depth	radius
U	13.9 MeV	2.83×10^{-13}
V	10.8	