

c9

Elementary Particles
Spring 1951

A - Field amplitudes as operators

A-1. The oscillator

$$\frac{p^2}{2m} + \frac{m\omega^2}{2} x^2 = H$$

$$E = \hbar\omega\left(n + \frac{1}{2}\right) \quad n \text{ quanta}$$

$$x(n \rightarrow n \pm 1) = \sqrt{\frac{\hbar}{2m\omega}} \begin{cases} \sqrt{n+1} \\ \sqrt{n} \end{cases} \quad p x - x p = \frac{\hbar}{i}$$

$$p(n \rightarrow n \pm 1) = \pm i \sqrt{\frac{\hbar m \omega}{2}} \begin{cases} \sqrt{n+1} \\ \sqrt{n} \end{cases}$$

Hence: Operators x, p create or destroy quanta

Linear combinations

$$a = \sqrt{\frac{m\omega}{2\hbar}} x + \frac{i}{\sqrt{2\hbar m\omega}} p \quad x = \sqrt{\frac{\hbar}{2m\omega}} (a + a^*)$$

$$a^* = \sqrt{\frac{m\omega}{2\hbar}} x - \frac{i}{\sqrt{2\hbar m\omega}} p \quad p = \sqrt{\frac{\hbar m \omega}{2}} \frac{a - a^*}{i}$$

$$a(n \rightarrow n-1) = \sqrt{n} \quad (\text{destruction operator})$$

$$a^*(n \rightarrow n+1) = \sqrt{n+1} \quad (\text{creation operator})$$

$$a a^* - a^* a = 1$$

$$a^* a = n$$

A-2. The electromagnetic field

Radiation field

$$\mathbf{H} = \nabla \times \mathbf{A} \quad \mathbf{E} = -\frac{1}{c} \frac{d\mathbf{A}}{dt}$$

Analysis in harmonic components

$$dN = \frac{\omega^2 d\omega}{\pi^2 c^3} \Omega$$

Component s

$$A_s = \varepsilon_s q_s \cos(\underbrace{f_s \cdot r + \beta_s}_{\alpha_s})$$

$$f_s = \frac{\omega_s}{c}$$

$$E_s = -\frac{\varepsilon_s}{c} \dot{q}_s \cos \alpha_s \quad H_s = (\varepsilon_s \times f_s) q_s \sin \alpha_s$$

$$W_s = \frac{\overline{E_s^2} + \overline{H_s^2}}{8\pi} \Omega = \frac{\Omega}{16\pi c^2} \dot{q}_s^2 + \frac{\Omega f_s^2}{16\pi} q_s^2 =$$

$$= \frac{m}{2} \dot{q}_s^2 + \frac{m \omega_s^2}{2} q_s^2$$

$$m = \frac{\Omega}{8\pi c^2}$$

$$\omega_s = c |f_s|$$

Superposition of solutions with energy additive

$$W_s = \hbar \omega_s (n_s + \frac{1}{2}) \rightarrow \hbar \omega_s n_s$$

n_s as number of photons.

Comment on infinite energy constant

$$\sum_s \frac{\hbar \omega_s}{2} = \int \frac{\hbar \omega}{2} \frac{\omega^2 d\omega \Omega}{\pi^2 c^3}$$

Since

$$A = \sum A_s$$

Matrix elements of A

$$\vec{A}(n_s \rightarrow n_s \pm 1) = \vec{\epsilon}_s \cos(f_s \cdot r + \beta_s) \sqrt{\frac{4\pi\hbar c^2}{\Omega \omega_s}} \begin{Bmatrix} \sqrt{n_s+1} \\ \sqrt{n_s} \end{Bmatrix}$$

Comment on standing waves versus directional photons

In latter case

$$\vec{A}(n_s \rightarrow n_s \pm 1) = \vec{\epsilon}_s \sqrt{\frac{2\pi\hbar c^2}{\Omega \omega_s}} \begin{Bmatrix} \sqrt{n_s+1} \\ \sqrt{n_s} \end{Bmatrix} e^{\mp \frac{i}{\hbar} \vec{p} \cdot \vec{r}}$$

Comments: $\begin{Bmatrix} \psi^* \\ \psi \end{Bmatrix}$ associated to $\begin{Bmatrix} \text{creation} \\ \text{destruction} \end{Bmatrix}$

Distinguish between position as c-number and field as q-number

Measurability of the field

$$\vec{A} = \sum_s \vec{\epsilon}_s \sqrt{\frac{2\pi\hbar c^2}{\Omega \omega_s}} \left(a_s e^{i f_s \cdot r} + a_s^* e^{-i f_s \cdot r} \right)$$

Comment on general electromagnetic field

A-3. Scalar field with mass
Klein Gordon equation

$$\nabla^2 \varphi - \frac{1}{c^2} \ddot{\varphi} - \frac{\mu^2 c^2}{\hbar^2} \varphi = 0$$

Plane wave solutions

$$\varphi \sim e^{\frac{i}{\hbar}(\mathbf{p} \cdot \mathbf{r} - Et)}$$

$$\mathbf{p}^2 + \mu^2 c^2 = \frac{E^2}{c^2}$$

Energy

$$\int \frac{1}{2} \left(\frac{\dot{\varphi}^2}{c^2} + (\nabla \varphi)^2 + \frac{\mu^2 c^2}{\hbar^2} \varphi^2 \right) d\Omega$$

Proceeding as before find

$$\varphi(\mathbf{r}) (n_s \rightarrow n_s \pm 1) = \frac{\hbar c e^{\mp \frac{i}{\hbar} \mathbf{p}_s \cdot \mathbf{r}}}{\sqrt{2\Omega \omega_s}} \begin{cases} \sqrt{n_s} \\ \sqrt{n_s + 1} \end{cases}$$

Comment on
Heaviside units

$$dN = \frac{\Omega \mathbf{p}^2 d\mathbf{p}}{2\pi^2 \hbar^3}$$

Charged field φ is complex (hence additional degrees of freedom)

$$\varphi = \sum_s \frac{\hbar c}{\sqrt{2\Omega \omega_s}} \left(b_s e^{i\mathbf{f}_s \cdot \mathbf{r}} + c_s^* e^{-i\mathbf{f}_s \cdot \mathbf{r}} \right)$$

$$\varphi^* = \sum_s \frac{\hbar c}{\sqrt{2\Omega \omega_s}} \left(b_s^* e^{i\mathbf{f}_s \cdot \mathbf{r}} + c_s e^{-i\mathbf{f}_s \cdot \mathbf{r}} \right)$$

$$b_s^* b_s = N_s^+ \quad (\text{mom} = \hbar \mathbf{f}_s)$$

$$c_s^* c_s = N_s^- \quad (\text{mom} = -\hbar \mathbf{f}_s)$$

$$\varphi(r)(n_s^{\pm} \rightarrow n_s^{\pm} \mp 1) = \frac{\hbar c}{\sqrt{2\Omega\omega_s}} e^{\pm \frac{i}{\hbar} p \cdot r} \begin{cases} \sqrt{n_s^+} \\ \text{or} \\ \sqrt{n_s^- + 1} \end{cases}$$

$$\varphi^*(r)(n_s^{\pm} \rightarrow n_s^{\pm} \pm 1) = \frac{\hbar c}{\sqrt{2\Omega\omega_s}} e^{\mp \frac{i}{\hbar} p \cdot r} \begin{cases} \sqrt{n_s^+ + 1} \\ \text{or} \\ \sqrt{n_s^-} \end{cases}$$

Comments on scalar, pseudoscalar, vector, ...

A-4 Pauli particles

General comments

No spin, no relativity

{ electron, positron
proton
neutron
muon
neutrino

A-4

$$\frac{\partial \psi}{\partial t} = \frac{i\hbar}{2m} \nabla^2 \psi$$

ψ as amplitude of probability and as position dependent operator

$$\psi(r) (0 \rightarrow 1) = \pm \frac{1}{\sqrt{2}} e^{\frac{i}{\hbar} p \cdot r}$$

$$\psi^*(r) (1 \rightarrow 0) = \pm \frac{1}{\sqrt{2}} e^{-\frac{i}{\hbar} p \cdot r}$$

Also when external potential

$$\psi(r) (0 \rightarrow 1) = \pm u_i(r)$$

$$\psi^*(r) (1 \rightarrow 0) = \pm u_i^*(r)$$

Dirac equation

$$\frac{1}{c} \frac{\partial \psi}{\partial t} = \alpha \cdot \nabla \psi + \frac{imc}{\hbar} \beta \psi$$

$$w_s = \pm \sqrt{c^2 p_s^2 + m^2 c^4}$$

Plane wave solutions

$$\frac{1}{\sqrt{2}} B_{\lambda}^{(s)} e^{\frac{i}{\hbar} p \cdot r}$$

$$\psi_{\lambda}(r) (1 \rightarrow 0) = \pm \frac{1}{\sqrt{2}} B_{\lambda}^{(s)} e^{\frac{i}{\hbar} p \cdot r}$$

$$\psi_{\lambda}^*(r) (0 \rightarrow 1) = \pm \frac{1}{\sqrt{2}} B_{\lambda}^{(s)*} e^{-\frac{i}{\hbar} p \cdot r}$$

ψ destroys electrons or creates positrons

B-Interaction of the fields

B-1

B-1 Electromagnetic interaction

$$\mathcal{H} = \int A \cdot J d\Omega \quad (\text{no } \varphi)$$

$$J = \frac{e\hbar}{2imc} (\psi^* \nabla \psi - \nabla \psi^* \psi)$$

$$\mathcal{H} = \frac{e\hbar}{2imc} \int (\psi^* \nabla \psi - \nabla \psi^* \psi) \cdot A d\Omega$$

Example:

$$\begin{array}{ccccccc} e_1 \rightarrow e_2 + \gamma & & & & & & \\ \frac{e\hbar}{2imc} & \psi^* & \nabla & \psi & A & & \\ \frac{e\hbar}{2imc} & \frac{(\pm 1)}{\sqrt{2}} & \frac{i}{\hbar} \vec{p}_1 & \frac{(\pm 1)}{\sqrt{2}} & \vec{\epsilon} \frac{\sqrt{2\pi\hbar c}}{\sqrt{\Omega\hbar\omega}} & \int e^{-\frac{i}{\hbar} \vec{p} \cdot \vec{r} + \frac{i}{\hbar} \vec{p}_1 \cdot \vec{r} - \frac{i}{\hbar} \vec{p}_2 \cdot \vec{r}} d\Omega & \\ & \sqrt{2} & & \sqrt{2} & & & \\ & & & & & \vec{p}_1 = \vec{p}_2 + \vec{p}_\gamma & \\ & & & & & \Omega & \end{array}$$

$$\frac{e}{2m} \sqrt{\frac{2\pi\hbar}{\Omega\omega}} (\vec{p}_1 \cdot \vec{\epsilon})$$

$$\mathcal{H}_{e_1 \rightarrow e_2 + \gamma} = \frac{e}{2} \sqrt{\frac{2\pi\hbar}{\Omega\omega}} (\vec{v}_1 + \vec{v}_2) \cdot \vec{\epsilon}$$

Theorem of momentum conservation

Rule for computing volume integrals

B-2 . Yukawa interactions

B-2

$$P \leftrightarrow N + \pi^+$$

$$N \leftrightarrow P + \pi^-$$

$$P \leftrightarrow P + \pi^0$$

$$N \leftrightarrow N + \pi^0$$

$$e_2 \int (P^* N \pi + N^* P \pi^*) d\Omega$$

$$e'_2 \int (\pi P^* \beta \alpha_1 \alpha_2 \alpha_3 N + \text{comp conj}) d\Omega$$

$$\frac{\hbar e''_2}{\mu c} \int (\nabla \pi \cdot P^* \frac{\alpha \times \alpha}{i} N + \text{comp conj}) d\Omega$$

$$\rightarrow e_2 \frac{1}{\sqrt{\Omega}} \frac{1}{\sqrt{\Omega}} \frac{\hbar c}{\sqrt{2 \Omega w}} \Omega = \frac{e_2 \hbar c}{\sqrt{2 \Omega w}}$$

$$\pi^{\pm} \longleftrightarrow \mu^{\pm} + \bar{\nu}$$

Quote experimental evidence. ν^2

$$e_3 \int (\pi \mu^* \nu + \text{conj conj}) d\Omega$$

antineutrinos!

$$\frac{e_3 \hbar c}{\sqrt{2\Omega\omega_0}}$$

$$N \longleftrightarrow P + e + \bar{\nu}$$

$$g_1 \int (P^* N e^* \nu + \text{c.c.}) d\Omega$$

$$\frac{g_1}{\Omega}$$

$$\mu \longleftrightarrow e + \nu + \bar{\nu}$$

Quote evidence

$$g_2 \int (\mu^* e \nu^* \nu + \text{c.c.}) d\Omega$$

$$g_2/\Omega$$

$$P + \mu^- \longleftrightarrow N + \nu$$

$$g_3 \int (N^* P \mu \nu + \text{c.c.}) d\Omega$$

$$g_3/\Omega$$

B-4. Golden rules

B-4

$$\text{Rate} = \frac{2\pi}{\hbar} |\langle \psi_{n0} | \hat{V} | \psi_{m0} \rangle|^2 \frac{dN}{dW}$$

Comments on $0 \times \infty$

$$dN = \frac{\Omega p^2 dp}{2\pi^2 \hbar^3} \quad dW = v dp$$

or

$$(v_1 + v_2) dp$$

$$\langle \psi_{n0} | \hat{V} | \psi_{m0} \rangle = \sum_m \frac{\langle \psi_{nm} | \hat{V} | \psi_{m0} \rangle}{W_0 - W_m}$$

Development parameters

$$(P) + \sum \frac{\langle \psi_{p0} | \hat{V} | \psi_{m0} \rangle}{\Delta W} (N + \Pi)_p$$

$$\sum \frac{|\langle \psi_{p0} | \hat{V} | \psi_{m0} \rangle|^2}{\Delta W^2} = \frac{e^2}{4\pi \hbar c} \int \frac{\eta^2 d\eta}{\pi (1 + \eta^2)^{3/2}}$$

$$\frac{g^2 m^4 c^2}{\hbar^6} \quad 10^{-12} \text{ to } 10^{-22}$$

C - Simple Interaction processes

C-1 - Pion decay

$$\pi \rightarrow \mu + \nu$$

$$\frac{e_3 \hbar c}{\sqrt{2\Omega \mu c^2}}$$

$$\frac{1}{\tau_\pi} = \frac{2\pi}{\hbar} \left(\frac{e_3 \hbar c}{\sqrt{2\Omega \mu c^2}} \right)^2 \frac{p^2 dp \Omega}{2\pi^2 \hbar^3 (\nu_\mu + \nu_\nu)} = \frac{e_3^2 p^2}{2\pi \hbar^2 \mu (\nu_\mu + \nu_\nu)}$$

$$cp + \sqrt{\mu^2 c^4 + c^2 p^2} = \mu c^2$$

$$\mu = 276 m$$

$$\mu_1 = 210 m$$

$$p = 58.1 mc \quad \nu_\nu = c \quad \nu_\mu = .27c \quad (4.1 \text{ MeV})$$

$$\frac{1}{\tau_\pi} = 3.8 \times 10^{37} e_3^2$$

$$\tau_\pi = 2.6 \times 10^{-8} \text{ sec}$$

$$e_3 = 10^{-15} \text{ esu} = 2 \times 10^{-6} e$$

C-2. Spontaneous muon decay

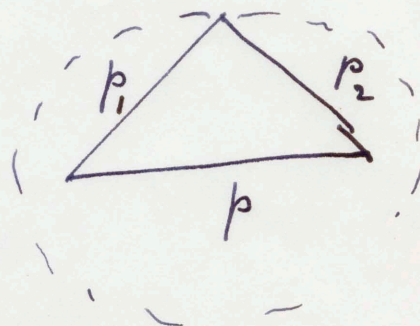
$$\mu \rightarrow e + \nu + \bar{\nu}$$

$$g_2 / \Omega$$

e relativistic

$$\text{Rate}(dp) = \frac{2\pi}{h} \frac{g_2}{\Omega^2} \frac{\Omega p^2 dp}{2\pi^2 h^3} \frac{dN}{dW}$$

$$|p_1| + |p_2| = \frac{W}{c} - p$$



Neutrino mom. space

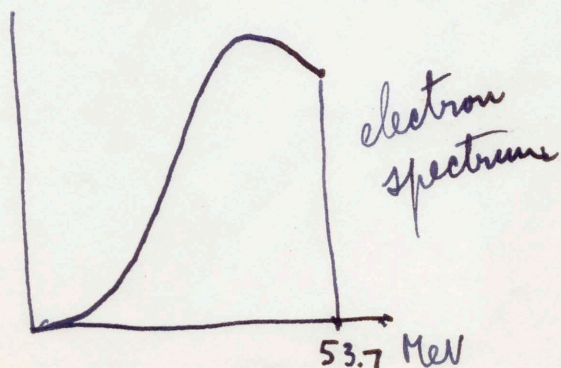
$$\frac{\pi}{6} \left(\frac{W^3}{c^3} - 3p \frac{W^2}{c^2} + 2p^2 \frac{W}{c} \right)$$

$$\frac{dN}{dW} = \frac{d}{dW} \left\{ \frac{\Omega}{8\pi^3 h^3} \frac{\pi}{6} \left(\frac{W^3}{c^3} - 3p \frac{W^2}{c^2} + 2p^2 \frac{W}{c} \right) \right\} \quad W = \mu_1 c^2$$

$$\text{Rate}(dp) = \frac{g_2^2 \mu_1^2 c}{48 \pi^3 h^7} \left(3 - \frac{6p}{\mu_1 c} + \frac{2p^2}{\mu_1^2 c^2} \right) p^2 dp$$

Integrating

$$\frac{1}{\tau_\mu} = \frac{7}{7680 \pi^3} \frac{g_2^2 \mu_1^5 c^4}{h^7} = \frac{1}{2.15 \times 10^{-6}} \text{ sec}^{-1}$$



$$g_2 = 3.3 \times 10^{-49}$$

C-3. Forced muon decay

generalities & experimental facts

 $Z=11$ forced = spontaneousProton & muon in Ω

$$\text{Rate} = \frac{2\pi}{\hbar} \left(\frac{g_3}{\Omega} \right)^2 \frac{\Omega p^2}{2\pi \hbar^3 (c + v_N)} = \frac{g_3^2}{\pi \hbar^4} \frac{p^2}{c + v_N} \frac{1}{\Omega}$$

$$210 mc^2 = cp + \frac{p^2}{2M}$$

$$p = 199 mc$$

$$v_N = .11 c$$

$$\frac{p^2}{2M} = 6 \text{ MeV}$$

$$\text{Rate} = 11,400 \frac{g_3^2 m^2 c}{\hbar^4 \Omega}$$

a) Z protons

$$b) \Omega \rightarrow \pi \left(\frac{a}{210 Z} \right)^3$$

c) $11,400 \rightarrow 5000$ (Pauli)

$$\frac{1}{\tau_3} = \frac{5000 (210)^3}{\pi} g_3^2 \frac{m^2 c Z^4}{\hbar^4 a^3}$$

$$\text{for } Z=1 \quad \tau_3 = 2.15 \times 10^{-6} \text{ yields}$$

$$g_3 = 1.3 \times 10^{-49}$$

C-4 - Beta rays

C-4

Various interactions

$$\text{I} \quad g_1 \int (P^* N e^* \nu + \text{c.c.}) d\Omega$$

$$\text{II} \quad g_\beta \int (\tilde{P}_\beta N \tilde{e}_\beta \nu + \text{c.c.}) d\Omega$$

$$\text{III} \quad g_\mu \int \left(\tilde{P} \frac{\alpha \times \alpha}{i} N \cdot \tilde{e} \frac{\alpha \times \alpha}{i} \nu + \text{c.c.} \right) d\Omega$$

$$g_\mu = 2 \times 10^{-49}$$

~~Allowed~~ Allowed transitions (Discuss

F & GT selection rules particularly I)

Selection rules

Nuclear matrix elements (non relativistic

$$\int \tilde{P} N d\tau \quad \text{or} \quad \int \tilde{P} \sigma N d\tau$$

Selection rules

Distortion of electron wave functions

$$\rightarrow \mathcal{H}_{no} = \frac{g_1}{\Omega} \int P^* N d\Omega = \frac{g_1}{\Omega} \mathcal{M}$$

$$\text{Rate}(dp) = \frac{2\pi}{h} \left| \frac{g_1 \mathcal{M}}{\Omega} \right|^2 \frac{\Omega g^2}{2\pi^2 h^3 c} \frac{\Omega p^2 dp}{2\pi^2 h^3} = \frac{g_1^2 \mathcal{M}^2}{2\pi^3 h^7 c} g^2 p^2 dp$$

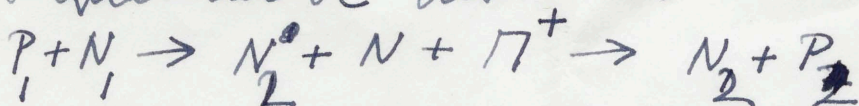
$$c q = \sqrt{m^2 c^4 + c^2 p^2} - \sqrt{m^2 c^4 + c^2 p_0^2} \quad \frac{1}{E} = \frac{g^2 \mathcal{M}^2 m^4 c^5}{2\pi^3 h^7} F(\eta_0)$$

$$F = -\frac{\eta_0}{4} - \frac{\eta_0^3}{12} + \frac{\eta_0^5}{30} + \frac{1}{4} \sqrt{1+\eta_0^2} \ln(\eta_0 + \sqrt{1+\eta_0^2})$$

D - Pions and Nucleons

D-1 - Yukawa theory

Qualitative discussion - Range from uncertainty



$$\frac{\frac{e_2 \hbar c}{\sqrt{2\Omega\omega}}}{-\omega} \times \frac{\frac{e_2 \hbar c}{\sqrt{2\Omega\omega}}}{-\omega} = -\frac{e_2^2 \hbar^2 c^2}{2\Omega(\mu^2 c^4 + c^2 p^2)}$$

$$\omega = \sqrt{\mu^2 c^4 + c^2 p^2}$$

$$p = p(p_1) - p(N_2)$$

$$= -p(N_1) + p(P_2)$$

Twice

$$\mathcal{H}(1 \rightarrow 2) = -\frac{e_2^2 \hbar^2 c^2}{\Omega(\mu^2 c^4 + c^2 p^2)}$$

$$V = U(r) Ex$$

$$\psi_1 = \frac{1}{\sqrt{\Omega}} e^{\frac{i}{\hbar} p_1 \cdot r_P} \frac{1}{\sqrt{\Omega}} e^{\frac{i}{\hbar} q_1 \cdot r_N}$$

$$\psi_2 = \frac{1}{\sqrt{\Omega}} e^{\frac{i}{\hbar} p_2 \cdot r_P} \frac{1}{\sqrt{\Omega}} e^{\frac{i}{\hbar} q_2 \cdot r_N}$$

$$r_P - r_N = r$$

$$V_{21} = \frac{1}{\Omega^2} \iint e^{-\frac{i}{\hbar} (p_2 \cdot r_P + q_2 \cdot r_N)} U(r_P - r_N) e^{\frac{i}{\hbar} (q_1 \cdot r_P + p_1 \cdot r_N)} d^3 r_P d^3 r_N$$

$$= \frac{1}{\Omega^2} \left(\int e^{\frac{i}{\hbar} (\vec{q}_1 + \vec{p}_1 - \vec{p}_2 - \vec{q}_2) \cdot r_N} d^3 r_N \right) \int U(r) e^{\frac{i}{\hbar} (q_1 - p_2) \cdot r} d^3 r$$

$$-\frac{e_2^2 \hbar^2 c^2}{\mu^2 c^4 + c^2 p^2} = \int U(r) e^{-\frac{i}{\hbar} p \cdot r} d^3 r$$

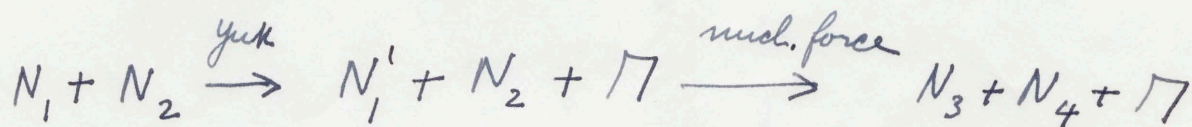
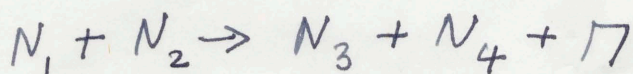
$$V(r) = \frac{1}{(2\pi\hbar)^3} \int \frac{-e^2 \hbar^2 c^2}{\mu^2 c^4 + c^2 p^2} e^{\frac{i}{\hbar} \mathbf{p} \cdot \mathbf{r}} d^3 r =$$

$$= -\frac{e^2}{2\pi\hbar} e^{-\frac{\mu c}{\hbar} r}$$

$$\frac{\hbar}{\mu c} = 1.4 \times 10^{-13}$$

Neutral pions contribute non exchange forces
 Discussion of the spin dependence

D-2. Production of pions in nucleon collision D-2



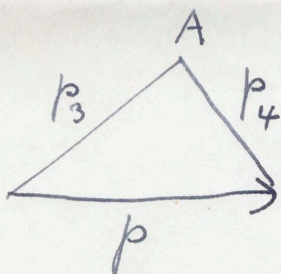
$$\frac{e_2 \hbar c}{\sqrt{2} \Omega \omega_\pi}$$

$$\times \frac{e_2^2 \hbar^2 c^2}{\Omega (\mu^2 c^4 + c^2 (p_4 - p_2)^2)}$$

$$\frac{\Delta W}{\Delta W} \approx$$

$$\approx \frac{\frac{e_2 \hbar c}{\sqrt{2} \Omega \mu c^2} \times \frac{e_2^2 \hbar^2 c^2}{\Omega \mu^2 c^4}}{\mu c^2} = \frac{f}{\Omega^{3/2}}$$

$$f = \frac{e_2^3 \hbar^3}{\sqrt{2} \mu^{7/2} c^4} \approx 10^{-61} \text{ erg} \times c^{3/2}$$



$$\frac{1}{2M} (p_3^2 + p_4^2) + \frac{1}{2\mu} p^2 = T$$

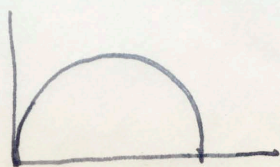
A on sphere of radius

$$\left\{ MT - \left(\frac{1}{4} + \frac{M}{2\mu} \right) p^2 \right\}^{1/2}$$

$$\frac{dN}{dW} = \frac{d}{dT} \left[\frac{\Omega}{8\pi^3 \hbar^3} \frac{4\pi}{3} \left\{ \right\}^{3/2} \right] = \frac{\Omega M}{4\pi^2 \hbar^3} \left\{ \right\}^{1/2}$$

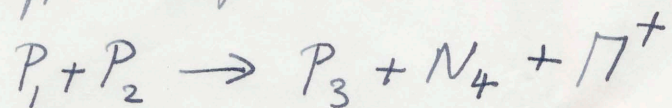
$$\sigma_{dp} = \frac{f^2 M}{4\pi^3 \hbar^7} \frac{1}{v} \left\{ \right\}^{1/2} p^2 dp$$

$$\sigma = \frac{f^2 M^3}{64\pi^2 \hbar^7 v} \frac{T^2}{\left(\frac{1}{4} + \frac{M}{2\mu} \right)^{3/2}}$$



D-3. Effect of nucleon binding

D-3



General comments

$$P_3, N_4 \quad \frac{1}{\sqrt{2}} e^{\frac{i}{\hbar} p_3 \cdot r_3} \frac{1}{\sqrt{2}} e^{\frac{i}{\hbar} p_4 \cdot r_4} =$$

$$= \frac{1}{\sqrt{2}} e^{\frac{i}{\hbar} (p_3 + p_4) \cdot r_D} \times \frac{1}{\sqrt{2}} e^{\frac{i}{\hbar} \frac{p_3 - p_4}{2} \cdot r(r_3 - r_4)}$$

$$r_D = \frac{r_3 + r_4}{2}$$

$$r = r_3 - r_4$$

$\rightarrow D(r)$

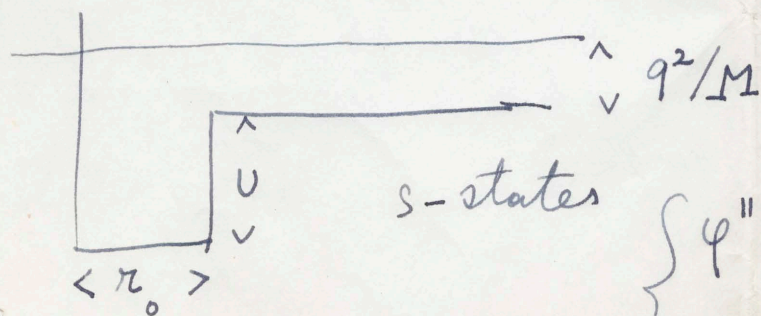
$$\frac{f}{\Omega^{3/2}} \rightarrow \frac{f}{\Omega^{3/2}} \frac{D(0)}{1/\sqrt{\Omega}} = \frac{f D(0)}{\Omega}$$

Prod. of D in bound state

$$D(0) = 3.2 \times 10^{18} \text{ cm}^{-3/2}$$

$$\sigma_D = \frac{f^2 D^2(0) M_R p}{\pi \hbar^4 v_{12}}$$

Continuum state



$$\varphi = r D(r)$$

$$\begin{cases} \varphi'' + \frac{q^2 + M U}{\hbar^2} \varphi = 0 & r < r_0 \\ \varphi'' + \frac{q^2}{\hbar^2} \varphi = 0 & r > r_0 \end{cases}$$

Inside: $\varphi = N \sin \frac{Q r_0}{\hbar}$

$$Q = \sqrt{q^2 + MU}$$

Outside: $\varphi = \frac{\sqrt{2}}{\sqrt{L}} \sin\left(\frac{q r_0}{\hbar} + \beta\right)$

$$N \sin \frac{Q r_0}{\hbar} = \sqrt{\frac{2}{L}} \sin\left(\frac{q r_0}{\hbar} + \beta\right)$$

$$\frac{Q}{\hbar} N \cos \frac{Q r_0}{\hbar} = \frac{q}{\hbar} \sqrt{\frac{2}{L}} \cos\left(\frac{q r_0}{\hbar} + \beta\right)$$

$$N^2 \left\{ \sin^2 \frac{Q r_0}{\hbar} + \frac{Q^2}{q^2} \cos^2 \frac{Q r_0}{\hbar} \right\} = \frac{2}{L} \frac{1}{2\pi L}$$

$$D^2(0) = \frac{N^2 Q^2}{\hbar^2} = \frac{Q^2}{2\pi L \hbar^2} \frac{1}{1 + \frac{Q^2 - q^2 \cos^2 \frac{Q r_0}{\hbar}}{q^2}}$$

States (dq)

$$\frac{\pi \hbar}{q} n \approx L$$

$$\frac{4\pi}{\hbar}$$

$$\frac{L dq}{\pi \hbar} \frac{2 q^2}{\hbar^2}$$

Number = $\frac{L dq}{\pi \hbar}$

$$D_{dq}^2(0) = \frac{q^2 dq}{2\pi^2 \hbar^3} \frac{q^2 + MU}{q^2 + MU \cos^2\left(\sqrt{q^2 + MU} \frac{r_0}{\hbar}\right)}$$

$$\sigma_{dq} = \frac{f^2 M_R p q^2 dq}{2\pi^3 \hbar^7 v_{12}} \frac{q^2 + MU}{q^2 + MU \cos^2\left(\sqrt{\frac{r_0}{\hbar}}\right)}$$

$$\frac{q^2}{M} + \frac{p^2}{2M_R} = T$$

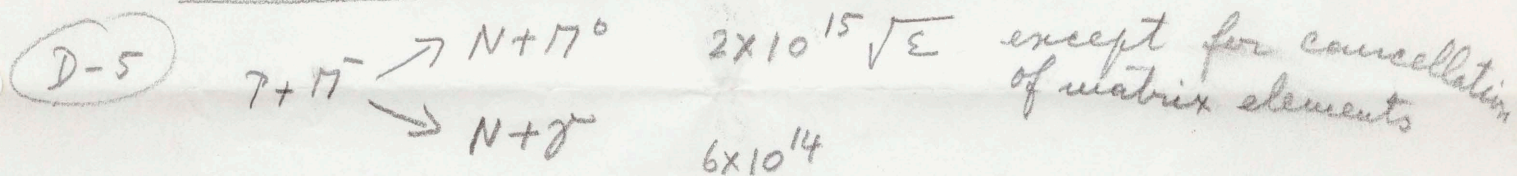
$$\frac{2 q |dq|}{M} = \frac{p |dp|}{M_R}$$

$$\sigma_{dp} = \frac{f^2 M q p^2 dp}{4\pi^3 \hbar^7 v_{12}} \frac{q^2 + MU}{q^2 + MU \cos^2\left(\sqrt{q^2 + MU} \frac{r_0}{\hbar}\right)}$$

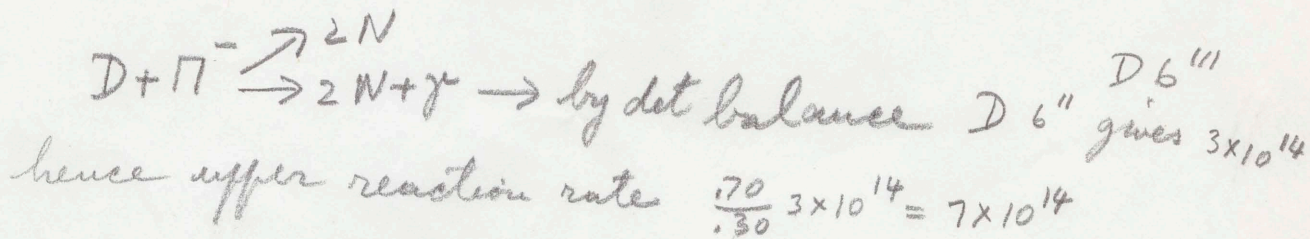
Angular distribution of $P+P \rightarrow D+\pi^+$
 condition for $\cos^2\theta$ distribution

$P+P$	$\rightarrow$	$D+\pi^+$	Scalar	P.s. scalar
1S_0		3S_1	no	3P_1
$^3P_{012}$		$^3P_{012}$	$^3P_{0,1,2}$	$^1S_0, ^1D_2$
1D_2		$^3D_{123}$		

Problem for the two cases find conditions
 for $\cos^2\theta$ emission



Discuss which is right by det bal of second reaction
 D-5'

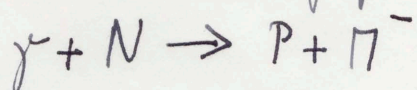


Scattering D-7

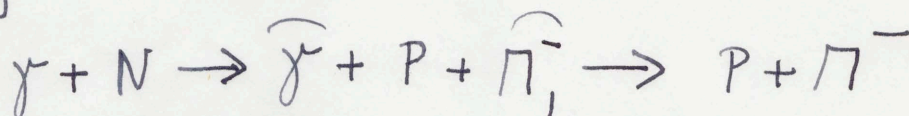
Annihilation

D-4 - Production of pions by γ 's

D-4



Process



$$\sigma = \frac{\frac{e_2 \hbar c}{\sqrt{2} \Omega \mu c^2} \times \frac{\sqrt{2} \pi e \hbar c}{\sqrt{\Omega \mu \hbar \omega}}}{\mu c^2} = \frac{\sqrt{\pi} e e_2 \hbar^{3/2}}{\Omega c \mu^{3/2} \omega^{1/2}}$$

$$\sigma = \frac{\sqrt{2} e^2 e_2^2}{c^3 \mu^{3/2}} \frac{\sqrt{\hbar \omega - \mu c^2}}{\hbar \omega}$$

for 335 MeV
 $\sigma \approx 3 \times 10^{-28}$

Discussion of other intermediate steps

Difference between positive and neg. pion production

Photo production of neutral pions

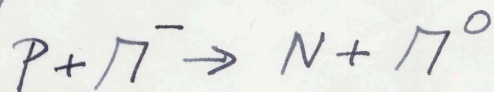
$$\frac{\pi^-}{\pi^+} \approx 1.3$$

D-5 Capture of π^- by hydrogen

D-5

Description of Panofsky experiment
 γ peaks at 60 to 70 MeV and 120 MeV

Possible processes



Cancellation of
matrix elements?

$$\text{Rate} = \frac{e_2^4 M^{3/2} \sqrt{\epsilon}}{2\sqrt{2} \pi c^4 \mu^{5/2} (M + \mu)^{3/2}} \frac{1}{\Omega}$$

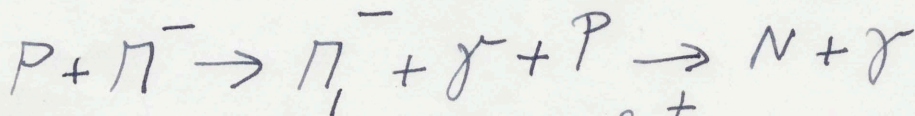
$$\Omega = \pi (a/276)^3$$

$$\rightarrow \approx 2 \times 10^{15} \sqrt{\epsilon_{\text{MeV}}}$$

Doppler effect indicates $\epsilon \approx 5 \text{ MeV}$

$$\rightarrow \approx 4 \times 10^{15}$$

Competing effect



$$\frac{\sqrt{2\pi} e \hbar}{\sqrt{2\mu}} \times \frac{e_2 \hbar}{\sqrt{2\Omega\mu}}$$

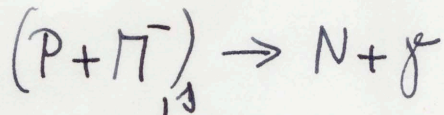
$$\mu c^2$$

$$\text{Rate} = \frac{1}{\pi} \left(\frac{276}{a} \right)^3 \frac{e^2 e_2^2}{\mu^2 c^3} \approx 6 \times 10^{14} \text{ sec}^{-1}$$

Discussion

Rate of

D-5'



from detailed balance

$$\sigma(P + \pi^- \rightarrow N + \gamma) = \frac{4}{2} \frac{p_\gamma^2}{p_\pi^2} \sigma(N + \gamma \rightarrow P + \pi^-)$$

$$\text{Rate}(\uparrow) = \frac{v_\pi}{\Omega} \sigma(P + \pi^- \rightarrow N + \gamma) =$$

$$= 2 \frac{v_\pi}{\Omega} \frac{p_\gamma^2}{p_\pi^2} \sigma(N + \gamma \rightarrow P + \pi^-)$$

$$\approx \frac{2}{\Omega} \frac{p_\gamma^2}{p_\pi} \sigma(N + \gamma \rightarrow P + \pi^-)$$

$$\approx \frac{2 \mu^2 c^2}{\Omega p_\pi} \sigma(N + \gamma \rightarrow P + \pi^-)$$

$$\frac{\text{Rate}(P + \pi^-)_{\text{free}}}{\text{Rate}(P + \pi^-)_{1s}} = \frac{\frac{1}{\Omega}}{\frac{1}{\pi a^3}}$$

$$\text{Rate}(P + \pi^-)_{1s} = \frac{\Omega}{\pi a^3} \frac{2 \mu^2 c^2}{\Omega p_\pi} \underbrace{\sigma(N + \gamma \rightarrow P + \pi^-)}_{10^{-28} \quad 10^{-28}}$$

from $\sigma(P + \gamma \rightarrow N + \pi^+) = 7 \times 10^{-29}$ at 180 MeV

$$\sigma(N + \gamma \rightarrow P + \pi^-) \approx 1.3 \times 7 \times 10^{-29} \approx 10^{-28}$$

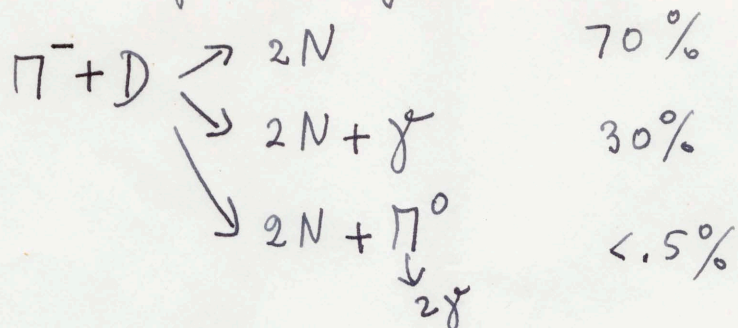
$$\rightarrow p_\pi = \sqrt{2 \times 2.5 \times 10^{-25} \times 30 \times 1.6 \times 10^{-6}} \quad (30 \text{ MeV})$$

$$= 5 \times 10^{-15}$$

$$\rightarrow = \frac{2 \mu^2 c^2}{\pi \left(\frac{5.3 \times 10^{-9}}{276} \right)^3} \frac{10^{-28}}{5 \times 10^{-15}} = 4 \times 10^{14}$$

D-6. Capture of π^- by D

D-6



If $\pi^- + D \rightarrow 2N$ from s-state π^- not scalar

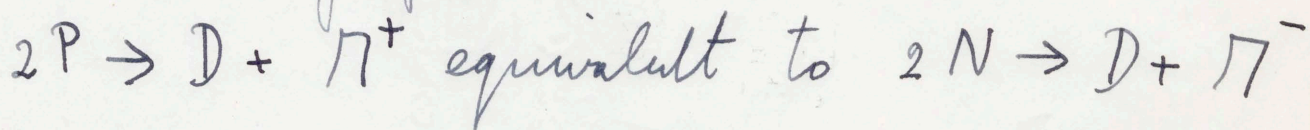
~~newer~~ s_0

$$s_1 \text{ even } \neq p_{012} \text{ odd}$$

d_2

Hence important to exclude capture in flight from p-orbit. Argument of Serber:

Det. balancing from reaction



$$\sigma \approx 2 \times 10^{-28} \text{ at } 340 \text{ KeV}$$

Assume: all σ due to capture into p state
matrix element $\sim \nabla \phi \rightarrow \nabla \pi(0)$

~~Free~~ For continuum state

$$|\nabla \pi|^2 = \frac{p^2}{\hbar^2 \Omega}$$

For bound hydrogenlike p state

$$|\nabla \pi|^2 = \frac{1}{3\pi a^5} \frac{n^2 - 1}{n^5} \quad a = \frac{\hbar^2}{\mu e^2}$$

From continuum Rate ($D\pi \rightarrow 2N$) = $\frac{\nu}{\Omega} \sigma_{D\pi}$

From bound state Rate ($D\pi \rightarrow 2N$)

$$\frac{\text{From bound state Rate } (D\pi \rightarrow 2N)}{\text{From continuum Rate } (D\pi \rightarrow 2N)} = \frac{\frac{1}{3\pi a^5} \frac{n^2-1}{n^5}}{\frac{p^2}{\hbar^2 \Omega}}$$

From detailed balancing

$$3 \sigma_{D\pi} p^2 = 2 \sigma_N p_n^2$$

Hence

$$\text{Rate } (D\pi_{\text{bound}} \rightarrow 2N) = \frac{\frac{1}{3\pi a^5} \frac{n^2-1}{n^5}}{\frac{p^2}{\hbar^2 \Omega}} \frac{\nu}{\Omega} \frac{2 \sigma_N p_N^2}{3 p^2} =$$

$$= \frac{2}{9\pi a^5} \frac{n^2-1}{n^5} \frac{\hbar^2 \sigma_N p_N^2 \nu}{p^4} =$$

$$\frac{p_N^2}{M} \approx \mu c^2$$

$$p \approx \mu v$$

$$= \frac{2}{9\pi a^5} \frac{n^2-1}{n^5} \frac{\hbar^2 \sigma_N M c^2}{p^3} \approx \frac{1.0 \times 10^{11} (n^2-1)}{n^5} \text{ sec}^{-1}$$

Transitions
from state

To $2N$

To $1s$ by
radiation

$2p$

$.94 \times 10^{10}$

17×10^{10}

$3p$

$.33$

5×10^{10}

$4p$

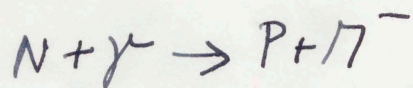
$.15$

2.2×10^{10}

Hence rad. transition much faster

Rate of $D + \pi^- \rightarrow 2N + \gamma$

from det. balance of photoprocess



estimate correction factor $2/3 = f$

Find

$$R(\pi^- + D \rightarrow 2N + \gamma) = f v_{\pi}^{\prime} \sigma(\pi^- + P \rightarrow N + \gamma) / \phi(0)^2$$

$$\sigma(\pi^- + P \rightarrow N + \gamma) = \frac{2 \mu^2 c^2}{p_{\pi}^{\prime 2}} \sigma(N + \gamma \rightarrow P + \pi^-)$$

$$\rightarrow = 2 f \mu c^2 / \phi(0)^2 \frac{\sigma(N + \gamma \rightarrow P + \pi^-)}{p_{\pi}^{\prime}}$$

Steinberger

$$\sigma(\gamma + D \rightarrow N + \pi^+) = 7 \times 10^{-29} \text{ at } 180 \text{ MeV}$$

correct

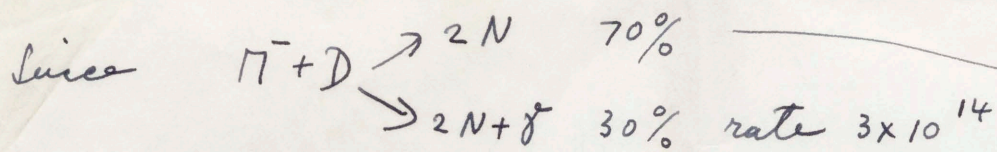
to 10^{-28} for $\gamma + N$

Assume

$\frac{\sigma}{p_{\pi}^{\prime}}$ energy indep

$\frac{\sigma}{p_{\pi}^{\prime}}$

$$R(\pi^- + D) = 2 \times \frac{2}{3} \times \mu c^2 \frac{1}{\pi a^3} \times \underbrace{2.5 \times 10^{-14}}_{\frac{\sigma}{p_{\pi}^{\prime}}} = 3 \times 10^{-14}$$



$$\rightarrow \text{rate} = \frac{70}{30} \times 3 \times 10^{14} = 7 \times 10^{14}$$

Det. balance of this reaction

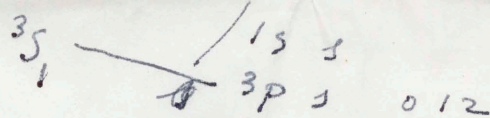
$$\text{Rate} = \frac{\Omega v}{\pi a^3 \Omega} \frac{2p_N^2}{3p_N^2} (\sigma_{NN})_s = \frac{2f}{3\pi} \frac{Mc^2}{a^3} \frac{\sigma_{NN}}{p_N} = 2.3 \times 10^{15} f$$

$\parallel$
 $f \sigma_{NN}$

$$7 \times 10^{14} = 2.3 \times 10^{15} f$$

$$f \approx .3$$

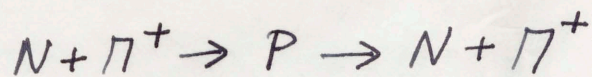
Why is $\pi^- + D \rightarrow 2N + \pi^0$ not observed



very low energy

D-7 Scattering of pions by nucleons

D-7



$$\frac{\left(\frac{e_2 \hbar c}{\sqrt{2\Omega \mu c^2}}\right)^2}{\mu c^2} = \frac{e_2^2 \hbar^2}{2\Omega \mu^2 c^2}$$

$$\frac{\left(\frac{1}{\sqrt{w}}\right)^2}{w} \approx \frac{1}{w^2}$$

$$\sigma = \frac{2\pi}{\hbar v} \left(\frac{e_2^2 \hbar^2}{2\mu^2 c^2}\right)^2 \frac{p^2}{2\pi^2 \hbar^3 v} \approx \frac{1}{4\pi} \left(\frac{e_2^2}{\mu c^2}\right)^2 = 1.6 \times 10^{-26}$$

Energy dependence

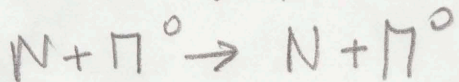
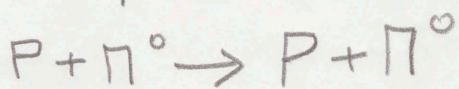
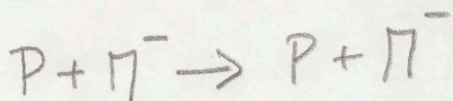
$$\frac{p^2}{v^2} \frac{1}{w^4} \rightarrow \frac{1}{w^2}$$

$$c^2 p = v w$$

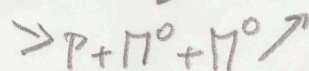
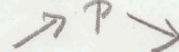
$$\frac{p}{v} = \frac{w}{c^2}$$

(Unreliable)

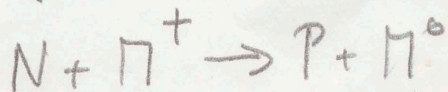
Discussion of similar processes



here possible destructive interference like



Scattering with exchange

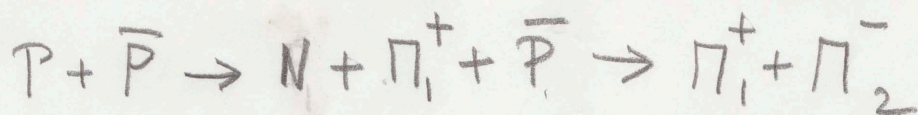


V particles

$$2165 \pm 20 \quad Q \approx 30 \text{ m c}^2$$

D-8 - Anti nucleon annihilation

D-8



$$\frac{\frac{e_2 \hbar c}{\sqrt{2\Omega\omega}} \times \frac{e_2 \hbar c}{\sqrt{2\Omega\omega}}}{\sqrt{2} \frac{Mc^2}{\hbar}} = \quad \omega = \frac{Mc^2}{\hbar}$$

$$\frac{\sqrt{2} e_2^2 \hbar^2}{2\sqrt{2} \Omega M^2 c^2}$$

2

$$\frac{1}{\Omega} = \eta$$

$$\frac{1}{\tau} = \frac{2\pi \sqrt{2} e_2^4 \hbar^4}{\Omega \hbar^8 M^4 c^4} \frac{M^2 c^2}{2\pi^2 \hbar^3 c \times 2} = \frac{e_2^4 \eta}{16\pi M^2 c^3}$$

Electromagnetic annihilation of $P + \bar{P}$ into 2γ 's

$$\frac{1}{\tau_{\text{em}}} = \frac{\pi e^4 \eta}{M^2 c^3}$$

For $\eta = 7 \times 10^{37}$

$$\frac{1}{\tau_{\text{em}}} \approx 1.5 \times 10^{17}$$

$$\frac{1}{\tau_{\eta}} = 1.5 \times 10^{20}$$

D-9 - Decay of $\pi^0 \rightarrow 2\gamma$ Generalities

D-9

$$\pi^0 \rightarrow P + \bar{P} \rightarrow P + \bar{P} + \gamma \rightarrow \gamma + \gamma$$

$$\frac{2\pi}{4\sqrt{2}}$$

$$\frac{\frac{e_2 \hbar}{\sqrt{2\Omega\mu}} \times \frac{\sqrt{2\pi} e \hbar}{\sqrt{\Omega\mu}} \times \frac{\sqrt{2\pi} e \hbar}{\sqrt{\Omega\mu}}}{(2Mc^2)^2} = \frac{\pi e_2 e^2}{2\sqrt{2} \Omega^{3/2} \mu^{3/2} M^2 c^4}$$

$$\times \frac{\Omega}{8\pi^3 \hbar^3} \frac{4\pi}{3} M^3 c^3$$

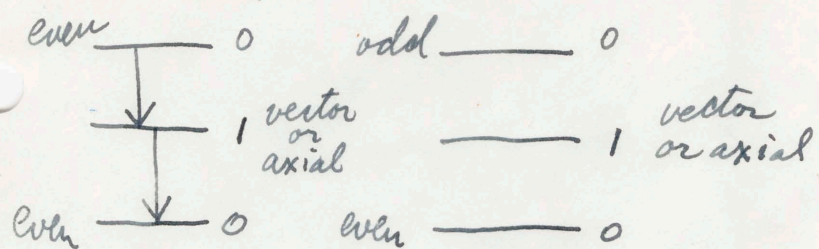
$$\Gamma^1 = \frac{e_2 e^2 M}{12\sqrt{2} \pi \mu^{3/2} c \sqrt{\Omega}}$$

$$\frac{1}{\tau} = \frac{e_2^2 e^4 M^2}{20000 \hbar^4 \mu c} \approx 10^{17}$$

Experiments 10^{-13} to 10^{-14}

Polarization of the γ 's

D-9'



D-10 V-particles

Rochester & Butler

Anderson

Blackett

2300

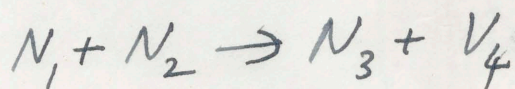
Thompson Cohen Fium

2170

30Life time $\approx 10^{-10}$ (instrumental?)

2 or 3 particle disintegration

Assume

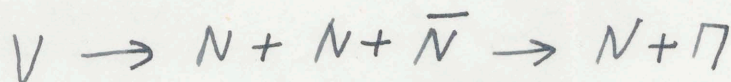


$$\sigma = \frac{g_4^2 p^2}{\pi \hbar^4 v_{12} v_{34}}$$

$$\sigma \approx 10^{-26}$$

$$g_4 \approx 10^{-43} \text{ ergs} \times \text{cm}^3$$

Spontaneous disintegration



$$\frac{g_4}{\Omega}$$

X

$$\frac{e_2 \hbar c}{\sqrt{2 \Omega \mu c^2}}$$

$$\frac{\Omega}{(2\pi\hbar)^3} \frac{4\pi}{3} (Mc)^3$$

$$\frac{\frac{g_4}{\Omega} \times \frac{e_2 \hbar c}{\sqrt{2 \Omega \mu c^2}}}{2 Mc^2} = \frac{e_4 \hbar c}{\sqrt{2 \Omega \mu c^2}}$$

$$\text{with } \frac{e_4}{e_2} = \frac{g_4 Mc^2}{12\pi^2 \hbar^3}$$

$$\frac{1}{\tau} = \frac{e_4^2 \mu c}{2\pi \hbar^2} \approx 3 \times 10^{22} \text{ s}^{-1} \quad 12.06$$

e_4 About 10^5 times too large for observed life time

Should be instead

$$e_4 = 3 \times 10^{-15}$$

Decay

D-10⁺

$$V \rightarrow N + \gamma$$

$$V \rightarrow N + \pi \rightarrow N + \pi + \gamma \rightarrow N + \gamma$$

$$\frac{e_4 \hbar c}{\sqrt{2} \Omega \mu c^2} \times \frac{\sqrt{2} \pi e \hbar c}{\sqrt{2} \Omega \mu c^2} \times \frac{e_2 \hbar c}{\sqrt{2} \Omega \mu c^2}$$

$$(\mu c^2)^2$$

$$V \rightarrow N + N + \bar{N} \rightarrow N + \gamma$$

$$\frac{g_4}{\Omega} \times \frac{\sqrt{2} \pi e \hbar c}{\sqrt{2} \Omega \mu c^2}$$

$$\frac{\Omega}{(2\pi\hbar)^3} \frac{4\pi}{3} (Mc)^3$$

$$\frac{g_4 e M^2 c}{12\pi^{3/2} \hbar^2 \mu^{1/2}} \frac{1}{\sqrt{2}}$$

$$\frac{1}{\tau_\gamma} = \frac{2\pi}{\hbar} \left(\frac{e g_4 M^2 c}{12\pi^{3/2} \hbar^2 \mu^{1/2}} \right)^2 \frac{p^2}{2\pi^2 \hbar^3 v}$$

$$\frac{p^2}{v} \approx \mu^2 c$$

$$\frac{1}{\tau_\gamma} \approx \frac{e^2 g_4^2 M^4 c^2 \mu}{144 \pi^4 \hbar^8} = 5 \times 10^{18}$$

angular momentum? $e \frac{i}{\hbar} p z \rightarrow$

Decay into muons

Relationships between constants

$$P + \bar{\mu} \rightarrow N + \bar{\nu}$$

$$\frac{g_3}{\Omega} \rightarrow P + \pi^- + \bar{\nu}$$

$$\frac{\frac{e_3 \hbar c}{\sqrt{2\Omega\mu c^2}} \times \frac{e_2 \hbar c}{\sqrt{2\Omega\mu c^2}}}{(\mu - \mu_1)c^2}$$

$$g_3 = \frac{e_2 e_3 \hbar^2}{2\mu(\mu - \mu_1)c^2}$$

$$e_2 = 10^{-8}$$

$$e_3 = 1.2 \times 10^{-15}$$

$$g_3 \downarrow = 4.9 \times 10^{-49}$$

instead of 1.3×10^{-49}

Reverse same argument

$$e_3 = \frac{e_2 g_3 \hbar^2 c}{12\pi^2 \hbar^3}$$

$$= 8 \times 10^{-16}$$

instead of 12×10^{-16}

However: no $\pi \rightarrow e + \nu$

Yukawa theory of β -rays

E - Statistical approach

E-1

E-1. General method

Generalities

Transitions between states of equal energy

$$\dot{a}_2 = \frac{i}{\hbar} H_{21} a_1$$

$$\dot{a}_1 = \frac{i}{\hbar} H_{12} a_2 \quad \ddot{a}_2 = -\left(\frac{|H_{12}|}{\hbar}\right)^2 a_2$$

Condition for ^{perturbation} equipartition

$$\frac{\tau |H_{21}|}{\hbar} \ll 1$$

Cond. for equipartition

$$\frac{\tau |H_{21}|}{\hbar} \gg 1$$

Collision of two fast nucleons
Concentration in $\frac{\hbar}{\mu c}$ size and its
rapid dissipation.

Instantaneous and frozen equilibrium.

$$V = V_0 \frac{2Mc^2}{W}$$

$$V_0 = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} \left(\frac{\hbar}{\mu c}\right)^3$$

Statistical not ~~is~~ probability for a state that
all its ptcls are within V .

E-1'

For each ptcl factor $\frac{V}{\Omega}$ yields

Comment: large V
favors hi multiplicity.

Example: Classical particles

Ellipsoid axes

$$\left(\frac{1}{2}\right)! = \frac{\sqrt{\pi}}{2}$$

$$\frac{\pi^{3/2}}{\frac{3}{2}!} = \frac{\pi^{3/2}}{\frac{\sqrt{\pi}}{2} \cdot \frac{3}{2}} = \frac{4}{3}$$

$$S(n) = \frac{m_1 m_2 \dots m_n}{\left(\frac{3n}{2} - 1\right)!} (kT)^{\frac{3n}{2} - 1} \left(\frac{V}{4\pi^2 h^3}\right)^n$$

In extreme relativistic case

$$S(n) = \frac{V^n W^{3n-1}}{\pi^{2n} h^{3n} c^{3n} (3n-1)!}$$

$$\frac{W}{c} = |p_1| + |p_2| + \dots + |p_n| = w$$

$$Q_1 = \frac{4\pi}{3} w^3$$

$$Q_2 = \int_0^w 4\pi p_1^2 dp_1 \frac{4\pi}{3} (w - p_1)^3$$

$$K_1 = \frac{4\pi}{3} \quad K_2 = \frac{4\pi^2}{45}$$

$$= \frac{16\pi^2}{3} w^6 \int_0^1 (1-x)^3 x^2 dx$$

$$1-3x+3x^2-x^3 \quad \frac{1}{3} - \frac{3}{4} + \frac{3}{5} - \frac{1}{6}$$

$$Q_n = K_n w^{3n}$$

$$K_{n+1} w^{3n+3} = 4\pi \int_0^w p_1^2 dp_1 K_n (w - p_1)^{3n}$$

$$\frac{K_{n+1}}{K_n} = 4\pi \int_0^1 (1-x)^{3n} x^2 dx$$

$$\frac{20 - 45 + 36 + 10}{60}$$

$$= 4\pi \int_0^1 y^{3n} (1-2y+y^2) dy =$$

$$\frac{4}{3 \times 15}$$

$$= 4\pi \left(\frac{1}{3n+1} - \frac{2}{3n+2} + \frac{1}{3n+3} \right)$$

$$= 4\pi \frac{9n^2 + 15n + 6 + 9n^2 + 9n + 2 - 18n}{(3n+1)(3n+2)(3n+3)}$$

$$= \frac{8\pi}{(3n+1)(3n+2)(3n+3)}$$

$$\frac{(8\pi)^n}{(3n)!}$$

$$a \frac{8\pi}{6} = \frac{4\pi}{3}$$

$$Q(n) = \frac{(8\pi)^n}{(3n)!} \frac{W^{3n}}{c^{3n}}$$

$$\frac{n W^{n-1}}{c^n} \left(\frac{V^n}{\pi^2 h^3} \right)^n \frac{1}{(3n)!}$$

E-1''''

Mixed classical & extreme relativistic case

$$S(s, n) = \frac{M^{3s/2} V^{n+s}}{2^{\frac{3s}{2}} \pi^{2n+\frac{3s}{2}} \hbar^{3s+3n} c^{3n}} \frac{(W - s M c^2)^{3n+\frac{3s}{2}-1}}{(3n+\frac{3s}{2}-1)!}$$

Role of conservation theorems

Momentum conservation

In classical case

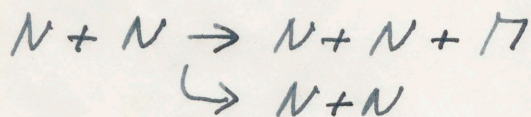
$$n \rightarrow n-1$$

$$m_1, m_2, \dots, m_n \rightarrow \frac{m_1 m_2 \dots m_n}{m_1 + m_2 + \dots + m_n}$$

Discuss also mixed case

E-2 Single pion production in nucleon collision

E-2



$$S_2 = \frac{\cancel{h} M^{3/2}}{4\pi^2 \hbar^3} \sqrt{T}$$

$$S_3 = \frac{V^2}{16\pi^3 \hbar^6} \left(\frac{M^2 \mu}{2M + \mu} \right)^{3/2} T_1^2 \approx \frac{V^2 M^{3/2} \mu^{3/2} T_1^2}{32\sqrt{2} \pi^3 \hbar^6}$$

$$\frac{S_3}{S_2} = \frac{\cancel{h} V \mu^{3/2}}{8\sqrt{2} \pi \hbar^3} \frac{(T - \mu c^2)^2}{\sqrt{T}} \approx \frac{V \mu (T - \mu c^2)^2}{8\sqrt{2} \pi \hbar^3 c}$$

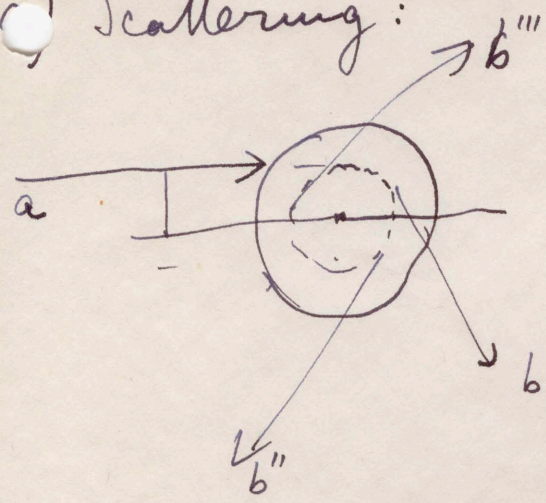
$$\frac{S_3}{S_2} = \frac{1}{6\sqrt{2}} \left(\frac{T}{\mu c^2} - 1 \right)^2 \frac{4\pi}{3} \left(\frac{\hbar}{\mu c} \right)^3$$

$$T = 165 \text{ MeV}$$

$$\frac{S_3}{S_2} = .004$$

Role of angular momentum conservation

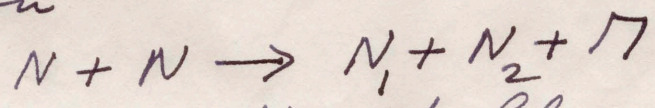
c) Scattering:



State a goes into states like b', b'', b''' with equal probability
Hence scattering $\approx d\theta dp$
Meridians $= \frac{1}{\sin \theta} d\omega$

Problem: Do same with partial waves

b) Reaction



barely above threshold

No spin

s-states

$$\sigma_{tot} = \pi \lambda^2$$

$$\frac{p^2}{2\mu} \approx \mu c^2 M$$

$$\lambda = \frac{h}{\sqrt{\mu M c}}$$

Competition: Scattering $r = N_1 - N_2$

$$\frac{1}{\sqrt{2\pi L}} \frac{\sin k r / p}{r}$$

Number $L = \pi \lambda n = \frac{\pi h n}{p}$
 $dn = \frac{L}{\pi h} dp$

$$\psi_0^2 R = \frac{R}{2\pi L}$$

$$\int_0^R \psi_0^2 dr = \frac{R}{L}$$

$$\frac{R}{L} dn = \frac{R dp}{\pi h} = \frac{R dE}{\pi h v}$$

Production

$$S_2 = \frac{R}{\pi h v} \approx \frac{R}{2\pi h c} \sqrt{\frac{M}{\mu}} = \frac{M}{4} v^2 = \mu c^2$$

$$v = 2c \sqrt{\frac{\mu}{M}}$$

$$S_3 = \frac{v^2 M^{3/2} \mu^{3/2} T^2}{32 \sqrt{2} \pi^3 h^4} =$$

$$\pi \lambda^2 \frac{S_3}{S_2} = \frac{\pi h^2}{\mu^2 c^2} \frac{1}{9 \sqrt{2}} \left(\frac{T}{\mu c^2} \right)^2$$

sol result $\frac{f^2 M^3 T^2}{64 \pi^2 h^7 v} \left(\frac{2\mu}{M} \right)^{3/2}$

Compare with old result

E-2''

$$\frac{f^2 M^3}{64 \pi^2 \hbar^7 v} \frac{T^2}{\left(\frac{1}{4} + \frac{M}{2\mu}\right)^{3/2}} \approx$$

$$\frac{M}{4} v^2 \approx \mu c^2 \quad v = 2c \sqrt{\frac{\mu}{M}}$$

$$\rightarrow \approx \frac{f^2 M^3}{64 \pi^2 \hbar^7} \frac{2^{3/2} \mu^{3/2}}{2c \sqrt{\frac{\mu}{M}}} \frac{T^2}{M^{3/2}} = \frac{f^2 M^2 \mu T^2}{32 \sqrt{2} \pi^2 \hbar^7 c}$$

Comparison yields

$$f = \frac{\pi^{1/2} \hbar}{\mu c} \frac{4^{1/2} 2^{1/2} \pi \hbar^{7/2} c^{1/2}}{3 \times 2^{1/2} \mu c^2 M \mu^{1/2}} = \frac{4 \sqrt{2} \pi^{3/2} \hbar^{9/2}}{3 M \mu^{5/2} c^{5/2}}$$

$$= 5.2 \times 10^{-62} \text{ erg} \times \text{cm}^{9/2}$$

$$f \approx 10^{-61}$$

$$6.5 \times 10^{-62}$$

Comments on ^{effect of} nuclear forces

E-3 . Hi energy collisions.

E-3

all particles extreme relativistic.

$$\frac{W_{\pi}}{V} = \frac{3}{2} \frac{6.494}{\pi^2 \hbar^3 c^3} (kT)^4$$

$$6.494 = 6 \sum_{n=1}^{\infty} \frac{1}{n^4} = \pi^4/15$$

$$\frac{W_N}{V} = \frac{8}{2} \frac{5.682}{\pi^2 \hbar^3 c^3} (kT)^4$$

$$5.682 = 6 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^4}$$

$$V = \frac{2 Mc^2}{W} V_0 \quad W = W_{\pi} + W_N$$

$$(kT)^4 = .152 \frac{\hbar^3 c^3}{Mc^2 V_0} W^2$$

Densities

$$n_{\pi} = .367 \frac{(kT)^3}{\hbar^3 c^3} \quad n_N = .855 \frac{(kT)^3}{\hbar^3 c^3}$$

Include factor .51 for conservation of ang. mom.

$$N_{\pi} = .091 \left(\frac{V_0 M W^2}{c \hbar^3} \right)^{1/4} = .54 \sqrt{W/Mc^2} \quad W \text{ in bary}$$

$$N_N = .21 \left(\quad \right) = 1.3 \sqrt{W/Mc^2} \quad \begin{array}{r} \frac{2}{3} \times .54 \quad .36 \\ .65 \\ \hline 1.01 \end{array}$$

$$N_{\text{charged}} = 1.2 (W'/Mc^2)^{1/4} \quad W' \text{ in lab sy}$$

Nucleons extreme relativistic only for $W' > 10^{13}$ eV
At lower energies

$$N_{\pi} = 1.34 \frac{(w-2)^{3/4}}{w^{1/4}}$$

$$w = \frac{W}{Mc^2} \approx \sqrt{\frac{2 W' 2}{2 Mc^2}}$$

E-4. Angular distribution

E-4

Experimental facts

Schein & Lord star - Assumption of two nucleon collision

$$W' = 30,000 \text{ BeV}$$

$$W \approx 280 \text{ MeV}^2$$

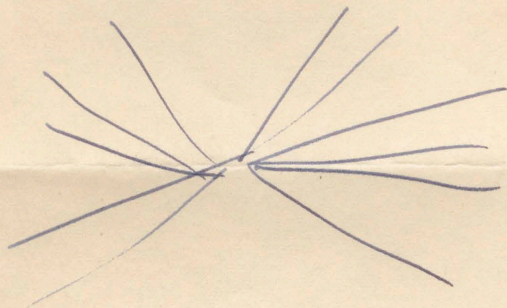
core $\approx .003$ radians

15 particles, increased to 25 for neutrals

Per particle $\approx 10 \text{ BeV}$

$$\lambda \approx 2 \times 10^{-15}$$

Distribution in barsy



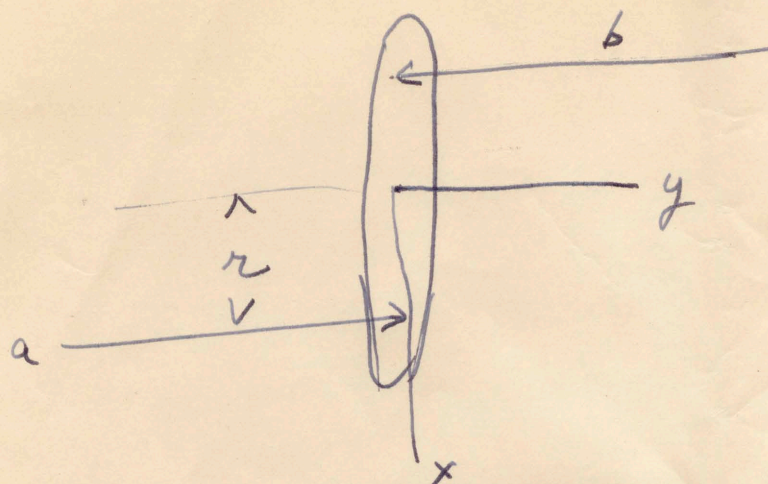
Role of angular momentum conservation
(Qualitative)

Statistics with conservation theorems

$$e^{-\beta w} \rightarrow e^{-\beta w + \lambda_0 Z + \mu S}$$

Determination of β , λ , S from total values of
 Σw , ΣZ , ΣS

Hence: no need to include ~~angular~~ momentum $\vec{L}$ and
vanishing components of angular momentum



$$Z = x p \cos \vartheta \quad \vartheta = \widehat{p y}$$

Distribution $\cos \vartheta = \eta$

$$\frac{1}{e^{\beta w - \lambda x p \cos \vartheta} - 1}$$

$$\frac{2 M c^2}{W} dx \pi (R^2 - x^2)$$

$$2 \pi \sin \vartheta d\vartheta p^2 dp =$$

$$= 2 \pi p^2 dp d\eta$$

$$\frac{1}{8 \pi^3 \hbar^3}$$

$$w = c p$$

$$dn = \frac{M c^2}{2 \pi \hbar^3 W} \frac{(R^2 - x^2) dx p^2 dp d\eta}{e^{\beta c p - \lambda x p \eta} - 1}$$

$\downarrow$
 $e^{\lambda p (1 - p \eta x / R)}$

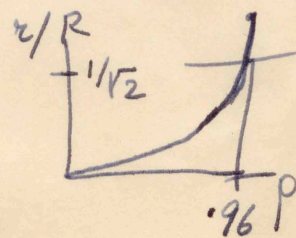
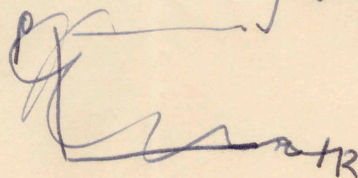
Compute

$$W = \int w dn$$

$$M = \int x p \eta dn = \frac{W \hbar}{c}$$

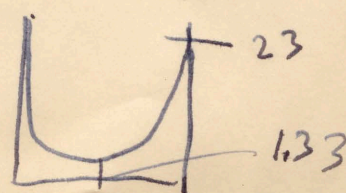
Determine x, p

Find eventually



$$\frac{dn}{d\eta} = \frac{a M c^2 R^3}{2 \pi W \hbar^3 \gamma^3} f_4(p \eta)$$

$$f_4(\alpha) = \frac{2}{\alpha^2(1-\alpha)} - \frac{1}{\alpha^3} \ln \frac{1+\alpha}{1-\alpha}$$

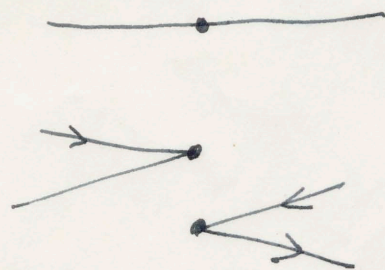


F. Feynman diagrams

F-1

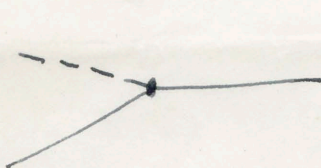
F-1 Feynman diagrams

Scattering: Electron in non quantized field

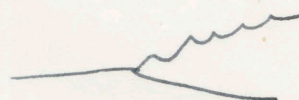
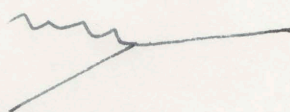


Comments on positron creation or annihilation

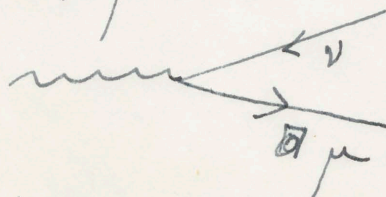
Electromagnetic interaction



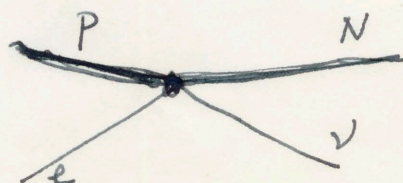
Yukawa

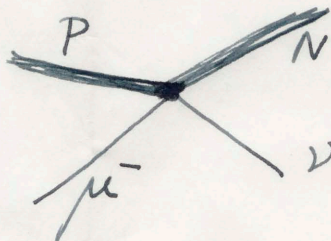
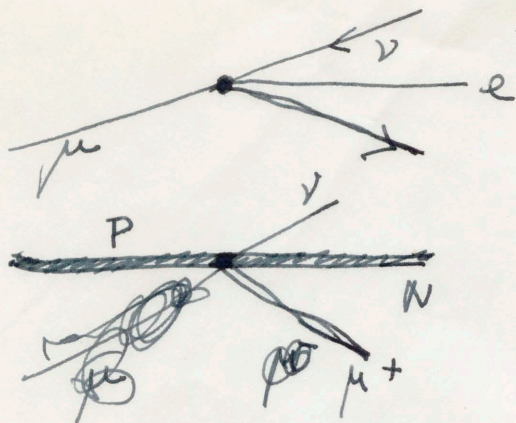


$\pi \rightarrow \mu + \bar{\nu}$



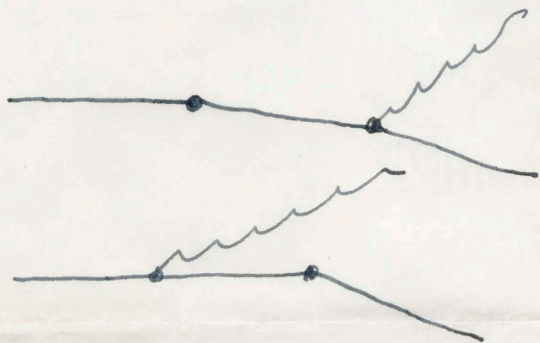
β interaction



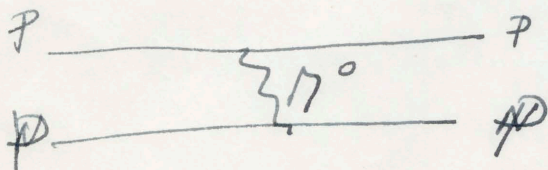
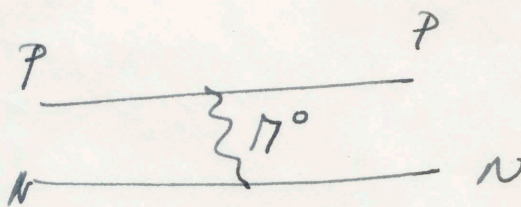
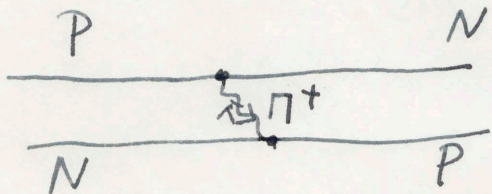


Continuity
of Pauli lines?

Emission of bremsstrahlung



Nuclear forces



Prod. of pions in nucleon collisions

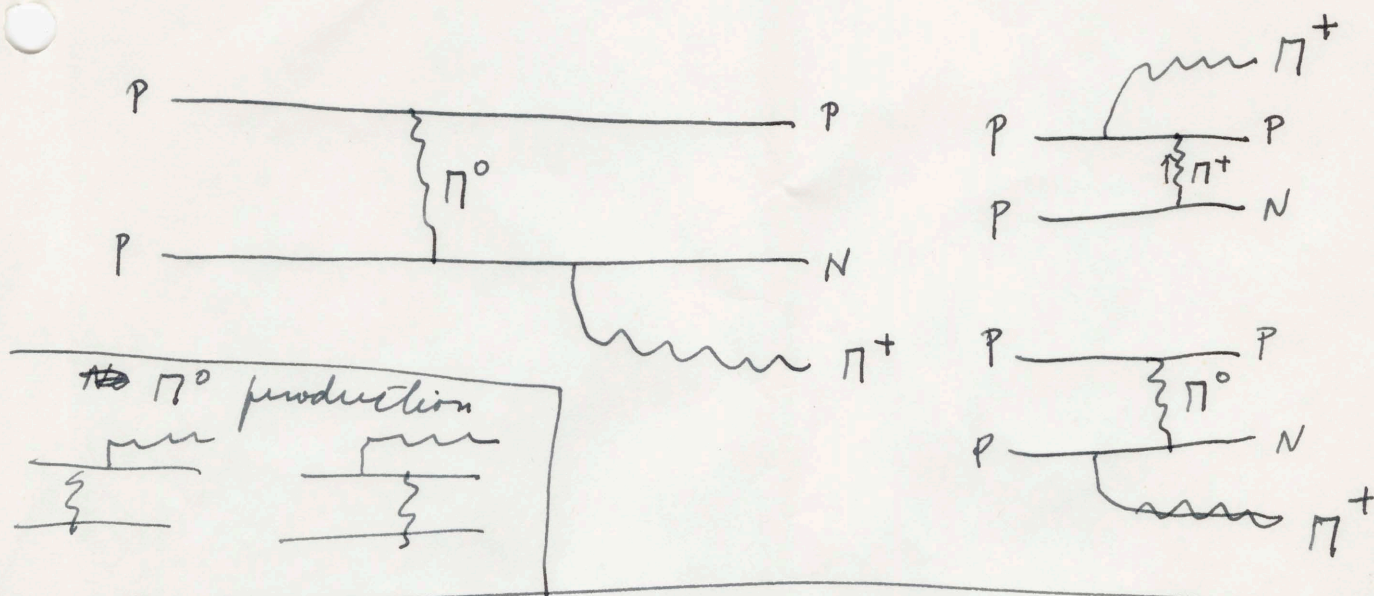
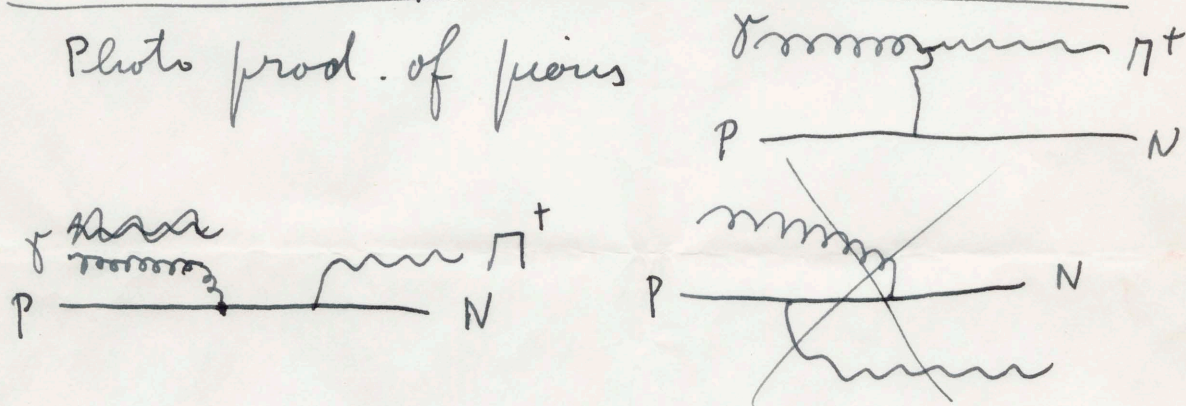
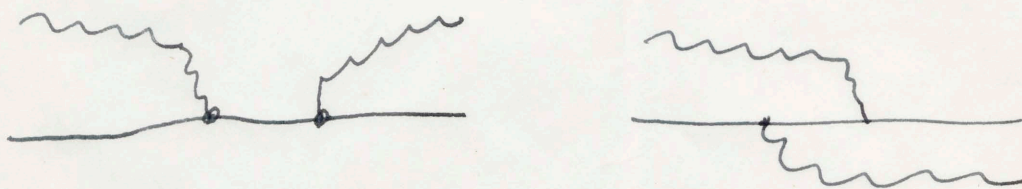


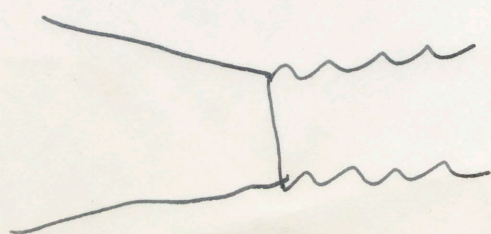
Photo prod. of pions



Scattering of pions by nucleons



Annihilation of nucleon antinucleon



decay of neutral pion



$$\gamma_4 = \beta \quad \gamma_1 = \beta \alpha_x$$

$$\gamma_4^2 = 1 \quad \gamma_1^2 = -1 \quad \gamma_5^2 = -1$$

$$\gamma_5 = \gamma_{1234}$$

$$a_\mu b_\mu = a_4 b_4 - a_1 b_1 - \dots$$

$$A = A_\mu \gamma_\mu$$

~~Let~~ N be a prod. of γ 's

$N^T = (\text{conj of coeff}) \times \text{prod in opposite order}$

$$\gamma_\mu^T = \gamma_\mu$$

$$q \not{b} + \not{b} q = 2 a_\mu b_\mu$$

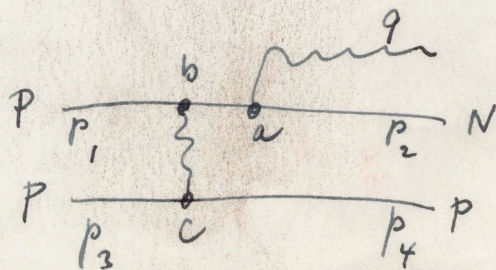
$$e = \sqrt{\frac{4\pi}{137}} = .304 \quad \hbar = c = 1$$

$$\text{Rate} = \frac{2\pi}{\pi(2E_i)} |M|^2 \frac{dN}{dE}$$

Normalize Pauli's

$$\psi^\dagger \psi = 2m$$

Then $\sum_{\text{spin}} \psi \psi^\dagger = \not{p} + m$



a) $e' \gamma_5 q$

(ab) $\frac{1}{\not{p}_2 + \not{q} - m}$

b) $e'' \gamma_5 (\not{p}_4 - \not{p}_3)$

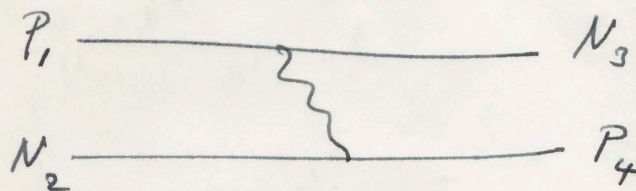
emission

c) $-e'' \gamma_5 (\not{p}_4 - \not{p}_3)$

absorption

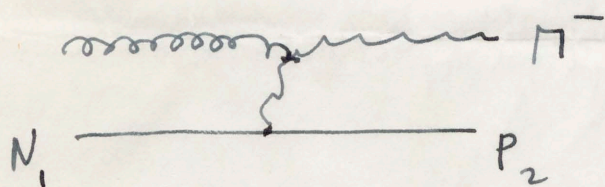
bc) $\frac{1}{(\not{p}_3 - \not{p}_4)^2 - \mu^2}$

Nuclear forces



$$\frac{e^2 N_3^+ \gamma_5 P_1 P_4^+ \gamma_5 N_2}{(p_4 - p_2)^2 - \mu^2}$$

Photoproduction



$$e^2 \frac{P^+ \gamma_5 N}{(p_2 - p_1)^2 - \mu^2} e \epsilon_\mu (p_1 - p_2 + q)_\mu$$