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THE SIGNIFICANCE OF THE MATHE- MATICAL ELEMENT IN THE PHILOSOPHY OF PLATO

A DISSERTATION

SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL OF ARTS
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INTRODUCTION.

PLATO took a deep interest in mathematics ; philosophy was his passion. These two interests, at first thought disparate, came into a relation of thoroughgoing intellectual interaction. Plato's mathematical studies had a different motive, aspect, and outcome from the fact that he was primarily a philosopher ; his philosophy had a different *quale*, from the fact that he was a devotee of mathematics.

It was significant for the progress of mathematics that when Plato turned his attention toward this science he looked with the eyes of a philosopher. Hence I shall discuss what it was that his philosophic insight saw in mathematics to attract him, and in what way the philosophic attitude of mind which he brought to bear on the study of this subject served to further the progress of the science.

On the other hand, the interaction of the mathematical and the philosophic elements was an important factor in the development of Plato's philosophic system. The main part of this book will be given up to the task of showing the influence of mathematics upon the formulation of philosophic problems, in the determination of method, and as affecting the content of philosophy.

In the first chapter I have put the mathematical element in the foreground with special reference to showing the significance to mathematics of the philosophical element. In the remaining chapters I have put the philosophical element into the foreground and have sought to show the influence upon it of the mathematical element. This has involved a duplication in the treatment of certain topics and considerable cross-reference at certain points. This element of repetition might have been avoided by a unification of treatment under the lead of the philosophical aspect, with the mathematical as incidental and subsidiary. But I have thought that the advantages of giving the mathematical element a more unified discussion on its own account counterbalanced the disadvantages from the other point of view.

No attempt has been made to deal with the so-called number theory of the Pythagoreans, into harmony with which it is sometimes said that Plato cast his philosophy later in life. The authority for setting up this relationship between mathematics and Plato's

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philosophy is very problematic, to say the most. It finds very little, if any, support in Plato's own writings. Again, the reader who is looking for solutions of the mathematical puzzles to be found in the dialogues will look in vain, except as some of these puzzles may find rational explanation from the point of view which is developed in this book—a point of view which is concerned with the movement of thought, and hence views the introduction of mathematical ideas, not alone from the side of their intrinsic character or worth, but primarily with reference to their bearing upon the philosophical problems in relation to which they stand.

CHAPTER I.

PLATO'S GENERAL ATTITUDE TOWARD MATHEMATICS.

THE dialogues of Plato abound in allusions and references to mathematics. It is not difficult to see that he is a great admirer of the mathematical sciences and has a keen appreciation of their value. Let us take up a little evidence of a general order before proceeding to details.

Mathematical study fascinates Plato by reason of its "charm."¹ It is through this quality that solid geometry is enabled to make progress, even though it is as yet undeveloped, generally unappreciated, and poorly taught.² Arithmetic is declared to have a "great and elevating effect."³ It is a "kind of knowledge in which the best natures should be trained,"⁴ being an essential to manhood.⁵ This latter conception of the value of mathematics is asserted very strongly in what is probably the very latest of Plato's dialogues — the *Laws*. There he argues that "ignorance of what is necessary for mankind and what is the proof is disgraceful to everyone." Some degree of mathematical knowledge is "necessary for him who is to be reckoned a god, demigod, or hero, or to him who intends to know anything about the highest kinds of knowledge."⁶ "To be ignorant of the elementary applications of mathematics is ludicrous and disgraceful, more characteristic of pigs than of men."⁷

In such high terms Plato expresses his appreciation and admiration of the mathematical sciences. Further and more detailed investigation will show more specifically the nature of his attitude toward this subject and the grounds upon which it rests. His estimate of the value of mathematical study grows out of a philosophical attitude of mind rather than a practical one. What his attitude toward utility was will be taken up in detail later. Suffice it to say here that his main interest in mathematics centered in its qualities, characteristics, the mental processes and methods involved, the possibilities which he saw in it of scientific procedure, and the suggestions and analogies which it furnished him in the field of philosophic processes, methods, and results.

¹ *Rep.*, 7 : 525.

² *Rep.*, 7 : 525.

³ *Rep.*, 7 : 522.

⁴ *Rep.*, 7 : 528.

⁵ *Rep.*, 7 : 526.

⁶ *Laws*, 7 : 818.

⁷ *Laws*, 7 : 819.

Of the qualities and characteristics of mathematical study which Plato regarded as valuable, one of the most important is its general disciplinary value. Anyone interested so much as he was in the cultivation of the reasoning processes could not help seeing the possibilities of mathematics in this respect, even fragmentary as the science was in his day. He observes that mathematical training makes one, even though otherwise dull, much quicker of apprehension in all other departments of knowledge than one who has not received such training.⁸ So much is he impressed with this fact that when he has once made the point in his discussion of arithmetic,⁹ he repeats it in his discussion of geometry.¹⁰ It is on account of the training which mathematics gives in the power of abstraction and in reasoning processes, aside from its idealistic tendency (to be discussed later), that Plato makes mathematical study a propædæutic to philosophy.¹¹

Though all sciences in the time of Plato were in a more or less embryonic stage of development, mathematics among them, yet this subject, by reason of the comparative simplicity of its elements, had advanced farther than the rest and stood out as rather conspicuous for the clearness of its procedure and the certainty of its results. Such a fact as this is of more interest to Plato than any utilitarian value that may arise from the exactness of mathematics. He has a philosophic appreciation of the fact that the arts which involve arithmetic and the kindred arts of weighing and measuring are the most exact, and of these those "arts or sciences which are animated by the pure philosophic impulse [*i. e.*, theoretical or pure mathematics] are infinitely superior in accuracy and truth."¹² The reason for this clearness and certainty was felt to lie in three important features: (1) the intuitive element in mathematics, (2) its more correct conception of definition, and (3) its method of procedure. As these points come up for further discussion later on in another relation, only the briefest elaboration of them will be undertaken here.

That Plato was impressed by the intuitive element in mathematics is certain from the reference in the *Meno*, if we had no other. When he wants an illustration of his doctrine of knowledge as recognition of that which was perceived in a state of being ante-

⁸ *Cf. Laws*, 5: 747.

⁹ *Rep.*, 7: 526.

¹⁰ *Rep.*, 7: 527.

¹¹ *Rep.*, 7: 521-33; see especially 533.

¹² *Phileb.*, 55-57.

cedent to this life, he turns to mathematics. The slave boy in the *Meno* is made to go through a demonstration in geometry where "without teaching," but by a process of questioning, he recovers his knowledge for himself.¹³ Whether Plato felt the full force of its significance or not, what he really brought out in this practical illustration was the intuitive element in mathematics. Is it unreasonable to think that it was this intuitive element in mathematics which either created or was a factor in creating the philosophical problem the solution of which Plato sought in his doctrine of recollection?

Mathematics had, to a higher degree than other subjects, also attained to a correct conception of definition. That one of the reasons for Plato's appreciation of mathematics is to be found in this fact is shown by the frequency with which he draws upon it for illustrations of what is requisite to a good definition. In the *Theætetus* the definitions of square numbers, oblongs, and roots are used to show that enumeration is inadequate as a principle of definition, and that definitions must be couched in general terms and must set off a class in accordance with a principle of logical division.¹⁴ In the *Gorgias*, rhetoric has been defined by one of the speakers as an art which is concerned with discourse. The looseness of this definition is immediately noted, and it is pointed out that rhetoric has not been defined in such a way as to distinguish it from all the other arts; for they, too, are concerned with discourse. To make the matter clear, an illustration is given from the sphere of mathematics. If arithmetic be defined as one of those arts which take effect through words, so also is calculation. Where, then, is the distinction? A difference must be pointed out—the difference being that the art of calculation considers not only the quantities of odd and even numbers, but also their numerical relations to one another.¹⁵

To the sort of certainty and clearness which comes from the intuitive element and from careful definition in mathematics Plato recognizes that there is to be added that which arises from the method of procedure.

Here is a science in which they distrust and shun all argument from probabilities.¹⁶ "The mathematician who argued from probabilities and likelihoods in geometry would not be worth an ace."¹⁷

There are hints that Plato was especially interested in mathematics for its suggestiveness in respect to a particular method of

¹³ *Meno*, 81-86.

¹⁵ *Gorg.*, 450-51.

¹⁴ *Theæt.*, 147-48.

¹⁶ *Phædo*, 92.

¹⁷ *Theæt.*, 162-63.

procedure — the method of analysis. There is evidence that he paid special attention to this method and developed it to a high degree. By tradition he is credited with being its inventor. In the *Meno* he suggests that it may be applied outside of the field of mathematics.

In arguing the question whether virtue can be taught, a hypothesis will be assumed as in geometry, and consequences deduced from it. If these consequences are contradictory to known facts, the hypothesis is rejected; if consistent with them, it is accepted.¹⁸

It is not to be wondered at that a man of philosophic temperament should have been struck with the beauty of mathematical procedure. At a time when fields of investigation had not been minutely specialized, when methods of scientific procedure were in the embryonic stage of development, here was a science which had something, at least, of a technique of its own. Starting with intuitive data of undoubted clearness and with concepts unambiguously defined, proceeding by methods which guarded at every step against error, that certainty of result might be achieved which stood in striking contrast to the vague probabilities of other sciences.

Closely connected with the qualities of clearness and certainty in mathematics are those of necessity and universality. These also are noticed by Plato and made a strong impression upon him.

In speaking of arithmetic, he says that "this knowledge may truly be called necessary, necessitating as it does the use of the pure intelligence in the attainment of the pure truth."¹⁹ This passage, however, is not conclusive. But in the *Laws* he points out with reference to mathematical subjects that "there is something in them that is necessary and cannot be set aside;" and he adds that "probably he who made the proverb about God had this in mind when he said, 'Not even God himself can fight against necessity.'"²⁰ In the *Theætetus* arithmetical notions are classed among universal notions,²¹ and in his scheme of education for the guardian class, described in the *Republic*, he makes it of great importance that attention be given to "that which is of universal application — a something which all arts and sciences and intelligences use in common — number and calculation — of which all arts and sciences necessarily partake."²²

Plato's attitude with reference to the utility of mathematics is an interesting study. In general he deprecates the demand for utility — at least in so far as utility (in the practical sense) is to be taken as

¹⁸ *Meno*, 86-87.

²⁰ *Laws*, 7: 818.

¹⁹ *Rep.*, 7: 526.

²¹ *Theæt.*, 185.

²² *Rep.*, 7: 522.

the real ground of the value of the subject. He makes its value rest chiefly on other grounds. He takes the philosophic points of view; he is always in the critical, reflective attitude of mind, or, at least, that attitude dominates over all others. There is abundant evidence of this.

He sneers at that class of people who will consider his words as "idle tales because they see no sort of profit that is to be obtained from them."²³ The kind of knowledge in which the guardians of his ideal state is to be trained is not to be found in the *useful* arts, which (from the educational point of view that he has in mind) are reckoned *mean*.²⁴ But they are to receive (among other things) a thorough training in mathematics. To this end, their arithmetic they are to learn "not as amateurs, nor primarily for its utility, nor like merchants or retail dealers, with a view to buying and selling." "Arithmetic, if pursued in the spirit of a philosopher, and not a shopkeeper," he regards as a charming science and one that is educationally advantageous.²⁵ From this point of view the purer and more abstract the mathematics the better. He accuses the mathematicians themselves of not being altogether free from the tendency to look upon their science too largely from the practical side. He scores them for "speaking in their ordinary language as if they had in view practice only." They "are always speaking in a narrow and ridiculous manner of squaring and extending and applying and the like—they confuse the necessities of geometry with those of daily life; whereas knowledge is the real object of the whole science."²⁶

While Plato decries the insistent demand for utility and maintains that there are higher values to be realized apart from the utilitarian standard, yet he does not fail to see the useful and significant place of mathematics both in the ordinary walks of life and also in relation to the career of the warrior. This twofold practical significance of mathematical study is appreciatively brought out in his advocacy of the teaching of children after the Egyptian fashion by means of mathematical games:

This makes more intelligible to them the arrangements and movements of armies and expeditions; and in the management of a household, mathematics makes people more useful to themselves, and more wide awake; and again in the measurement of things which have length and breadth and depth they free us from the natural ignorance of all these things which is so ludicrous and disgraceful.²⁷

²³ *Rep.*, 7: 527.

²⁵ *Rep.*, 7: 525.

²⁴ *Rep.*, 7: 522.

²⁶ *Rep.*, 7: 527.

²⁷ *Laws*, 7: 819.

Let us now take up these two points separately, beginning on the side of civic life. The practical importance of mathematics to the arts is pointed out. It is by reason of the mathematical element that they are enabled to rest upon a more secure basis than empiricism. Plato clearly sees that measure, the objective application of the principle of quantity, lies at the very foundation of all fruitful technical procedure.

The arts are said to be dependent upon mathematics; "all arts and sciences necessarily partake of them."²⁸ "If arithmetic, mensuration, and weighing be taken away from any art, that which remains will only be conjecture and the better use of the senses which is given by experience and practice, in addition to a certain power of guessing, which is commonly called art, and is perfected by attention and pains."²⁹

In other words, he might have said that all arts are nothing but "cut and try" methods until application of mathematics has been made to them.

In the *Republic* a great deal is made of the fact that mathematics is of practical value to the military man.

The art of war, Plato urges, like all other arts, partakes of mathematics.³⁰ The principal men of the state must be persuaded to learn arithmetic for the sake of its military use.³¹ The warrior should have a knowledge of this subject, if he is to have the smallest understanding of military tactics.³² "He must learn the art of number or he will not know how to array his troops."³³ While Plato views knowledge as the real object of the whole science of geometry, as over against its practical value, yet he includes among "its indirect effects, which are not small, the military advantages arising from its study."³⁴ In the scheme of education for the guardians, he says that "we are concerned with that part of geometry which relates to war; for in pitching a camp, or taking up a position, or closing or extending the lines of an army, or any other military manœuvre, whether in actual battle or on the march, it will make all the difference whether a general is or is not a geometrician."³⁵

It is sufficiently proved that it is not from any lack of understanding or appreciation of the practical value of mathematics that Plato decries the study of the subject for the sake of its utilitarian

²⁸ *Rep.*, 7: 522.

²⁹ *Rep.*, 7: 522.

³⁰ *Rep.*, 7: 522.

³¹ *Rep.*, 7: 527.

³² *Philebus*, 55.

³³ *Rep.*, 7: 525.

³⁴ *Rep.*, 7: 525.

³⁵ *Rep.*, 7: 526.

value. He does it in order to throw the emphasis where he thinks it more truly belongs. He would not have the higher value ignored for the merely practical, which he regards as of less worth. We might say that with him the value of theoretical mathematics is primary and fundamental, that of practical mathematics secondary, incidental, and to be taken for granted. The practical value of mathematics is something which he points out by the way, in passing; while theoretical mathematics makes an appeal to his deepest intellectual needs. One reason for his exalting this theoretical study is certainly to be found in his conception of the nature of knowledge and of being. The discussion of that will come later; we are concerned here more especially with the fact—with his attitude toward the subject of mathematics. In this connection there is another important point yet to be made.

It was in connection with the theoretical study of mathematics that Plato saw the possibility of scientific procedure, which was lacking in the empirical or merely practical. From this point of view we find him insisting on a sharp line of distinction between the scientific and the practical, the philosophic and the popular, the pure and the impure in mathematics. He is interested in the pure, philosophic, or theoretical *because it can be scientific*. This fundamental distinction comes out over and over again in Plato's writings.

Knowledge, he says, is divided into educational and productive, the latter into pure and impure.³⁶ Sciences in general are divided into practical and purely intellectual.³⁷ Arithmetic in particular is of two kinds, one of which is popular and the other philosophical.³⁸ As an illustration, we may take the distinction between arithmetic [scientific] and calculation [popular]. Arithmetic treats of odd and even numbers [*i. e.*, properties]; calculation, not only the quantities of odd and even numbers, but also their numerical relations to one another [*i. e.*, utilitarian values].³⁹ Philosophical mathematics demands more careful discriminations than popular mathematics; quantities which are incommensurable, for example, must not be confused with those which are commensurable; their natures in relation to each other should be carefully distinguished.⁴⁰ Another illustration of what is meant by the scientific study of mathematics as distinct from the practical or popular may be found in Plato's account of the properties of the number 5,040, this number

³⁶ *Philebus*, 55 ff.

³⁸ *Philebus*, 56.

³⁷ *Statesman*, 258.

³⁹ *Gorgias*, 451.

⁴⁰ *Laws*, 7: 819-20.

being prescribed in the *Laws* as the proper number of citizens for a city. The number 5,040 has the property of being divisible by fifty-nine different integral numbers, and ten of these divisors proceed without interval from one to ten.⁴¹

Investigations into the properties of numbers yielding such striking results as the one just cited must have profoundly impressed the minds of primitive thinkers. This may be at the basis of a great deal of the mysticism of Pythagoreanism. Plato advocates most strenuously the scientific study of mathematics—the study of the nature and properties of numbers and figures—that in mathematics which is exact, unchanging, absolute. Of the mathematical arts and sciences he maintains that those which are animated by the pure philosophic impulse are infinitely superior in accuracy and truth.⁴² He would make it necessary, however, for mathematical studies to be gone through with scientifically by a few only.⁴³

In proportion as Plato admired the qualities and characteristics of mathematics and its possibilities in the way of achievement, through careful and definite methods of procedure, of certainty and universality; in that same proportion he also deplored the amount of ignorance of mathematical subjects that prevailed among the Greeks. Even the mathematicians themselves, he thinks, lack the full appreciation of the value of them when pursued in a thoroughly scientific manner. But he recognizes that mathematics is a difficult study.

In speaking of arithmetic, he remarks that “you will not easily find a more difficult study and not many as difficult.”⁴⁴ The difficulty of mathematics, the demand of mental rigor which it makes when pursued scientifically, may account for the ignorance of the subject which he characterizes as “habitual.”⁴⁵ One of the chief characters in the *Laws* is represented as hearing with amazement of the Greek ignorance of mathematics and is “ashamed of all the Hellenes.”⁴⁶ They are so inaccurate that they are accustomed to regard all quantities as commensurable, being ignorant of incommensurables—a sort of knowledge not to know which is disgraceful. Also they are ignorant of the nature of these two classes of quantities in their relation to one another.⁴⁷

Three particular lines of investigation are pointed out where little

⁴¹ *Laws*, 5:737-38, 745-47; 6:771; cf. 6:756.

⁴² *Philebus*, 57.

⁴⁴ *Rep.*, 7:526.

⁴⁶ *Laws*, 7:819.

⁴³ *Laws*, 7:817, end.

⁴⁵ *Laws*, 7:818.

⁴⁷ *Laws*, 7:819-20.

really scientific work in mathematics has yet been done. Little seems to be known about solid geometry; no director can be found for it, and none of its votaries can tell its use. The subject is declared to be in a "ludicrous state."⁴⁸ Secondly, the mathematical study of the heavens is a work, he says, infinitely beyond our present astronomers;⁴⁹ and thirdly, in the study of harmony, even by the Pythagoreans, the procedure is not mathematical enough, for problems are not attained to.⁵⁰ All of these subjects, Plato feels, are as yet too empirical.

The philosophic point of view here as well as elsewhere dominated his attitude. He lent the whole weight of his influence to the development of these subjects along theoretical, scientific lines.⁵¹

Judged by the standard of original solutions, doubtless it is a correct estimate of Plato to say that he was not a mathematician, yet he has a positive contribution to make, and that too of a character which ought to rank with the extension of the science by means of original solutions. This contribution was made through the reaction of his *philosophic* insight upon the technique of mathematics. The critical faculty of the philosopher was very much needed just at that time in this field of investigation. We must remember that both arithmetic and geometry were in a very fragmentary condition. It was before the time of Euclid's *Elements*. Mathematics could not with propriety be said to be *organized*. It was still decidedly crude. Some difficult and very complex problems had been solved, to be sure. This is rather a basis for admiration of Greek genius and the intellectual power of some few individual mathematicians than for inference as to the high development of mathematical science. Mathematics, and Plato felt this, though the most exact and consistent of any body of knowledge, yet was scarcely worthy of the name of science, so much was it a body of empirical results and *dissecta membra*.

The progress of mathematics does not consist alone in lines of investigation which lead to new solutions of problems. These themselves depend upon modes of procedure. These modes of procedure are at first not differentiated from the solutions in which they occur, they are not generalized. Each problem has, as it were, an independent character—its solution is particular and peculiar to itself. Reflection upon the process reveals general principles and leads to

⁴⁸ *Rep.*, 7: 528.

⁵⁰ *Rep.*, 7: 530-31.

⁴⁹ *Rep.*, 7: 530.

⁵¹ See note at end of this chapter.

the formulation of method. This is the work essentially of the philosophic mind. It is just here that the greatest significance of Plato to mathematical science comes in. To a genuine interest in and familiarity with mathematics he added the philosophic interest just at the time when mathematics had progressed to that stage of development in which the next step necessary to further progress was the analysis of its concepts and processes and the formulation of its technique. This technique, when developed, could be directed back by the specialist upon the great unsolved problems with an added power which enabled him to secure further and more striking results in his field of original mathematical investigation. In this way the philosopher equally with the mathematician becomes a contributor to the advance of mathematical science, and it is difficult to determine which of the two is more truly the mathematician. Certainly it was Plato's philosophic temper of mind that made him "the maker of mathematicians."

The "Eudemian Summary" states that "Pythagoras changed the study of geometry into the form of a liberal education; for he examined its principles to the bottom, and investigated its theorems in an immaterial and intellectual manner."⁵² Even if we can rely upon this statement as authoritative, still it is true that there remained a great work to do in the way of putting mathematics upon a thoroughly scientific basis. Even with the Pythagoreans there remained much of the mystical element, which drew attention away from the natural fields of mathematical investigation and was a hindrance to legitimate scientific development. Outside of Pythagoreanism rational and empirical results were apt to be very loosely discriminated, and to the empirical result there was attached a blind and unjustifiable worth. This may be illustrated by the old Egyptian method of finding the area of an isosceles triangle, which, among other rules drawn from the Ahmes papyrus, passed current in Greece. According to this rule, the area of the isosceles triangle was found by taking one-half the product of the base and one of the equal sides. Of course this would be an exaggeration of the condition of affairs as it existed in the time of Plato. But we may judge from the scoring which he gives empirical methods that instances of procedure of this sort were still frequent enough.

Now, Plato was especially enthusiastic over the scientific possibilities of mathematics. From this point of view definition was of

⁵² *Gow*, p. 150.

great importance. Plato had learned from Socrates the importance of analyzing and defining concepts in ethics. He applied the principle to mathematical science, insisting upon the most careful investigation of its fundamental concepts, resulting in a more rigid and precise formulation of its definitions and axioms. Whether Plato actually completed any considerable amount of this work or not, there can be little doubt that his influence in the matter was a decisive factor in that reconstruction of geometry which soon culminated in Euclid's *Elements* — a formulation so exact and comprehensive that for many centuries it remained the text-book of the civilized world and is able still to infuse its spirit into every modern school text in geometry.

To this result Plato contributed largely in another important respect. Noting the possibilities of exactness, rigidity, and necessary conclusions in mathematical procedure and reflecting upon and universalizing its processes, "he turned the *instinctive logic* of the early geometers into a *method to be used consciously* and without misgiving."⁵³ It is worthy of note that Plato seems to be equally, if not more, interested in methods than in results. One cannot read carefully the demonstration with the slave boy in the *Meno*⁵⁴ without noticing this fact. Plato is intensely interested in the reasoning process. This point will be emphasized again in another relation (see p. 47). Moreover, in this passage in the *Meno* he points out the mathematicians' use of hypothesis, which is none other than the method of analysis. It will later be shown (see p. 57) how fully conscious of the essential elements of this method Plato became. Whether the invention of the method be attributed to Plato or not, there is little doubt that the tradition which ascribes it to him rests upon the fact that he successfully developed and used the method as a powerful instrument of investigation.

The tendency of all this improvement in the direction of rigor of definition, careful sifting and clear statement of postulates, analysis and generalization of process, and formulation of logical methods, was to give mathematics a technique and make it more scientific. In this same direction tended the determination and limitation of fields of investigation. Problems in geometry were limited to those capable of construction by ruler and compass. The study of solid geometry was encouraged.⁵⁵ Astronomy and also harmony were to be made mathematical in character.⁵⁶ The reaction of philosophy upon

⁵³ *Gow*, p. 175.⁵⁴ *Meno*, 81-86, 86-87.⁵⁵ *Rep.*, 7: 528.⁵⁶ *Rep.*, 7: 530-31.

mathematics in Plato was certainly an important factor in making this subject scientific in character. In Plato you find no patience with empirical methods and empirical results. Nor do you find in him any but the slightest traces of a tendency to make a mystical use of mathematics.⁵⁷ His demands are over and over again for the theoretical, pure, and scientific as against the practical, popular, and empirical. He scores the mathematicians themselves for not being scientific enough, the students of astronomy and harmony for not being mathematical enough. Whether he was a mathematician himself in the ordinary sense of the word, or not, he certainly made a contribution to the subject of mathematics from the philosophical point of view and set the ideal which mathematicians had henceforth to follow in the pursuit of their science.

The writer originally worked out in considerable detail the question of Plato's relation to the mathematicians of his time and the extent and character of his influence upon the progress of mathematics. But this ground has been so thoroughly covered by the great historians of mathematics that he has thought best to give only a brief general statement of the significance of Plato to mathematics from the philosophical point of view. For further details as to mathematics proper the reader is referred to the bibliography at the end of the book.

⁵⁷ For these instances see *Rep.*, 8: 546; *Timæus*, 35-36, 38 ff., 43, 53 ff. These may be less mystical than they appear to be. See p. 40 of this book.

CHAPTER II.

THE FORMULATION OF PHILOSOPHICAL PROBLEMS.

THE philosophy of Plato grows out of a highly complex situation involving many mutually interacting factors both personal and environmental. In analyzing out a few of the most significant and determining strands of his thought, it is not necessary to assume that all of them thus analyzed out were consciously determining in the mind of Plato himself. Quite commonly the most fundamental factors in a man's thought are so much a part of his whole attitude and integral mode of reaction that he is entirely unaware of them as determining in his mental processes. Yet another, viewing them from the outside, may clearly see, interpret, and point out their psychological and logical bearing. Whatever attitude the reader may take with reference to the point of view running through this book, the character of the work as an attempt to analyze *after the fact* must not be overlooked. Isolation of parts for the sake of getting their bearing and seeing their significance gives both organization and emphasis which did not belong necessarily to the work of Plato as he conceived it himself.

The problems of no great thinker arise in his consciousness *ab externo* and *ex abrupto*; they have some connection with his immediate environment, social and intellectual. A period of reaction to or against the philosophic ideas of others naturally precedes definite and conscious formulation of one's own. Such reaction is both the stimulus to initiation and the condition for progress. The discussions of the dialogues show that Plato familiarized himself with all the leading historical and contemporary philosophical systems that found currency in Greece. The details of these systems are not up for our consideration here; a certain familiarity with them will have to be assumed. We can touch only upon certain characteristic concepts as they affect the understanding of our special problem.

The speculations of the earlier philosophers had resulted in fixing attention upon certain great limiting concepts. Especially did the great opposing attitudes of the Eleatics and the Heracliteans dominate thought in such a masterful fashion that no serious and far-reaching reflection was possible without taking into account the

problems involved in the antitheses of being and becoming, of the one and the many, of permanence and change, of essence and genesis, of sensation and thought, of opinion and knowledge, of appearance and truth. At first interest centered largely in the external and objective world, the problems were those of cosmology and ontology. The problems of man—questions regarding the soul, the mental processes, human activities and conduct—were incidental. When discussed, the tendency was to treat them from the point of view of man as a part of the cosmos. They were taken up from the same objective point of view which dominated the nature-philosophy. The *raison d'être* of interest in these problems seems to have been very largely that without reference to them the cosmological account would have been incomplete. This was as true of atomism and other mediating systems as of Eleaticism and Heracliteanism.

The profound social and political disturbance incident to the Persian wars disrupted the routine of the old Greek life and shifted the center of attention and of interest from cosmology to human life. The significance of man was brought to consciousness—his achievements, his powers. The growing importance and scope of the political activity carried in its train a great stimulus to the study of rhetoric and eloquence. Problems of human mind and of human conduct were brought to the focus of attention. Quite naturally, with the rise of a new set of problems, the intellectual tools forged in dealing with the old questions were tried upon the new ones. Points of view, fundamental distinctions, working concepts characteristic of the departing age, were drawn upon in the attempt to define and solve the problems of the new era.

So far as the particular Platonic problem of this book is concerned, the first movement along the new line to demand our attention is that which has come to be quite ambiguously associated with the name "Sophists." What I have in mind is the philosophy of relativity, by whatever name called, or with whatever individual associated in thought—the "flowing philosophy," as Professor Shorey¹ has quite aptly styled it—an outgrowth of the Heraclitean doctrine of "flux" and the sensationalistic psychology of Protagoras. I shall hereafter refer to this type of philosophy as Protagoreanism.

Protagoras applied the Heraclitean principle of motion to the analysis and explanation of perception. The result was a thorough-going doctrine of the subjectivity and relativity of sense-perception.

¹ Unity of Plato's Thought.

Man has already been identified with a particular phase of the cosmos. The principle that had explained the universe Protagoras extends more fully than his predecessors to the explanation of man. World-process and mental process are identified throughout the whole of man's mental life. Sensation and thought, opinion and knowledge, are continuous phases of one world-process, the resultants of the interactions of continually shifting motions. Knowledge is perception; the relativity of perception is the relativity of knowledge. In bringing to consciousness the principle of subjectivity and in viewing the psychical life from the side of process, Protagoras made a very significant contribution to psychology; but he failed to find within the process any solid basis for the validity of thought. When the Protagoreans applied the principles of this "flowing" philosophy to the concepts of ethics, the fixity and permanence of the solid structure of habit, custom, and tradition, in which the morality of the age inhered, was reduced to a fleeting, fluctuating stream of mere convention. Ethics, like knowledge, was subjective and relative. The world of things, the world of experience, the world of conduct, were all alike subject to the Heraclitean law of "flux," genesis, or becoming.

Socrates was not deeply interested in the speculative problems of physics or of ontology. There is no reason to think that he reacted especially against the Protagorean philosophy. But he did react against the situation of ethical confusion and moral relaxation of his day, which found aid and comfort in the negative and relativistic type of philosophy. His moral earnestness could not endure the destruction of the ethical concepts. These must be restored. If the moral sanctions inherent in faith in the old régime had been loosed from their moorings, then they must be grounded anew. Socrates sought to give the virtues a securer basis than convention or habit by grounding them in knowledge. Not everything was under the law of change; there were such things as universals. This he sought to show not upon the basis of any theory or speculation, but upon the basis of an examination and analysis of the facts of human conduct. He found that the artisan, at least, had a standard of the good. The shoemaker, the harness-maker, the shipbuilder, the maker of weapons, etc.—in fact, every artisan—worked toward some standard of excellence, even though he may not have set that standard for himself with reference to a more ultimate end. The success of these men in attaining the good within their limited and circum-

scribed sphere depended upon their having *knowledge*; this one thing they *knew*. With them their knowledge and their virtue, or excellence, were one. The great trouble with the politician, or statesman, was that he did not know what was the good of the state, for he did not know the nature and the end of the state. The great trouble with people in general was that they acted upon the basis of convention or habit, unconscious of the principle in accordance with which they were acting, thinking themselves wise when they were really ignorant. So Socrates conceived it to be his mission to question people till he could show them their ignorance and make them seek to become wise. The great ethical significance of Socrates lies in the fact that he made morality a personal thing, not a conventional thing. Knowledge of ends, not imitation, or tradition, or custom, was its basis. He recognized the subjective factor, but not in the same way as the Protagoreans.

As with Socrates, so with his pupil Plato, his dominant interest was ethical and practical. This point of view I would maintain in spite of the fact that Plato devotes much time and space to the discussion of many abstract and abstruse metaphysical questions. He had, undoubtedly, a fondness for theoretical questions; but, as a rule, their discussion is for the purpose of throwing greater light upon some ethical or other practical human problem. Plato took up the ethical point of view of Socrates which made virtue a function of knowledge. But he pushed the analogy of the arts much farther. Nor was he content to let the theoretical question raised by the Protagoreans go untouched. If the virtues rest on a basis of knowledge, as Socrates contends; and if at the same time knowledge is sense-perception and a relative thing, as the Protagoreans contend, then Socrates is in as sorry a plight in the matter of finding secure ethical sanctions as when he began. The basis of ethics is insecure as long as it abides within the sphere of becoming. The ethical demand, on logical *a priori* grounds, is for knowledge which is of the eternal and abiding. The question for Plato is, then: Is there any such knowledge? The solution of the ethical problem leads him over into the epistemological question.

Protagoras, under the impulse of the Heraclitean factor, has identified sense-perception and knowledge. Plato, in order to give ethics a secure logical foundation, will again recognize the Eleatic factor and set up a distinction between sense-perception and knowledge, bringing both factors within his own system, with a decided

emphasis on the value for knowledge of the Eleatic factor. He admits in general the inadequacy of sense-perception and seeks to find elsewhere a more secure basis for knowledge. But as for the total relativity of sensation, there is at least an intimation in the *Theætetus* (171) that he does not think that the doctrine is true. In the *Republic* (7:523) he has this positive statement: "I mean to say that objects of sense are of two kinds; some of them do not invite thought because *the sense is an adequate judge* of them; while in the case of other objects, sense is so untrustworthy that further inquiry is imperatively demanded." The nature of this further inquiry will be taken up later. What I want to bring out here especially is the fact that Plato does give the senses some positive function, but at the same time he would not make sense-perception the equivalent of the whole knowledge-process. There is the question of the adequacy and the inadequacy which must be settled by some higher function. A distinction has to be set up between the lower and the higher, between sensation and thought.

Furthermore, Plato contended that there is knowledge which does not come through the senses. This point he works out in the *Theætetus*. The senses are specific—the eye being concerned with seeing, the ear with hearing, etc. But the common notions which we have are not thus specific in character. Our knowledge contains ideas of being, or essence, and of non-being, of likeness and unlikeness, of sameness and difference. Ideas such as these—abstract, universal, or embodying the results of comparison—cannot have come through any bodily sense; they have been perceived by the soul.² Thus there must be a distinction made between the senses and the intellect. Knowledge is not necessarily identical with sense-perception; it may have its basis in a higher faculty and have a character of permanence and stability characteristic of the world of being as opposed to that world of becoming which finds conscious expression through the process of sensation.

We are now at the point where we can begin to study specifically the significance of the mathematical element in Plato's thought. On logical grounds, the ethical demand is for a source of knowledge not subject to the Heraclitean law of "flux," or becoming. The Protagorean position of the identity of sensation and knowledge must then be overthrown. This is done by setting up a distinction between sensation and knowledge. It has been argued that there is a kind of

² *Theæt.*, 184-86.

knowledge which is not of sensational origin, and also that where sensation is involved the basis of knowledge may lie in the exercise of a higher faculty. Both of these points Plato followed up by an appeal to mathematics. It may even be that it was mathematics which gave him his first clue to this line of argumentation. Certainly the bringing in of the argument from mathematics made the justification of his position so clear and striking that it had all the force of a new proof rather than one of the same nature.

The best place to make a beginning of the mathematical argument will be a passage in the *Republic*. This may be summarized as follows: Objects of sense are of two kinds: (1) those of which sense is an adequate judge, and which hence do not invite thought; (2) those of which sense is not an adequate judge, and which hence do invite thought. The second case is that of receiving opposed impressions at the same time from the same object; *e. g.*, to the sense of touch hard and soft at the same time; or to the sense of sight great and small. Thus a conflict is created. This sense-conflict marks the beginning of an intellectual conflict. Since two qualitatively distinct and opposed impressions have been received, the problem arises as to whether they can come from one and the same object, or whether there are not two objects. The soul is put to extremity and summons to her aid calculation [an intellectual principle] to determine whether the objects announced are one or two; and hence arises the distinction between the perceptible and the intelligible. When mind has come in to light up, analyze, and interpret the conflicting manifold, of which sense is not an adequate judge, the conception of the one and the many both arise, and thought is aroused to seek for unity.³

According to Plato, then, the distinction between the senses and the intellect arises through a process of reflection stimulated by a sense-perception situation involving contradictory and conflicting experiences. This situation can be resolved only by the introduction of the mathematical process. But when this process is once introduced the distinction between sense and intellect is already under way. The mathematical thinking does not begin so long as there is only a confusion of sense-experience, but only when an intellectual conflict has been provoked and the mind has been put into the inquiring attitude. Furthermore, these mathematical notions, though brought to light under the stimulus of a certain type of sense-experience, are not themselves of sense-origin. They could not be;

³ *Rep.*, 7: 523-25.

for the senses are specific. Plato finds no separate sense organ for them, and his conclusion is that they are perceived by the soul alone.⁴

Thus mathematical thinking originates and necessitates the distinction between the senses and the intellect; for no mathematical thought would be possible without such distinction. But it would never occur to the mind of Plato to doubt that we do have a genuine knowledge-process in mathematical thinking. The logical *a priori* demand for the overthrowing of the Protagorean position by a reassertion of the distinction between sense-experience and rational process receives specific content when Plato turns to the study of mathematical thought and observes what takes place there. It is found to be justifiable and necessary from the point of view of an accepted and undoubted realm of knowledge. Yet we are not warranted on the basis of this passage from the *Republic* in saying that Plato conceived of the distinction as an absolute one in the broadest sense of the word "knowledge."

Another passage in the *Republic* is clearer still in showing the distinction between the sensible and the intelligible as the effect of the mathematical element. Also it throws some light upon the working nature of the distinction. This passage may be summarized as follows:

The body which is large when seen near appears small when seen at a distance. And the same objects appear straight when looked at out of the water and crooked when in the water; and the concave becomes convex, owing to the illusion about colors to which the sight is liable. Thus every sort of confusion is revealed within us. But the arts of measuring and numbering and weighing come to the rescue of the human understanding, and the apparent greater or less, or more or heavier, no longer have the mastery over us, but give way before calculation and measuring and weight. And this surely must be the work of the calculating and rational principle in the soul. And when this principle measures and certifies that some things are equal, or that some are greater or less than others, then occurs an apparent contradiction. But such a contradiction is in reality impossible—the same faculty cannot have contrary opinions at the same time about the same thing. Then that part of the soul which has an opinion contrary to measure is not the same with that which has an opinion in accordance with measure. The better part of the soul [*i e.*, intellect, or reason] is likely to be that which trusts

⁴ *Theætetus*, 185.

to measure and calculation. And that which is opposed to them [*i. e.*, sense-perception] is one of the inferior principles of the soul.⁵

Here it is shown that the mathematical principle of measure in its various forms brings in the element of intellectual control, and that where this control is introduced we have greater certainty than can be derived merely from the senses. This ordering and controlling function of mathematics will receive further discussion later in connection with the analogy of the arts. I point it out here merely to suggest that this gives us an indication that Plato works out the distinction between the senses and the intellect, not merely for the sake of maintaining a rigid separation between the sensible and the intelligible, but that he may find a higher principle by which to judge and control the lower. The cognitive aspect of that which takes mathematical form is very different from the cognitive aspect of that which takes merely perceptual form. We can see that in the mind of Plato not only does mathematics *effect the distinction* between the sensible and the intelligible, but he also intimates that the presence of the mathematical element is *criterion of the value* of a thing as *knowledge*. When Plato once gets this view of mathematics, it transforms his whole conception of the arts and sciences, as we shall see later. It also has very significant ontological implications; for in Plato epistemology and ontology are very closely bound up together. I may point out in passing that the view of the mathematical element as criterion of value for knowledge serves as a basis for the doctrine of idealism.

So closely interwoven are the strands of Plato's thought that from this point on we might follow them up in any one of several different ways and our problem work out very much the same. However, as the analogy of the arts plays such a fundamental part in all his thinking, it may be well to work that out in part at this point. It was in connection with the problem of ethics that the analogy of the arts made such a profound impression upon the mind of Socrates. This was also the most vital spring of Plato's interest in the arts and the artisan class. The point which is significant for us at the present is that both Socrates and Plato saw definitely in the arts the realization of the good dependent upon some measure of knowledge — knowledge at the very least of immediate ends. Socrates stated the principle, but we cannot tell how far he worked out its rationale and technique. Probably not very far. Plato pressed the analogy of

⁵ *Rep.*, 10: 602-3.

the arts to take in more and more remote ends, and he also worked out the means side of the problem of the arts in both its practical and its epistemological aspects. It will be easier, because more natural, to get at the logical significance to Plato of the analogy of the arts by beginning with the practical aspect. Following up the ontological problem for a little while will throw light upon the logical, or epistemological, one.

On the ontological side, what is common to all the arts is the fact that they are concerned with production. This is true not alone of the simple industries, but also of the ruler of the state. His art, too, is concerned with production.⁶ Now, production involves motion, the destruction of that which exists in one form by the breaking of it down or dividing it up and making new aggregations or some change in the relation of parts so as to produce a change of form. It is a process of becoming. The arts seem to fall wholly under the Heraclitean law of "flux," yet here Plato will find an Eleatic element of the abiding. On the lowest level that which is produced may come to be what it is by some chance, or by the happy guess of somebody, but this is not art.⁷ Art involves the exercise of some principle of control. The "cut and try" process is not art, nor is mere routine art. Production as an art is not a random matter, but is in accordance with mathematical principles.⁸ All arts and sciences necessarily partake of mathematics.⁹ Mathematics introduces the element of intellectual control into the process of production. The flowing sense-world is subjected to measure in all its various forms, and thus made subject to a higher world of order, beauty, and harmony. We do not have merely a world of becoming in all its ungraspableness, nor a world of unitary pure being in all its lonely grandeur. In the arts the two limits are brought together through the mathematical element into one ordered whole.

Now, we want to get the intellectual significance of this. The arts, all the processes of production, are concerned immediately or remotely with the satisfaction of human wants. The word "want" is ambiguous, and in its very ambiguity it is true to the situation to which it applies. There is both a physical and a psychical implication. On the lowest level the satisfaction of a want involves a need and the meeting of that need all within the unity of the same act without any process of intermediation between the two limits. But when the want is not satisfied by an immediate response to stim-

⁶ *Statesman*, 261.

⁷ *Philebus*, 55.

⁸ *Statesman*, 284.

⁹ *Rep.*, 7: 522.

uli, then the need takes the psychological form of consciousness of a lack. In ontological terms this would correspond to not-being. The tension of this situation may be relieved by some chance or random activity or by some process of external imitation. This repeated gives habit or routine. The process of production under these circumstances would be wholly empirical. There is a certain sense in which we would then have arts. There is a certain way which the workman has of reaching an end; there is a certain sense in which he may be said to know how to get a certain result. Yet Plato would not call this art. The process is not intellectually controlled. It is mathematics which introduces this control. Response then does not follow immediately upon stimulus, nor does it flow off without attention into some routine channel. On the psychological side as well as on the physical, the process of production is mediated and controlled with reference to an end. The states of consciousness are not a mere "flux." They become ordered, arranged, in accordance with a principle. We have technique instead of routine; control, or power, instead of chance; rational method instead of habit, custom, and imitation. On the psychical side as well as on the physical, the processes of production are no longer mere becoming, but arts (*τέχνη*). This involves not only knowledge of an end, but also that form of intellectual control which takes up means and end and consciously identifies them within one process through the intermediation of a regular series of steps. This is what mathematics enables one to do with the process of production. It gives knowledge and control of process with reference to ends.

Whether or not Plato was able to work out the complete psychology of the technical arts, he certainly did get a great deal of their intellectual significance. Through a comprehension of the meaning of the mathematical element the analogy of the arts became less of a mere analogy to him than it had been to Socrates. He had in his mind a clear working image of a type of intellectual control—an image rich in suggestion as to the possibility of a knowledge higher than sense, which could hold in its grasp and unify the fleeting and fluctuating sense manifold.

On the basis of the psychology of the industrial arts, involving the intellectual principle of mathematical control, Plato would arrive again at the conception of a distinction between the senses and the intellect—this time in a clearer and more concrete form than that already pointed out in the illustration of the mathematical element

coming in to settle the conflict of sense (see p. 26). The world of experience, on its cognitive side, would fall into two main divisions¹⁰ — all that which is matter of opinion (δόξα) and all that which is a matter of rational process or intellect (νόησις). On the objective side this distinction would correspond to that which is the object of sense-perception (τὸ ὁρατόν — the visible used here as a symbol for all the perceptible), as over against that which is the object of thought. (τὸ νόητον). Let us see how this worked out from a study of the arts. In the first place, those who worked by routine or rule of thumb could give no reason for their method of procedure; they could not see it in the light of any rational principle. From this point of view, they were entitled only to an opinion. They were either following the rule of another, or being guided by a series of associations of sense-experiences through which they had passed before in getting the same result. Even though they might be engaged in a real art which had a technique, which had already been brought under the law of intellectual control through the introduction of the mathematical element, yet that technique might be, so far as their own consciousness was concerned, mere routine, and they might not themselves be conscious of any rational principle of control. If so, they could not be said to have knowledge in the higher sense, but only opinion. This was the case with the vast majority of the artisan class, and it was this fact that made Plato rank them so low as he did. Their art, to be sure, was based on principles of the higher knowledge, but they themselves had not this knowledge. It could be seen that in the arts knowledge involved not merely the ability to reach a certain end, but also insight into the process by which that end could be reached, the ability ideally to construct that process with direct reference to the end and to intellectually control it. Opinion, even at its best, is not knowledge. The man may have true opinion in the matter of his process of production, yet there is a distinction between the cognitive aspect of his consciousness and that of the man who knows the rationale and can construct for himself the method. The former is a perceptual type, the latter a noetic type, of consciousness.

We have gone far enough now to see how Plato came to the conception of a type of knowledge higher than sense-perception. We have also seen how large a part mathematics played in his work of transcending the Protagorean-Heraclitean epistemology. This work

¹⁰ Here I use the terminology of *Republic*, 6:508-11.

is, indeed, not yet complete; but, as the ethical and the logical problems are so closely intertwined in the thought of Plato, it may be well to gather up some of the ethical implications at this point. In fact, by so doing we shall the better see what was the impetus which drove Plato to carry out his epistemology to the limit of dialectic.

According to the Socratic formula, moral conduct, the attainment of the good, is a function of knowledge. This he illustrated by the analogy of the arts. Plato has taken up this analogy and done two things: (1) he has shown that the knowledge on which the arts rest is of a higher type than that of sense-perception, involving, as it does through the mathematical element, the power to judge and control the sense manifold; (2) he has analyzed the process of attainment of the good in production, and has found that the significance of the cognitive factor involved consists in the fact that here is used a rational method, or technique, made possible by mathematics, for the intelligent adaptation of means to an end. Applying the results of this analysis to the problem of ethics, it is not enough to say that virtue is knowledge, not even if you say knowledge of ends. Scientific ethics must meet a further demand. Conduct, if it is to be regarded as ethical in the scientific sense, must involve that higher type of knowledge which is conscious of its own technique, and can hence control the elements of a situation in such a way as to *be sure* of producing the good, and not merely guess at it, or run the risk of failure through the breaking down of habit or routine.

Socrates and Plato both observed all around them the good existing in isolation from any principle of propulsive power, not brought under the control which comes from knowledge. Charmides was temperate, but he did not know what temperance was; Lysis was a friend, but he could not define friendship; etc. Thucydides and Aristides were noble in their deeds, but they did not know how to impart nobility to their sons, Melesias and Lysimachus.¹¹ Why could they not teach it? Socrates maintained that virtue is knowledge; and, if knowledge, then it can be taught. Plato showed the conditions which must be satisfied in order for virtue to be taught. It must be a virtue that is not wholly imbedded in habit, routine, or custom. Its rationale must be known, the technique of its process must be worked out. Education is a process of production; the teaching of virtue, like the teaching of an art, implies the ability to control a process, and control of the process, in any scientific sense of

¹¹ *Laches*, 178-79.

the word, implies knowledge of its technique. The practice of virtue involves the same principle. The man whose moral conduct is regulated merely by convention is on the same level of opinion and empiricism as the artisan who depends wholly upon routine. Each, in his own sphere, may attain the good; but that result is uncertain, insecure, liable to all sorts of error. Only the man who can control the process has virtue in any true or scientific sense of the word. I may have gone beyond the words of Plato, but I have tried to interpret his problem in the spirit of his words.

There is still one other knowledge-factor in ethical conduct besides knowledge of the technique. That is knowledge of the end. Socrates brought this out, and Plato emphasized it. The artisan, even when he rises to the higher plane of having a rational understanding of the technique upon which his art rests, still may be on the lower ethical plane. He is limited in respect to his knowledge of ends. What he makes he may make with intelligent adaptation of means to ends, so that, with reference to the end that he has in view, it may be perfectly good. But whether it is good in any further, more remote, or ultimate sense he does not know. What he makes he turns over to another to use. He may make the shoe, but whether it is good to wear a shoe is outside of his province. The physician may by his art know how to save the life, but he does not know whether it was better for the man to have lived or to have died. The pilot may carry you safely across the water, but it may have been better that you should have drowned. Instances of this sort Plato multiplies almost without number. A completely scientific ethics must take into account both knowledge of ends and knowledge of process. From this point of view, we can understand Plato's numerous thrusts at the sophists, the lawyers, and the politicians. The sophist professes to teach rhetoric, eloquence, virtue; but when examined he knows neither the nature, the true end, nor the technique of these. The politician would make laws for the state; but he does not know what justice is nor the process of attaining it. The ambitious man would rule, but he knows less about the nature of the kingly art than the cobbler does about making shoes. We call in specialists to judge of a musical instrument, a piece of armor, a case of sickness; but we are asked to turn over the larger interests of education in morals and the conduct of the state to men who know neither the process nor the end of the art which they are willing to undertake. Plato's ethical demand is that virtue shall rest upon

knowledge through the whole length and breadth of human activities. The system of human relations, social, industrial, and political, should fall into an order in which the highest class should be an embodiment of the ideal of the completely scientific standard of ethical conduct. Illustrations of some of the above points may be found in *Gorgias*, 455 and 511. ¹

Complete and reciprocal knowledge of ends and technique is not found in the case of any of the arts, neither has it been revealed in the mathematical element which lies at their basis as a principle of intellectual control. The mathematical element has revealed only the possibility of intellectually controlling the process of realizing an end when it is already known. It cannot tell us whether or not the end is good in light of a further principle. The epistemological problem has not yet been pushed far enough for Plato fully to establish and ground ethics upon the rational and scientific basis which he demands. We have come to the end of the ethical problem until we can push farther the logical problem by taking up the method, or technique, of knowledge in general.

Before passing on to the method, it may be well to go back and work out some of the further implications, already hinted at, of Plato's conception of the significance of the mathematical element. There are three principal topics which will come up for consideration: the influence of mathematics upon his conception of the sciences, the place of mathematics in his cosmology, and the relation of mathematics to idealism.

The sciences in Plato's day were in their infancy, the organization of knowledge could scarcely be called scientific anywhere except in some parts of mathematics, and even in mathematics there was much that was wholly empirical. Yet from mathematics Plato got the conception of what intellectual control of material meant. We have already pointed out that (see p. 28) this intellectual control involved an Eleatic factor in knowledge, which made it superior to the law of the "flux" of the senses. Through the mathematical element something abiding and valid and universal was attained. The organization of other departments of knowledge, Plato conceived, could be made scientific, if procedure was based on mathematical principles, if measure and number were introduced. It was from this point of view that he criticised the study of harmony and of astronomy as it was conducted in his day. Astronomy must be something more than star-gazing in order to be scientific. The

heavenly bodies are conceived as themselves moving according to mathematical laws. The proper method of arriving at astronomical truth is by attacking the subject from the side of mathematical problems. The same is true of harmony. Empirical methods, relying upon the ear alone, are not adequate. Absolute rhythm, perfect harmony, is a matter of the relation of numbers; the method of its attainment is a mathematical problem.¹²

It has already been pointed out that Plato conceived of all the arts and sciences as resting upon a mathematical basis (see p. 29). Now, he holds further that the more the arts make use of the mathematical element, the more they partake of the nature of knowledge, and the more exact and scientific they become. In fact, the arts can be graded up and arranged in order on the basis of the extent to which they avail themselves of mathematics. His position in these respects can be illustrated from a passage in the *Philebus*, which I will summarize:

In the productive or handicraft arts, one part is more akin to knowledge, and the other less; one part may be regarded as pure and the other as impure. These may be separated out. If arithmetic, mensuration, and weighing be taken away from any art, that which remains will not be much. The rest will be only conjecture, and the better use of the senses which is given by experience and practice, in addition to a certain power of guessing, which is commonly called art, and is perfected by attention and pains. Music, for instance, is full of this empiricism; for sounds are harmonized, not by measure, but by skilful conjecture; the music of the flute is always trying to guess the pitch of each vibrating note, and is therefore mixed up with much that is doubtful and has little which is certain. And the same will be found good of medicine and husbandry and piloting and generalship. The art of the builder, on the other hand, which uses a number of measures and instruments, attains by their help to a greater degree of accuracy than the other arts. In shipbuilding and housebuilding, and in other branches of the art of carpentering, the builder has his rule, lathe, compass, line, and a most ingenious machine for straightening wood. These arts may be divided into two kinds — the arts which, like music, are less exact in their results, and those which, like carpentering, are more exact.¹³

¹² *Rep.*, 7: 529-31.

¹³ *Philebus*, 55-56.

It is interesting to note that he goes on to sift out from these more exact arts that element — arithmetic, weighing, and measuring — on which their exactness depends, and to examine that with reference to its cognitive and scientific character. Having got from mathematics the conception that the scientific character of a body of knowledge was dependent upon the power of exercising intellectual control through the principle of quantity, he went to work and applied that conception to mathematics itself. He demanded that mathematics be made scientific through the rigid application of its own principles. This latter point does not come out specifically in the passage here in the *Philebus*, but the principle of distinction involved in it is employed. He notes the wide difference between popular arithmetic and philosophical. In the former, reckoning is done by the use of unequal units; “as, for example, two armies, two oxen, two very large things or two very small things.” That is, units are used which are not determined on the basis of the principle of measuring. “The party who are opposed to them insist that every unit in ten thousand must be the same as every other unit.” This same difference in accuracy exists between the art of mensuration which is used in building and philosophical geometry, also between the art of computation which is used in trading and exact calculation. The conclusion of the matter is that those arts which involve arithmetic and mensuration surpass all others, and that where these enter in their pure, or scientific, form there is infinite superiority in accuracy and truth.¹⁴ Mathematics itself, then, if it is to be made scientific, must be based upon its own rigid principles. It is only when it is pursued in the spirit of the philosopher that it attains to its true cognitive function, that it reaches scientific knowledge.¹⁵

In concluding the discussion of the influence of mathematics upon Plato's conception of science, we may say two or three things by way of summary. He regards every art as having its scientific aspect, even the art of war, which we have not specifically discussed.¹⁶ This scientific aspect varies with the extent to which the art has been reduced to intellectual control through the use of mathematics. Also with reference to the sciences proper, they are to be deemed such by reason of the fact that they are bodies of knowledge the accuracy and validity of which are secured by the use of mathematical methods of procedure.

¹⁴ *Philebus*, 56-57.

¹⁵ *Rep.*, 7: 525-27.

¹⁶ See *Rep.*, 7: 522, 525, 526, 527.

In Plato's cosmology, mathematics plays the same instrumental and intermediary part as in the arts. The cosmological problem is only a broadening out of the ontological one. We have already discussed to some extent the ontological problem in relation to the analogy of the arts (see p. 29). There it was taken up not so much for its own sake as for the light which it threw upon the problem of knowledge and of ethics. There we saw that the arts are concerned with production, and that the intermediation between becoming and being was effected by the mathematical element. The Eleatic element of permanency was maintained as that which held in control the shifting stream of becoming. We now have to take up the same problem in its more general form.

The Heraclitean-Eleatic opposition of becoming and being had already been resolved by philosophers who held to the doctrine of elements and by others who postulated atoms, as the ultimate permanent and unchanging being. Generation and decay were accounted for on the basis of the integration and disintegration of complexes of these original elements. As in the case of production in the arts, so in the general case Plato saw something more in becoming than this. In the *Phædo* he intimates his dissatisfaction with the explanation of generation and decay by separation and aggregation, by any principle of mere increase or decrease. He narrates how he had a youthful enthusiasm for the problem of generation and corruption (96), but that soon he got into all manner of difficulties. This is his account of the experience:

There was a time when I thought that I understood the meaning of greater and less pretty well; and when I saw a great man standing by a little one, I fancied that one was taller than the other by a head; or one horse would appear to be greater than another horse; and still more clearly did I seem to perceive that ten is two more than eight, and that two cubits are more than one, because two is the double of one.

I should be far enough from imagining that I knew the cause of any of them, by heaven I should; for I cannot satisfy myself that, when one is added to one, the one to which the addition is made becomes two, or that the two units added together make two by reason of the addition. I cannot understand how, when separated from the other, each of them was one and not two, and now when they are brought together, the mere juxtaposition or meeting of them should be the cause of their becoming two; neither can I understand how the division of one is the way to make two; for then a different cause would produce the same effect—as in the former instance the addition and juxtaposition of one to one was the cause of two, in this the separation and

subtraction of one from the other would be the cause. Nor am I any longer satisfied that I understand the reason why one or anything else is either generated or destroyed or is at all, but I have in mind some confused notion of a new method, and can never admit the other. (96-97.)

This is a very difficult passage to interpret. It is evident that from one point of view he is leading up to the problem of ends. He is feeling his way toward a final cause in the matter of the physical world. This is evident from his elaboration of the principle of the good in the passage immediately following. But within this whole problem of final cause there falls this one of generation and decay, the problem of becoming. Just as he is going to be dissatisfied with the causal explanation of the physical universe that has been given by Anaxagoras and others, so he is also dissatisfied with the explanation of the process of becoming that views it solely from the side of aggregation and juxtaposition. He takes up this problem in its most acute form—where it affects one's conception of relations. Plato seems to indicate, when he comes to resolve these contradictions (101), that they arise from the fact that the principle of explaining generation and decay—namely, that of aggregation and juxtaposition on the one side, and disintegration and division on the other—was not a mathematical principle. If it had been, it would not have caused so much confusion and contradiction in the case of dealing with relations. Participation in number is an essential to the process of becoming. Whether we agree or not with the form in which Plato expresses this mathematical principle underlying the process, the essential point to note here is that he seems to be making for mathematics a function in the whole process of becoming.

In the arts, order and determination were introduced into the process of production through the mathematical principles of number, measure, and weight. Passages in the *Timæus* would show that Plato had much the same conception of the whole cosmological process. We cannot go into the details of cosmology as outlined in the *Timæus*, but only strike at a few of the most significant points for our purpose. The mathematical element is made very prominent. Two or three illustrations will be enough to exhibit the principle.

We will start with his conception of the elements of the physical universe (53-57). He begins with the traditional four elements of earth, water, air, and fire. The old physical philosophers had explained becoming on the basis of the transformations of these elements, but they had no adequate technique of that process. Plato

undertakes to explain the process by working out a technique for it on a mathematical basis. Each of these elements is itself made up of triangles, the particular mathematical principle employed being that of the construction of the regular solids — the regular pyramid, the octahedron, the icosahedron, and the cube. The cube is the form of the element earth; the icosahedron, water; the octahedron, air; and the regular pyramid, fire. The stability, mobility, or decomposibility of these various elements is dependent upon their form and the relation of the triangles involved in their composition. These can all be expressed by a mathematical formula. The assumption of the truth of the account of the nature of the elements rests upon "a combination of probability with demonstration." The principles which are prior to the triangles "God only knows, and he of men who is the friend of God." Thus it will be seen that the triangles are not themselves regarded as ultimate, they are instrumental and intermediate. As in the arts, so here the mathematical element comes in as the factor of control, as that which makes technique possible, which gives the power of controlling means with reference to ends, of bringing forth being out of becoming, that is, making becoming not merely a random, ceaseless streaming, or process of "flux," but actually a process of *becoming*.

It is interesting to note also that the four elements themselves stand in a mathematical relation to one another (31-32). Between the densest and the rarest two means are inserted as a bond of union — fire is to air as air is to water as water is to earth. The creation of the universal world-soul was also conceived to have been by the taking of the elements of same, other, and essence, and combining them into a compound upon the basis of certain proportions with which Plato was familiar as lying at the basis of harmonies (35-36). The motions of the heavenly bodies, with all their diversity and complexity, were yet explained on the basis of a structure which rested upon mathematical principles (38-40).

In the *Laws*¹⁷ there is also an intimation that the processes of growth and decay involve mathematical principles. Plato speaks, in this connection, of the proportional distribution of motions, and he also uses a geometrical figure in describing the process of creation by increase from the first principle up to the body which is perceptible to sense. In another place¹⁸ he defends himself against the charge of impiety for holding to a mathematical conception of the universe.

¹⁷ *Laws*, 10: 893-94.

¹⁸ *Laws*, 12: 966-67.

Now, the upshot of all this is that when Plato used mathematical terms and mathematical figures in the discussion of cosmological questions, he did not use them as mere figures, or in a mystical sense either. He was carrying out, as best he could, in its application to the problems of the physical universe that conception of the significance of mathematics which he had learned from the arts as engaged in processes of production. Mathematical principles were as necessary to the creative activity of the deity as to the constructive activity of man. God and man were alike under necessity in this respect,¹⁹ and that necessity is the necessity of intelligence, or mind.²⁰

This interpretation of the place of mathematics in Plato's cosmology, and in his thought generally wherever the question is one relating to becoming, genesis, or production, enables us to view as serious and intelligent some passages otherwise very perplexing. Among these are certain passages in the *Laws*, already referred to in another connection.²¹ In these passages we can see an attempt to organize the state upon scientific principles, to introduce the element of rational control by the application of the principles of mathematics. Deference is paid to the mathematical relations involved in nature. In other cases, as in the use of the number 5,040, the particular number is probably not important, but the principle which it illustrates.

A further curious instance of the same sort is found in the *Republic*.²² Here it is conceived that the perpetuity of the state could be indefinitely secured, provided the rulers had the wisdom to understand the mathematical law governing births; for then they would have control of the birth of good and evil, and consequently could permit only those births which would be for the interest of the state.

Further development of Plato's cosmology will be postponed for the present. It will be touched upon again after some discussion of method (see p. 91). Before turning to the problem of method, there remains a brief discussion of the relation of mathematics to Plato's idealism.

Two distinctions have been brought out as necessitated by mathematics: one, on the side of content, or object of knowledge, namely, "the sensible" (τὸ ὁρατόν) and "the intelligible (τὸ νόητον); the

¹⁹ *Laws*, 7: 818.

²⁰ *Laws*, 12: 967.

²¹ See *Laws*, 5: 737, 738; 745-47; 6: 771; and cf. 6: 756.

²² *Rep.*, 8: 545-47.

other, on the side of faculty, mental activity, or process, namely, "sense" (δόξα) or opinion and "intellect" (νόησις) (see p. 26). A distinction of value also comes in (see p. 28) which tends toward idealism, namely, the minimizing of sense and the exaltation of reason. It is a familiar fact of Plato's philosophy that he exalts that which comes through mental function, whether reason or direct intuition of the soul, to the highest rank, and regards nothing as partaking of scientific character and worthy to be called knowledge which comes through sense alone or is empirically derived.²³ This transition to the idealistic point of view is equally bound up in the logical *a priori* point of view and in the mathematical. It is only in the interaction of the two points of view that the mathematical element gets its deepest significance. It is because the idealistic interpretation of things appeals so strongly to Plato as the direction in which to look for the solution of his philosophic problems that he is charmed and fascinated by the idealism of mathematics and so eagerly points it out and snatches at it.

The idealism of mathematics clarifies, illumines, gives force and content to Plato's idealistic demand. This comes out both on the process and the content side of the subject. Take the following statements as evidence:

Masters of the art of arithmetic are concerned with those numbers which can only be realized in thought, necessitating the use of the pure intelligence in the attainment of the pure truth.²⁴ Arithmetic must be studied until the nature of number is seen with the mind only.²⁵ "The art of measurement would do away with the effect of appearances, and, showing the truth, would fain teach the soul at last to find rest in the truth."²⁶ Arithmetic compels the soul to reason about abstract number, and rebels against the introduction of visible and tangible objects into the argument.²⁷ Geometricians, "although they make use of visible forms and reason about them, are thinking, not of these, but of the ideals which they resemble; not of the figures which they draw, but of the absolute square and the absolute diameter and so on—the forms which they draw or make, and which have shadows and reflections of their own in the water, are converted by them into images, but they are really seeking to behold the things themselves."²⁸

To return to the discussion, these passages show that in the mind

²³ *Rep.*, 6: 510; 7: 527, 529, 530-31, 523.

²⁵ *Rep.*, 7: 525.

²⁷ *Rep.*, 7: 525.

²⁴ *Rep.*, 7: 525-26.

²⁶ *Protog.*, 356.

²⁸ *Rep.*, 6: 510.

of Plato mathematics has continually the double process-result idealistic function. Its superiority as knowledge lies in the fact that it is most largely free from the sense-element. Objects of sense are to be distrusted. On the side of process, mathematics is engaged in getting away from them. It is exercising the mind, leading the soul away from the realm of sense. Although the mathematician may start with the data of sense as suggestive of his problem, these data are only the images of the absolute realities lying behind them, and his problem becomes truly mathematical only when he has made the transition to data that are purely ideal. On the side of content or result, mathematics furnishes to Plato the most conspicuous instance of a science which deals with absolute realities. Through starting with data which have been stripped by abstraction of their sense-elements and then have been ideally transformed, and then drawing conclusions by processes wholly rational or intuitive, the results attained are absolute, unchanging, necessary. They serve as the idealistic model for all scientific knowledge. Any subject of study in order to become scientific must, according to Plato, yield itself to this movement. This is brought out in his discussion of astronomy and harmony in the *Republic*.²⁹

Both astronomy and harmony are very rich in the sense-element, but Plato feels that so long as this is not transcended and left behind we do not get the realities involved in them. These subjects must be made rational rather than empirical, and they become rational only by being made mathematical.

Plato ridicules the idea that star-gazing is astronomy. This is "seeking to learn some particular of sense," and "nothing of that sort is matter of science." The spangled heavens may be glorious and beautiful to the sense of sight, but the geometrician "would never dream that in them he could find the true equal, or the true double, or the truth of any other proposition." In the study of astronomy the gift of reason must be made use of, the mathematical method must be applied, the proper procedure in the solution of problems. That which is eternal and subject to no variation must be sought; but nothing that is material and visible can be eternal and subject to no deviation.³⁰ The empirical study of harmony is also held up to the same sort of ridicule, and for the same reason — that sense-perception is placed before reason and that absolute realities are not attained. Failure here, too, is due to not applying

²⁹ *Rep.*, 7: 529-30 and 530-31.

³⁰ *Rep.*, 7: 529-30.

the mathematical method. The empirical students "set their ears before their understanding." Even the Pythagoreans "are in error, like the astronomers; they investigate the numbers of the harmonies that are heard, but they never attain to problems."³¹

These two discussions — one on astronomy, the other on harmony — both illustrate very strikingly the distinction of value for knowledge which Plato makes between the sense-perception element and the intellectual element, and how through mathematical procedure this distinction leads over into idealism. Plato demands of knowledge that which is absolute, eternal, invariable. In the fields of astronomy and harmony he finds this demand met only through mathematical procedure. The truth, the reality which cannot be found on the side of sense-perception, can be found in the results of the rationalistic mathematical process.

In always playing this double part of going through processes that lead over into the realm of ideas and of furnishing results that belong to that realm, mathematics, as it were, both furnishes the *stimulus to* idealism and *is* idealism. Certainly in the building up of Plato's idealistic philosophy mathematics, though not the only factor, is a very important one. The idealism of mathematics furnishes him with one of the strongest arguments by analogy for a universal idealism. Just as the ultimate reality with which the mathematician deals does not spring out of data of sense by any empirical process, but is both in respect to its real data and in respect to its final results something absolute and transcending sense-perception; so with the ultimate reality behind all phenomena, it is the ideas, something in harmony with the rational principle of the soul, not subject to change, to the flux of the imagery of sense-perception. Only, in mathematics the process by which the material of sense is transcended and ideas are reached is capable of being exhibited, whereas in other realms it is not. This feature of mathematics is one key to the understanding of the importance which Plato attaches to mathematical training as a preparation for the study of philosophy. Without such training, on the one hand, the problem of philosophy, the problem of being or essence, cannot be adequately understood; nor, on the other hand, a suggestion as to the process, or technique, of its solution arise.

The problem of philosophy for Plato is to know true being. The function of the philosopher is to find through reason the absolute truth, the eternal being, lying behind and controlling all the phe-

³¹ *Rep.*, 7: 530-31; cf. *Phileb.*, 55-56.

nomena of sense. But on logical *a priori* grounds this knowledge cannot come through the channel of sense-perception; for the senses are inadequate. Ordinarily "the eye of the soul is literally buried in an outlandish slough of sense."³² Some preliminary training is needed in the idealistic process before the soul can rise to that height of freedom and power and self-control where she can gaze on absolute being and attain true knowledge. Mathematics serves the function of giving that training; here intellect has found ultimate realities which are abiding, absolute, necessary, ideal.

The philosopher must be an arithmetician, studying the subject until the nature of number is seen with the mind only, and for the sake of the soul herself; this will be the easiest way for her to pass from becoming to truth and being.³³ The true use of number is simply to draw the soul toward being.³⁴ Geometry also gives the same valuable idealistic training. It tends to make more easy the vision of the idea of the good, compelling us to view being and not becoming only. Its real object is knowledge, and the knowledge at which it aims is knowledge of the eternal and not of aught perishing and transient. Then geometry will draw the soul toward truth and create the spirit of philosophy.³⁵ The study of harmony is useful to the same end, if it be made mathematical and be studied "with a view to the beautiful and good."³⁶

It is thus seen that Plato feels that the mind which has accustomed itself in the realm of mathematics to make the transition from the exercise of the senses to that of the intellect, and has acquired the power of abstraction and of centering its attention upon purely ideal elements, is the only mind fit to philosophize. The training of mathematics is positive, direct, and necessary in preparing the mind for that point of view which seeks the ultimate reality of all things in ideas as over against the products of sense-perception. While Plato seems to reserve specifically to dialectic the power to reveal the absolute truth, the ultimate reality, yet he feels that it can reveal this only to one who is a disciple of the mathematical sciences, which are used as "handmaids and helpers" in the work of uplifting the soul.³⁷

Before leaving this discussion of the idealism of mathematics it will be necessary to take account of an important passage in the

³² *Rep.*, 7: 533.

³³ *Rep.*, 7: 525.

³⁴ *Rep.*, 7: 523; see also 7: 521-23 and 523-25.

³⁵ See *Rep.*, 7: 526-27.

³⁶ *Rep.*, 7: 521.

³⁷ *Rep.*, 7: 533.

Republic — the famous figure of the divided line.³⁸ From this passage it would appear that Plato does not place mathematical notions on a level with the ideas. In this passage there is a discussion of the stages of knowledge, or, in ontological terms, the degrees of being. First, two main divisions are made: Opinion (*δόξα*), the lower, which is concerned with the visible world (*τὸ ὁρατόν*) and has to do with becoming (*γένεσις*); and Intelligence or Thought (*νόησις*), which is concerned with the intelligible world (*τὸ νόητον*) and has to do with Being (*οὐσία*). Opinion (*δόξα*) is itself divided into two stages: Conjecture (*εἰκασία*), which has to do with images (*εἰκόνες*) in the nature of shadows and reflections; and Belief (*πίστις*), which has to do with things — the animals which we see and everything that grows or is made. Intelligence is also divided into two parts: Understanding (*διάνοια*), which Plato makes clear, works with images of things, but to which he does not make clear that there is any corresponding *distinct* object of knowledge or being; and Reason (*νοῦς*), which has to do with the Idea (*ιδέα*) or eternal Being. Schematizing this, it would be something as follows, without attaching any significance to the length of the lines used.

| | | | | |
|---|------------------|---------------|-------------------------|-------------|
| <div style="display: inline-block; vertical-align: middle;"> <i>Faculties</i> \updownarrow <i>Objects</i> </div> | { <i>δόξα</i> | | { <i>νόησις</i> | |
| | <i>εἰκασία</i> | <i>πίστις</i> | <i>διάνοια</i> | <i>νοῦς</i> |
| <hr/> | | | | |
| | { <i>εἰκόνες</i> | | { <i>μαθηματικά</i> (?) | |
| | <i>τὸ ὁρατόν</i> | | <i>τὸ νόητον</i> | |

On the side of faculty, it is clear that there are four divisions, of which the third is Understanding (*διάνοια*), and that this third faculty is the one concerned with mathematical thinking.³⁹ Now, in making a distinction on the side of process between mathematical thinking and reason (*διάνοια* and *νοῦς*), it is a question whether Plato wishes to make a distinction in the nature of their objects — mathematical truths (*μαθηματικά*) and Ideas (*ιδέαι*). Milhaud shows at considerable length that such an interpretation does not hold. He points out the fact that the fundamental line of division with Plato is the twofold one, that according to this line of division mathematical notions and ideas belong together as not different in essence, and that it is more in harmony with the spirit and usage of Plato to identify than to separate them.⁴⁰ This proof of Milhaud would have

³⁸ *Rep.*, 6: 508 ff.

³⁹ *Rep.*, 6: 511.

⁴⁰ In MILHAUD see especially pp. 242, 244, 263, 267, 270-72, 274 ff., 277-79.

been still clearer if he had observed carefully the distinction which Plato makes between faculty, or method, and object. On the side of method (a point which will be discussed further later on, p. 84) the distinction between mathematical thinking (Understanding, or *διάνοια*) and pure Reason (*νοῦς*) is clear; on the side of object it is not clear; hence the ambiguity, the problem, the apparent inconsistency of Plato. On the side of process four divisions in all, on the side of object only three, or at least the distinction between a third and a fourth merely formal. That the distinction on the side of object between mathematical essences and Ideas is formal and apparent rather than real⁴¹ in the mind of Plato, take for proof this passage on the identity of their nature right from the midst of the discussion of the different divisions:

And do you not know that, although they [the mathematicians] make use of their visible forms and reason about them, they are thinking not of these, but of the ideals which they resemble; not of the figures which they draw, but of the absolute square and the absolute diameter, and so on — the forms which they draw or make, and which have shadows and reflections in water of their own, are converted by them into images, but they are really seeking to behold *the things themselves*, which can only be seen with the eye of the mind?⁴²

The mathematician may start with data of sense, but these he ideally transforms and transcends, and comes out with absolute and ideal results.

⁴¹ I am speaking in ontological terms here. On the side of cognition, I hold that the distinction is a real one in the mind of Plato, but not absolute. See p. 87.

⁴² *Rep.*, 6: 510.

CHAPTER III.

METHOD, OR THE TECHNIQUE OF INVESTIGATION.

PLATO'S INTEREST IN METHOD.- 19

It has already been suggested that Plato is as much interested in mental processes as in mental results (see p. 19). I hold that we do not get the full significance of Plato's dialogues when we view them from the side of literature alone, nor from the side of their philosophical content alone. Either, or both together, is a mistaken view. In Plato we have a most remarkable exhibition of a man who reveals the psychological and logical processes by which he reached his conclusions. This he did with such literary skill and power, and the problems with which he dealt were of such magnitude, that we are apt to get lost either in the literary side of his productions or in the realm of their ideas. Plato might have given us only the results of his search; but, whether purposely or otherwise, in many of his dialogues he has given more of the method than of the actual result. This quite likely is not an unconscious matter of chance with him. He was a master of the pedagogical method of stimulating and awakening interest in the problems which he discussed before going on to give his own views. He recognizes the significance of the principle of "shock" which comes from conflict and contradiction.

Do you see, Meno, what advances he has made in his power of recollection? He did not know at first, and he does not know now, what is the side of a figure of eight feet; but then he thought he knew, and answered confidently as if he knew, and had no difficulty; now he has a difficulty, and neither knows nor fancies that he knows. Is he not better off in knowing his ignorance? If we have made him doubt, and given him the "torpedo shock," have we done him any harm? We have certainly, as it would seem, assisted him in some degree to the discovery of the truth; and now he will wish to remedy his ignorance, but then he would have been willing to tell all the world again and again that the double space should have a double side. But do you suppose that he would ever have inquired into or learned what he fancied that he knew, though he was really ignorant of it, until he had fallen into perplexity under the idea that he did not know, and had desired to know.¹

It is very noticeable that often before getting down to the most

¹ *Meno*, 84.

serious discussion of a subject he plays, as it were, with the ideas of his interlocutors. This, too, he sometimes does quite eristically, as shamelessly from the point of view of logical principles as any Sophist, but after all in a manner which is most often on a level with the logical processes of his respondents or opponents, and which is calculated to show by the contradictions into which he leads them the inadequacy and shallowness of their thinking. This accomplishes two things: (1) it stimulates an interest in the real rather than the superficial aspects of problems; (2) it prepares the way for the consideration of better methods and the introduction and appreciation of more rigid logical processes.²

We do injustice to Plato if we fail to see that he has a genuine pedagogical interest in method, and so come to overlook the fact that there are many long digressions in his dialogues which are not to be regarded as significant so much for their content as for the illustration and explanation of some method or principle of procedure. As a sample of such a digression take the discussion of the principle of contradiction in *Theætetus*, 155, and in *Republic*, 4:436-39. Large portions of the *Republic*, the *Sophist*, the *Statesman*, and the *Parmenides* come under this same head. The larger part of the *Republic*, while undoubtedly expressing sociological ideas which Plato wished to discuss, is from another point of view explicitly a point in methodology. From this point of view the ideal state is only a construction devised for the purpose of working out a psychological analysis of justice. The first book has failed to issue in a definition of justice. A suggestion is then made to proceed to another method.

Seeing then, I said, that we are no great wits, I think we had better adopt a method which I may illustrate thus: Suppose that a short-sighted person had been asked by someone to read small letters at a distance; and it occurred to someone else that they might be found in another place which was larger and in which the letters were larger — if they were the same and he could read the larger letters first, and then proceed to the lesser — this would have been thought a rare piece of good fortune.³

The ideal state is nothing but the letters writ larger in a larger place. The analysis of justice here gives the key to the analysis of justice in the individual.⁴

This same principle of analyzing a plain and simpler case for the sake of its application to the more complex and obscure is again enunciated in the *Sophist* (218) and in the *Statesman* (286). So

² Cf. *Soph.*, 230.

³ *Rep.*, 2: 368.

⁴ See *Rep.*, 4: 434 ff.

clearcut and significant is the statement in the *Sophist* that it may be well to quote it:

Now the tribe of the Sophists which we are investigating is not easily caught or defined; and the world has long ago agreed that if great subjects are to be adequately treated, they must be studied in the lesser and easier instances of them before we proceed to the greatest of all. And as I know that the tribe of Sophists is troublesome and hard to be caught, I should recommend that *we practice beforehand the method* which is to be applied to him on some simpler and smaller thing.⁵

The case of the angler is taken up in great detail, illustrating definition through a long process of division of species. When this is complete, then "following this pattern" they "endeavor to find out what a Sophist is."⁶ Is it too much to think that not only this illustration, but also the whole dialogue, has for one of its great purposes an exposition of method—the method of logical division whereby mutually exclusive alternatives may be secured? (See pp. 48, 58, 61.)

In the *Statesman* there is again the explicit recognition of digression from the main line of argument for the sake of methodological reasons. Take the passage where the art of weaving has been analyzed by the process of division and has led up to the discussion of the art of measurement. Now that this discussion is completed the proposition is made "to consider another question, which *concerns not only this argument, but the conduct of such arguments in general.*" Attention is called to the fact that in teaching a child the letters which go to make up a word the aim is not merely to improve his grammatical knowledge of that particular word, but of all words. So, in the case of the analysis of the art of weaving, it has not been done for its own sake, but for the sake of the training in giving and accepting a rational account of things.⁷

Exposition of method and training in method is one of the motifs of the dialogues "of search."⁸ Even positions are taken in which the speaker does not necessarily believe, for the sake of the argument. Thrasymachus says: "I may be in earnest or not, but what is that to you?—to refute the argument is your business."⁹ Glaucon maintains the cause of injustice, though confessedly not believing it, in order that he may see how the position can be refuted.¹⁰ In the *Gorgias* (462) Socrates gives Polus a lesson in the method of argu-

⁵ *Soph.*, 218.

⁶ *Soph.*, 218-21.

⁷ *Stat.*, 285-86.

⁸ I use this term to apply more widely than to the minor dialogues alone.

⁹ *Rep.*, 1: 349.

¹⁰ *Rep.*, 2: 358.

mentation. In the *Phædo* (100 ff.) there is quite a long digression on method. The *Parmenides* is from one point of view, taken as a whole, an exposition of method. When Socrates is involved in certain difficulties regarding the Ideas, Parmenides explains that this arises out of his attempting to define the beautiful, the just, the good, and the ideas generally, without sufficient previous training. The special lack in his training is pointed out, and it is indicated wherein he could get better training. Parmenides is asked to give an illustration of the method which he has indicated. This he proceeds reluctantly to do. On the side of form, the dialogue *Parmenides* is the outcome.¹¹

PLATO'S DOGMATISM.¹²

It may seem a little inappropriate at first thought to discuss dogmatism under the general head of method. But there seems to be no better place for its discussion in this book without involving repetitions. The dogmatism of Plato has a bearing on method, even if it cannot be said to be a part of method. The term "dogmatic" is often used as a term of reproach. Plato was a dogmatist in the better sense of the word. *He was in full sympathy with the process of investigation*; he longed for the truth above all things. But he also believed that the process of search was capable of resulting in valid conclusions. His confidence in knowledge is well illustrated in a passage in the *Meno*, where the following declaration is put in the mouth of Socrates:

Some things I have said of which I am not altogether confident. But that we shall be better and braver and less helpless if we think that we ought to inquire than we should have been if we indulged in the *idle fancy that there was no knowing and no use in seeking to know*; that is a theme upon which I am ready to fight in word and in deed to the utmost of my power.¹³

Compare this with the statement of *Meno*, 98:

That knowledge differs from true opinion is *no matter of conjecture with me*. There are not many things which I profess to know, *but this is most certainly one of them*.¹⁴

How significant is this language which describes the assumption that there is no knowledge as an "idle fancy," and the statement "that knowledge differs from true opinion is no matter of conjecture with me, but one of the things which I most certainly know!"

¹¹ *Parm.*, 135-36.

¹² I use dogmatism as opposed to skepticism, not as opposed to criticism.

¹³ *Meno*, 86.

¹⁴ See also *Phædo*, 99, 100, and 107.

On *a priori* grounds Plato was a dogmatist. It seems to be claiming too much, as Milhaud does, to trace this dogmatism wholly to the mathematical influence. But given a dogmatic tendency and problems so vital that they demand a positive solution, then we might expect that any branch of study which lent itself in support of that dogmatic tendency would be of great significance in clarifying and strengthening it. In the interaction of the dogmatic tendency of Plato's mind, as it swept out on the *a priori* movement of thought in the search for truth with the dogmatic element in mathematics, this mathematical element received a significance for him which it otherwise would not have had. Plato's unswerving confidence in knowledge coincides with what we should expect from the mathematician. The conclusions of the mathematician strike the mind with all the force of necessity. They seem absolute, irrefutable, independent of the fluctuations of the sense manifold. Mathematics is a realm where we actually have knowledge. Descartes, Locke, and Hume felt this. Kant made the *a priori* synthetic judgments of mathematics and pure physics the starting-point of his thought. There seems to be good reason for believing that mathematics is a fundamental factor, though not the only one, in Plato's dogmatism. The dogmatism of mathematics must have been a powerful support and stimulus at all times to that natural dogmatism which made Plato expect to find a secure basis for knowledge because on logical grounds ethics demanded it.

Plato's dogmatism, I have said, has a bearing on his method which justifies its treatment at this point. It gives his processes of thought a firm basis of reaction; it is the fulcrum, as it were, upon which his method turns. It gives stimulus and emphasis to the process of search. In the *Phædo* he issues a warning not to distrust arguments merely because they fail — not to hate ideas, *but to search more diligently*.¹⁵ It is from this point of view that Plato lays so much stress on the principle of contradiction. If knowledge is possible, and not merely a highly relative and fleeting product of the senses, then any procedure which would reduce knowledge to the test of a standard which is not absolute is inherently wrong. The possibility of knowledge is the rock against which all else breaks. There is a significant passage in the *Sophist* bearing on this point.

And surely contend we must in every possible way against him who would

¹⁵ *Phædo*, 89-91; cf. 99-100.

annihilate knowledge and reason and mind, and yet ventures to speak confidently about anything.¹⁶

It is the dogmatic attitude which gives vitality to the principle of contradiction, so that we are not content to rest with contradictory conclusions, but insist on continuing the search with the feeling that there must have been something wrong with the method employed or with the premises used. The principle of contradiction occupies a central position in Plato's thought. Let me give two passages from the *Gorgias* in illustration of his feeling on this point:

I would rather that my lyre should be inharmonious, and that there should be no music in the chorus which I provided; aye, or that the whole world should be at odds with me, and oppose me, rather than that I myself should be at odds with myself, and contradict myself.¹⁷

For my position has always been, that I myself am ignorant how these things are, but that I have never met anyone who could say otherwise, any more than you can, and not appear ridiculous.¹⁸

MATHEMATICS AND METHOD

In connection with the dogmatic attitude of mind and its relation to the principle of contradiction, it has been pointed out that mathematics plays a part in determining Plato's method. This phase of the mathematical influence now needs to be worked out along other lines. The significance of mathematics as a factor in determining Plato's method is usually overlooked or underestimated. Doubtless one reason for this is that the influence of Socrates upon Plato has been regarded as decisive. This is an easy inference to make; for Plato adopts the question-and-answer form of discussion, which appears on the surface to be Socratic. Again, he is much engaged in the Socratic task of analyzing concepts. We do not deny that Plato's method is Socratic in its early stages, and that it always retains Socratic elements. But the ethical problem took such a form with Plato that he was obliged to transcend his teacher and seek a more rationalistic method of procedure. This he worked out by combining with the Socratic principles the logical *a priori* demand and certain principles of mathematical reasoning. The outcome was a method which in all that is most essential to it is wholly un-Socratic. Let us take up in more detail some of the leading factors which entered into the Platonic reorganization of method.

In the first place, there was a marked difference in the way in

¹⁶ *Soph.*, 249.

¹⁷ *Gorgias*, 482.

¹⁸ *Gorgias*, 509.

which Socrates and Plato approached the ethical problem. We may regard the ethical interest as fundamental with both of them, but in different ways. After the destructive criticism of the materialistic Protagorean school, which did away with the ethical sanctions inherent in custom and the traditional polytheistic religion, these two men sought to save Athenian society by refounding ethics upon new and more secure moral sanctions. The ethical concepts underwent searching empirical and inductive examination and investigation by Socrates, with a view to showing their validity in actual experience. He was not so much concerned with the problem of the *possibility* of ethical standards, or universals, as with the *fact* of their *existence* as *factors real and operative in human life*. This he offsets against Protagorean *theory*. But Plato goes a step farther and meets *theory* with *theory*. He has a living interest in theoretical problems. This is decidedly a non-Socratic impulse. It undoubtedly grows out of the conflicting conclusions of the philosophical systems with which he was familiar. To Socrates these conflicts indicated the folly of speculative philosophy and the necessity of abandoning it and of confining himself to practical problems. But to Plato, under the influence of the dogmatic temperament and a mathematical training, these conflicts could not lead to skepticism. They were to him the stimulus to further investigation and final solution.

In accordance with these theoretical interests, Plato came to the logical demand for the distinction between the senses and the intellect, and hence for universals recognized by the mind alone and untainted by any sense-elements. At whatever time this logical demand became operative in his thought, whether with vague or with clear consciousness, whether alone or in conjunction and interaction with the mathematical line of thought, there must have begun a feeling of dissatisfaction with the Socratic universals. They were grounded ultimately in the world of sense-perception. They were derived empirically, through the analysis of the opinions of men, and could have no content except such as they found there. So long as they were tainted with the fleeting and inadequate sense-element, and not grounded in reason alone, they could not meet the supreme test of knowledge in accordance with the logical *a priori* demand.

In connection with this movement of thought it is significant that Plato was a student of mathematics. Socrates held theoretical mathematics (together with other theoretical subjects) in contempt. In so far as he did care for mathematics, it was solely on the practical side,

and that, too, in a very limited field. Plato, on the contrary, held mathematics in the highest esteem, especially theoretical mathematics, insisting everywhere and always on the superiority of abstract, or pure, mathematics in respect to scientific accuracy and truth. This is a decisive, non-negligible factor in the determination of his method. On the basis of the logical *a priori* demand, the Socratic universals are seen to be unsatisfactory. Mathematics led to the same conclusion. But more important than its coinciding with the other movement of thought in this respect is its clarification and grounding of that position. In mathematics knowledge is unquestionably found, and in this field its characteristics are plainly to be discerned. They are clearness, certainty, necessity, universality, rationality, and especially rigor of derivation. These then are the demands which concepts must meet if they are to be called knowledge. On *a priori* grounds knowledge must be wholly rational, free from the taint of the sense-element. This demand receives specific content from the realm of mathematics. In that realm we see clearly what the conditions are which knowledge must satisfy. These are the very same demands which must be met in every other field. When insight into the nature of mathematics has made these plain, the concepts of Plato can no longer be the inductively derived concepts of Socrates. Nor can they be these concepts merely hypostatized, if we mean by hypostatization the taking of them as they are, giving them an objective existence, and setting them up arbitrarily as independent of the phenomenal world. On this basis they would no more meet the demands of knowledge than before their hypostatization. Besides, it would be a reversal of the order which Plato gives to them; for it would be deriving the ideal from the phenomenal, instead of the phenomenal being a copy of the ideal, the former serving only as a stimulus to the search for the ideal. The ideality of the Platonic universals, if mathematical notions may be taken as an illustration of them, is not an ideality arbitrarily *thrust* upon them *merely* to meet an *a priori* logical demand. It is an ideality inherent in the very nature of knowledge. In mathematics, the demands of knowledge are met because of the ideality of its realities. The method by which such realities are attained in mathematics should be suggestive of the conditions which must be fulfilled in order to attain them in other realms.

The procedure of mathematics is especially marked by the fact that it starts out with data which it receives through intuition

(direct mental vision) or through intellectual determination, and having found such data it then works under such rigid rules of inference as to exclude any consequences which might creep in through premises inadvertently brought in from the outside. The whole process is intellectually controlled in such a manner as to exclude the introduction of empirical elements or empirical processes at any stage of the procedure. There is abundant evidence that the essential features of this process appealed strongly to Plato.

Plato's appreciation of the intuitive element in mathematical method has already been brought out (see p. 10) by the incident cited from the *Meno*.¹⁹ It was the *a priori* character of mathematics, the non-empirical origin of its truth, that impelled Plato to evolve the doctrine of recollection. At least, the proof that he gives of it is based most largely on the intuitive element in geometry. It seems altogether probable that Plato's doctrine of recollection²⁰ and his doctrine of philosophic love²¹ vaguely play the part in his philosophy that *a priori* synthetic judgments consciously do later in Kant's philosophy—the undoubted existence of mathematical truths settling dogmatically the question of the possibility of knowledge, these doctrines serving the function of showing *how* it is possible, through the acquisition of non-empirical elements with which to start.

In addition to intuitive data, there are data intellectually determined. Plato regards careful definition as important in any method of philosophic procedure.²² In all his dialogues of search it may be noticed that every new discussion is prefaced by a skirmish as to the meaning of terms, or at least a basis of agreement.²³ Some dialogues seem to be almost wholly given up to the process of sifting tentative definitions until a satisfactory one—*i. e.*, a rigorous one—can be found; or, if this is impossible, the search is abandoned with the clear consciousness that the definitions given are inadequate. Plato's conception of definition is the outcome of the interaction of the twofold movement of thought already described—the logical *a priori* and the mathematical. Milhaud claims too much for the mathematical element in the elevation of Plato's conception of definition above the Socratic point of view.

Plato views definition as something more than the matter of a

¹⁹ *Meno*, 81-86; see also *Phædo*, 73 ff.

²⁰ *Meno*, 81 ff.; *Phædo*, 73 ff.; *Phædrus*, 249 ff.

²¹ *Phædrus*, 249-53.

²² See *Phædrus*, 237.

²³ Cf. *Phileb.*, 20.

name.²⁴ Definition is concerned with the general idea,²⁵ with the nature of a thing rather than any description of its qualities.²⁶ It must not be expressed in terms of itself,²⁷ nor in terms as yet unexplained or unadmitted.²⁸ This latter point he takes from mathematics. Enumeration is inadequate as a principle of definition. This is worked out in non-mathematical instances, such as the definition of clay, as well as in the case of mathematical terms. The trouble with the definition which names kinds is that it does not reveal the nature of a thing in the abstract.²⁹ Definition must distinguish the class of things referred to from all others. This requisite is worked out through starting with an illustration from mathematics, followed up by several illustrations not mathematical, with a return to mathematics.³⁰ Both in the point made with reference to enumeration and in this latter point it is evident that Plato might have come to his conception of definition entirely apart from mathematical considerations. But that mathematics had for him a significance in this respect that should not be ignored seems evident when we observe how frequently he turns to mathematics for illustrations of principles in definition, and especially how he lingers over these illustrations and develops their significance. That Plato's conception of definition is very different from the Socratic conception of it as the expression of the common quality³¹ is most clearly seen from a passage in the *Phædrus*. Two important principles are there enunciated: (1) The comprehension of scattered particulars in one idea. This is illustrated in the definition of love given, "which, whether true or false, certainly gives clearness and consistency to the discourse. The speaker should define his several notions and so make his meaning clear." (2) Natural division into species. Socrates is made to say that he is "a great lover of these processes of division and generalization; they help him to speak and to think."³² These two principles both point in the direction of intellectual control of the sense manifold through logical processes centering in an idea. In so far as it does point that way, it is in harmony with a characteristic of mathematical definition. Here is a realm where the very essence of definition is intellectual determination, comprehending the scattered

²⁴ *Soph.*, 218.²⁷ *Phædo*, 105.²⁵ *Euthyphro*, 6.²⁸ *Meno*, 79.²⁶ *Gorg.*, 448 ff.²⁹ *Theæt.*, 146-48; see also *Meno*, 71-76.³⁰ *Gorg.*, 451-53.³¹ *Laches*, 192.³² *Phædrus*, 265-66; cf. *Soph.*, 253.

particulars in one idea by the free activity of the mind in mental construction, rather than selecting some common quality that is discovered to run through and attach to all the particulars and defining on the very doubtful (for Plato because sensational) basis of that. Plato feels that what philosophic method needs is what mathematics is actually coming to have in his day—clear, unambiguous, intellectually determined definitions; and he is very fond of showing up the absurd conclusions which can be derived from accepted premises merely by playing upon slight ambiguities in the meaning of terms employed. No rigorous conclusions in philosophy, he feels, can be obtained except as in mathematics; *i. e.*, by employing a method of procedure which shall continuously work within the limits of concepts which are either intuitive in character or intellectually determined, and by means of rational processes which are carefully guarded at every step.

THE METHOD OF ANALYSIS.

When we go down deeper still into the very form and structure of Plato's argument, we find, in so far as the use of the method of analysis is conclusive, that his thought is even more profoundly modified by the influence of mathematics than appears from those passages where mathematical terms or mathematical usage enter expressly into the discussion. We find whole dialogues in the discursive and constructive period of his thinking where the movement of his thought takes the form of the mathematical method of analysis. He talks a great deal about dialectic, but he actually uses the method of *reductio ad absurdum*. This method, in its earlier and less rigid state of development, connects itself so closely in form with Socratic analysis that the student may not easily notice the transition from one to the other, wide apart as they are when the transition is once made. But before that which is distinctive in Plato's analysis can be made plain, it will be necessary to discuss mathematical analysis more in detail.

In the time of Plato the method of analysis in mathematics was coming into pretty clear consciousness. Plato himself is said to have been the inventor of the method. But in simple cases it is such a natural movement of thought that it is more than likely that it had already been employed vaguely and loosely by the mathematicians, and that Plato only brought it into full consciousness *as a method*.

Then, having developed it in the field of an exact science—mathematics—he sought to apply it also in the field of philosophy. It will be convenient for our purpose to discriminate two phases in this method and to bring out the logical implications and significance of each of them. These two phases I shall call the positive phase and the negative phase.

1. *The positive phase of analysis.*—A certain proposition is up for investigation. It is assumed as true. Consequences are deduced on the basis of this assumption. If these consequences are known from some other source to be true, and if we can then start with them and reverse the process with the result of getting back to the original proposition, that proposition is proved to be true. If the original proposition cannot be thus deduced, nothing can be inferred either as to its truth or its falsity. Positive analysis with positive outcome is thus often inconclusive, and usually so in problems involving considerable complexity. If the conclusion, however, is known to be false, especially if it is in direct contradiction with some known truth or of accepted data (and hence is absurd), we have a right to infer that the original proposition is false; for by correct processes of reasoning we cannot deduce from a true proposition consequences which are false. Thus positive analysis, whether with positive or with negative outcome, admits of no inference as to the truth or falsity of any proposition other than the original one from which we start, and for that proposition is successful most frequently only in proving its falsity. It is not, then, by itself of much use in the *search for truth*.

2. *The negative phase of analysis.*—This is the complement of the positive phase, the two together forming a complete logical method. Here as before we have a proposition up for investigation. But we start with the assumption that it is not true—a proposition logically contradictory with the original one, or else with such an alternative proposition that the original proposition and it are mutually exclusive. If, in reasoning from the assumption, we arrive at a conclusion known to be false—*i. e.*, in contradiction to known truths or of positions granted (and which is thus absurd)—then we have to reject the supposition which led to this conclusion, as in (1). But this proposition is so related to the original one that the rejection of it as false carries with it necessarily the acceptance of the truth of the original. Here, then, negative outcome of one proposition has with reference to a related proposition a positive significance. This

procedure is then a very powerful instrument of investigation in all cases where two mutually exclusive propositions can be stated, in which the truth of either involves the falsity of the other and the falsity of either involves the truth of the other. The method may be extended, of course, so as to apply to cases of three or more alternatives where the proved falsity of all but one, through analysis to contradictory or absurd conclusions, is proof of the truth of that remaining one.

The above, I think, is a fair account of what is involved in the method of analysis, whether applied in mathematics or outside of mathematics. There would naturally be more variations in the details of its application in outside fields. These variations come in especially in the matter of starting the analysis, not with the proposition under discussion, but with other accepted truths. These are analyzed and conclusions drawn which are at variance with the original proposition. The inference, then, is that there is something wrong with that proposition.

SOCRATIC ANALYSIS.

There is no reason to suppose that Socrates was under the influence of mathematical thought. Yet by discussing his method with reference to the outline given above we can get a clearer conception of the relation between his analysis and Platonic analysis, and especially of the differences between them. If we follow Jowett and Windelband, we may take as most representative of Socrates's own thought, and as least modified by the introduction of any reactive element on the part of Plato himself, the group of dialogues comprising *Charmides*, *Lysis*, *Laches*, *Protagoras*, *Euthyphro*, *Apology*, and *Crito*, with the addition of *Hippias Minor* and *Alcibiades I.*, which are classed as of doubtful genuineness. I shall omit the latter two from the discussion. However other scholars might arrange the Socratic dialogues, they would agree substantially on this group as fairly typical.³³

The most uniform characteristic of this group of dialogues on the side of content is in their dealing with the problem of the virtues. This most frequently takes the form of an attempt at definition; e. g., *Charmides*, the definition of temperance; *Lysis*, of friendship;

³³ Jöel and Shorey emphasize the Platonic element in them. My main argument does not depend on either point of view. It would, however, harmonize better with that of Jöel and Shorey. See p. 77.

etc. Let us take them up and investigate them on the side of method. In *Charmides* five definitions of temperance are taken up in succession. In every case there is a process of analysis to contradictions or absurdities. There is no case of negative analysis. There is no statement of true alternatives. There is one instance (160) which might at first be taken as such, where the statement is made: "And of two things, one is true—either," etc. But closer examination shows that it is an inadequate opposition of quietness and quickness. In the *Lysis*, again, it is positive analysis with negative outcome. There is a clear statement of the assumption on which such analysis rests: "If we had been right, we should never have gone so far wrong" (213). Some attempt is made at setting off alternatives,³⁴ but they are not true alternatives and are used ambiguously in the course of the argument. In the *Laches* the analysis is all positive with negative outcome, and there is no explicit attempt to set off alternatives. The thought of the *Protagoras* moves in the form of positive analysis with negative outcome. There are no true alternatives. There is an appearance of the use of alternatives in several places in the setting of opposites against each other. In one place this effect is heightened by playing with the double meaning of the negative: if the virtues are distinct and separate, then "holiness is of the nature of the not-just, and therefore of the un-just; and the un-just is unholy;" *i. e.*, holiness is unholy. The opposites in 332–33 are not used as alternatives, but for the purpose of showing that two opposites to the same thing are identical; *i. e.*, in the course of the argument temperance has been shown to be the opposite of folly, and wisdom has been shown to be the opposite of folly; therefore temperance and wisdom are identical. In 359–60 recurs practically the same thing with the addition of one step to the process. Courage has been shown to be the opposite of cowardice, wisdom the opposite of ignorance, and cowardice identical with ignorance; therefore courage and wisdom are identical. In the *Euthyphro* also we have the positive analysis with negative outcome. There is an apparent recognition of the principle of alternatives in the statement: "Then either we were wrong in our former assertion; or if we were right then, we are wrong now." "One of the two must be true." But these alternatives are not used as pivots on which the analysis swings. They are given merely for the sake of emphasizing a contradiction, not for the sake of passing from it to a truth. One

³⁴ Cf. *Lysis*, 216 and 222.

line of analysis, starting with the definition of piety as "that which is dear to the gods," has led to contradictions; another line, starting with a different hypothesis, has led straight back to the definition again of piety as "that which is dear to the gods." This is confusion confounded, and the statement of the alternatives has the effect of heightening this confusion. The *Apology* and the *Crito* are more dogmatic and deductive in form, and from the nature of their subjects are hardly to be classed as dialogues of search.

A common feature of the dialogues just discussed, and one inherent in their method, is their negative outcome — no satisfactory definition is found. The real argument starts with an accepted, or suggested, or tentative definition. This is analyzed, and from it consequences are deduced. These consequences are found to involve absurdities and contradictions. Or, if the start is made from other accepted truths, the consequences are found to be in contradiction with the definition or its consequences. Hence the suggested definition is rejected, at least in its present form, and a new start has to be made with a new definition or some modification of the old.

Socrates was a master of that kind of analysis which got at meanings empirically. He could also show the absurd consequences deducible from assumed data among loose thinkers, and thus it was that he was able to take the conceit out of the windbags of knowledge. The logical connection between the absurd consequences and the falsity of the original proposition he seems to have grasped. But he had not developed the method far enough to take advantage of its power in the derivation of *new truth*. The negative outcome of the Socratic dialogues is explained when we remember that they are cast in the form of positive analysis, and hence that *logically the contradictory consequences do not allow of any inference to a new truth, but only to the falsity of the original proposition*. The function of negative analysis and the necessity of working out from mutually exclusive alternatives have not yet been seen. There are, however, some gropings after the complete form of analysis. Opposites are frequently set off against one another, and alternatives are stated. But these are very loosely used, not being true opposites or mutually exclusive alternatives, or not being used in the forward and positive movement of thought. We do not mean to say that, apart from this, the Socratic analysis did not have a forward movement. But its movement toward the truth was by a series of hitches. A proposition was analyzed to an absurd conclusion and rejected. A new

start was then made by choosing a new proposition or by modifying the original one in accordance with some suggestion received in the preceding process. This procedure might be kept up in some cases until some proposition was found the analysis of which did not necessitate as the outcome its rejection. But in this phase of the method it will be easily observed that *there is no adequate control in the determination of the direction in which to look for the truth*. It was a sort of logical "cut and try" procedure, more or less empirical in character, and not truly scientific.³⁵ It cannot be determinate in the search for truth until negative analysis is introduced and mutually exclusive alternatives are worked out. Plato came to see this need and he devoted whole dialogues, so far as their method is concerned, to the exposition of it.

ZENO'S ANALYSIS.

Zeno's analysis also seems to be lacking in the element of intellectual control, although there are intimations that he was vaguely reaching out for some positive significance to the negative outcome of his analysis. There seems to be a vague feeling that, because the Heraclitean doctrine of motion leads to paradoxical conclusions, somehow the contradictory consequences of Eleaticism are minimized. In the *Parmenides*, Plato has Zeno say:

The truth is, that these writings of mine were meant to protect the arguments of Parmenides against those who make fun of him and seek to show the many ridiculous and contradictory results which they suppose to follow from the affirmation of the one. My answer is addressed to the partisans of the many, *whose attack I return with interest by retorting upon them that their hypothesis of the being of many, if carried out, appears to be still more ridiculous than the hypothesis of the being of one*.³⁶

Zeno's argument in no way shows, however, that the absurd conclusions deducible from the Heraclitean hypothesis *involve the truth* of the Eleatic doctrine. Plato seems to feel that, while Zeno's analysis is significant, yet it is lacking in the element of intellectual control. It is an easy matter to get all sorts of contradictions when you are operating in the realm of sense-perception. The Zenonian paradoxes lose their significance because they are not worked out rigorously and exclusively in the intellectual realm, the world of ideas.³⁷

³⁵ For another statement of the value and the limitations of Socratic analysis, see WINDELBAND, *History of Ancient Philosophy*, pp. 128-30.

³⁶ *Parmenides*, 128.

³⁷ *Parmenides*, 135.

PLATONIC ANALYSIS.

The method of analysis can be found in all its phases and stages of development in the properly Platonic dialogues. It would be unnecessarily tedious to take up all the cases of its use, but I shall take up numerous illustrations in detail for the purpose of showing its significance clearly and making apparent the advance which it marks over the Socratic analysis.

The Gorgias.—Ostensibly the subject of this dialogue is rhetoric. It opens with a presentation of Gorgias, the rhetorician, who praises his art. Socrates is drawn into an argument with him, and soon catches him in a contradiction. But the turn which the dialogue quickly takes leads one to think that Plato's real interest in writing the dialogue was the ethical one. There is an assumption on the part of Gorgias and his friends that rhetoric is a fine and useful art because of the advantage that it gives to a man to have the power of attaining his ends through persuasion, even though he may be inferior in knowledge. This view of rhetoric seems to Plato not alone superficial, but above all immoral; for it makes the art the tool of the unrighteous against the righteous. He attacks the point of view of Gorgias by the assertion of a series of Socratic paradoxes. The real and vital argument of the dialogue centers about the following paradoxical positions of Socrates; (1) that to do injustice is worse than to suffer it (469, 473, 474); (2) that when a man has done evil he is happier if he be punished than unpunished (473). As incidental to the proof of these there arise two subsidiary paradoxes: (a) bad men do what they think best, but not what they desire, for the desire of all is toward the good;³⁸ (b) to be, and not to seem, is the end of life.³⁹ The position of his chief opponents, Polus and Callicles, may be summed up in the following statements: (1) might makes right (483–84); (2) law is nothing but the combination of the many weak against the strong (483); (3) the pleasant is the good (492). Now, while these two sets of propositions cannot literally be set off against each other as mutually exclusive alternatives, yet ethically considered the points of view which they represent are, taken as a whole, felt by the participants to be antagonistic and non-inclusive. In accordance with this feeling, the method employed in the defense and establishment of the Socratic paradoxes is indirect.

³⁸ *Gorg.*, 466, cf. 509, end.

³⁹ *Gorg.*, 511 to end of dialogue. See JOWETT, *Introduction to the Gorgias*, 2: 270 ff. and 303 ff.

The *opposite* point of view is attacked and shown to be shot through with contradictions when its implications and consequences are drawn out. Thus the negative of the Socratic position is proved untenable. The outcome of this negative conclusion of the negative analysis is of positive significance in establishing the truth of the Socratic position. The evidence for my interpretation of the method in which the Socratic paradoxes were proved is not alone internal, but also the explicit statement at the close of the argument upon them. The statement is as follows:

These truths, which have been already set forth as I stated them in the previous discussion, would seem now to have been fixed and riveted by us, if I may use an expression which is certainly bold, in words which are like bonds of iron and adamant; and unless you or some other still more enterprising hero shall break them, there is no possibility of denying what I say. For my position has always been, that I myself am ignorant how these things are, but that I have never met anyone who could say *otherwise*, any more than you can, and not appear ridiculous.⁴⁰

The establishment of the ethical paradoxes of Socrates has a connection with the ostensible subject of the dialogue, namely the function of rhetoric. The pointing out of this connection is a device which preserves the literary symmetry of the work. At the same time, it is done in such a way as to be an integral part of the method of analysis. When the truth of these paradoxes is accepted, the much boasted usefulness of rhetoric vanishes on any supposition of its advantage in helping to attain unjustly one's ends; for nothing can be useful except in so far as it helps one to do justly, or is used by him to persuade to his own punishment when he does wrong (480). Again, unless rhetoric is not to be a true art, but only a sort of flattery (462-67), the only alternative (implied) is the "position, which, according to Polus, Gorgias admitted out of modesty, that he who would truly be a rhetorician ought to be just and have a knowledge of justice."⁴¹

Within the total movement of thought, several interesting instances of the use of the method of analysis might be pointed out. In the passage 492-95, Callicles maintains the identity of pleasure and the good, a position which Socrates attacks by showing the disagreeable consequences which follow from it, clinching his argument with an explicit statement in 495: "I would ask you to consider whether pleasure, from whatever source derived, is the good; *for*

⁴⁰ *Gorg.*, 509.

⁴¹ *Gorg.*, 508.

if this be true, then the disagreeable consequences which have been darkly intimated must follow, and many others." Callicles still holds to his position, and Socrates attacks him through the mutually exclusive opposites of good and evil, identifying pleasure (on Callicles's hypothesis) with the good and pain with the evil; but pleasure and pain can coexist, then good and evil can coexist, which is contrary to the hypothesis that they are mutually exclusive opposites (495-97). There is a still further analysis from another point of view in 497-99. In 475 there is a case of the explicit statement of alternatives, followed by the elimination of all but one, whereupon that one is regarded as proved true. Similarly in 477 and in 478.

Here, then, within the limits of one dialogue are found both positive and negative analysis, the use of alternatives implicit and explicit, and the attainment of positive conclusions—though there is some vagueness and lack of rigor in the use of the complete method.

The Meno.—Early in the *Meno* virtue has been defined as "the desire of things honorable and the power of attaining them." Desire of the honorable is identified with desire of the good. Then begins an analysis of this definition. The very specification seems to imply that there are some who desire the evil also. This is admitted. Further analysis of this idea leads to the conclusion that they desire to be miserable and ill-fated, which cannot be held to be true. Then on this basis the definition has to be rejected. A return is then made to the definition, and it is attacked from a different point of view. The two parts of the definition are taken up separately: first as to the desire of things honorable, and secondly as to the power of attaining them. Analysis of the first leads to the unacceptable conclusion that one man is no better than another in respect to virtue. Before analyzing the second, the qualification "with justice" is added to "the power of attaining them." But justice is a part of virtue, and we have the unsatisfactory conclusion of virtue defined in terms of a part of itself. On three different counts, then, Meno's definition of virtue has to be rejected (77-79). This reduction to absurd consequences from several points of view is quite characteristic of Plato's analysis. There are two illustrations of *reductio ad absurdum* in the famous geometrical demonstration with the slave boy. His answer that the side of the square of double the value of the square whose side is two feet will be double that of the given square is followed out so that he sees that such a square will have an

area of sixteen square feet, whereas by hypothesis it ought to have only eight. Hence his answer is wrong and he must try again. His second answer that its side should be three feet is treated in like manner (82-83).

In the cases given from the *Meno* thus far the analysis is positive with a negative outcome. No alternatives have been stated by means of which inference could be made to the truth of the other proposition on the basis of the falsity of the one examined. There are passages, however, where there is a working with alternatives. A new attempt is made to define virtue by identifying it with knowledge (87). If this is correct, then virtue can be taught. The difficulties of this position are analyzed out at some length, in the process of the analysis several subordinate points being made by working through alternatives (88, 89, 96). The contradictions to which the definition of virtue as knowledge leads calls for its rejection. But there is an alternative proposition to this, namely, that virtue is right opinion; for there are only these two guides to action—knowledge and right opinion (97, 99). This alternative proposition is regarded as true⁴² by reason of the falsity of the other. This in turn has a bearing on the question whether virtue can be taught. Virtue is either natural, or acquired, or a God-given instinct (98-100). Whether it is right opinion or knowledge, it is not natural (98); if it is to be acquired, this must be because it is knowledge, a view which has already been rejected (98-99); virtue then is neither natural nor acquired, hence it must be what it is in order to be right opinion, a God-given instinct (99-100).

The Euthydemus.—This dialogue appears to some trivial and meaningless. It is not so when one has firmly grasped the idea that Plato uses the method of analysis, not for the purpose merely of landing one in hopeless contradictions, as the eristics did, but as having some positive significance, even if that positive significance be not explicitly pointed out. The destruction of one point of view, with him, meant the acceptance of another. The *Euthydemus* is a satire of eristic, but it is more than that. It is an illustration of the absurd and contradictory consequences which can be drawn where definition is not careful and words are used ambiguously. This has its significance in teaching indirectly that the symbols of language

⁴² That is, from the point of view of this discussion. Plato's own view seems to be that virtue in the highest sense is identical with wisdom in the highest sense. See pp. 32-34, 89, 91 of this book.

are functional with reference to thought, and not necessarily fixed and unambiguous. Furthermore, the *Euthydemus* is a *reductio ad absurdum* of that view of judgment which gives the predicate an existential force or makes the judgment an identical proposition.

The Republic.—In the first book of the *Republic* the discussion centers about the definition of justice. Cephalus defines justice “to speak the truth and pay your debts” (331). The first half of the definition is analyzed out to contradictory conclusion and abandoned. The second half is likewise analyzed out to conclusion which is absurd; it is then remodeled, when again absurd conclusions are derived which make out justice to be useless. This results in still further modifications of the definition, which upon analysis again result in contradictions (331–36). Thrasymachus defines justice as “the interest of the stronger.” This is reduced to contradiction with his own statement that it is just for the subjects to obey their rulers; for the rulers may themselves err as to what is their interest (338–39). But Thrasymachus maintains that no artist or ruler *qua* artist or ruler is ever mistaken. In opposition to this it is then shown that the ruler in his capacity of ruler merely is interested in the welfare of his subjects—that is his sole business *qua* ruler. Justice then is their interest and not his, the interest of the weaker and not of the stronger—a conclusion which is contradictory to the original definition which Thrasymachus proposed (340–42). Thrasymachus now, defeated in the argument, expounds at length the advantages of injustice (343–45). Put in the form of a proposition, his contention is that the life of the unjust is more advantageous than the life of the just; and, further, that injustice is virtue and wisdom, justice the opposite. Through an intermediate proposition which Thrasymachus accepts the consequences are deduced that the just is wise and good; the unjust evil and ignorant (347–50). The second half of Thrasymachus’s position has, then, to be rejected. Before taking up the first half, a little piece of negative analysis is introduced. Taking the conclusion just reached, the position can now be refuted that injustice is stronger and more powerful than justice; for perfect injustice is shown to be self-destructive in its effects, defeating its own ends (351–52). Returning to the first half of Thrasymachus’s position, through a doctrine of ends the conclusion is reached that justice is the excellence of the soul and injustice the defect; the just is happy and the unjust miserable. But happiness and not misery

is profitable. Therefore injustice can never be more profitable than justice (352-54).

It will now be seen that the essential movement of thought in the first book of the *Republic* is through positive analysis, unaccompanied by alternative propositions. The outcome is negative, the position of Thrasymachus and his friends is overthrown; but no definition of justice is established in the place of those proposed. The definition of justice is reserved until its nature has been seen in the analysis of the ideal state.

It may be noted that in the fourth book of the *Republic* there is a very clear case of the use of negative analysis. The alternatives are stated as follows: "Which is the more profitable, to be just and act justly and practice virtue, whether seen or unseen of gods and men; or to be unjust and act unjustly, if only unpunished and unreformed?" In the light of the previous discussion (all that follows Book I), the question is now declared to be absurd; for analysis of the second alternative shows that through injustice the very essence of the vital principle is undermined and corrupted, and under that condition it is inconceivable that life is worth the having. The first alternative, then, must be accepted.

The Phædo.—Some cases of the use of the method of analysis are found in the *Phædo*. One is found in connection with the argument for the pre-existence of the soul. Alternatives are worked out (75) and stated (76): We come into life having knowledge; or knowledge is recollection. The first alternative is taken up for examination. If we come into life having knowledge, we ought to be able to give an account of it from the beginning, which we cannot do. The first alternative is then untrue, and the second is proved, namely, that knowledge is recollection. It is felt that this proof of the pre-existence of ideas carries with it the proof of the pre-existence of the soul (76, 77). But what about the soul's living after death? It is said that the soul is a harmony. Then just as the harmony dies with the perishing of the strings, so the soul passes away with the dissolution of the body. This argument is refuted from three different points of view in succession. (1) It is shown that this view of the soul leads to a conclusion which is contradictory to the previously proved and accepted doctrine that knowledge is recollection (91-92). The conclusion to be drawn from this is clearly stated: "Having, as I am convinced, rightly accepted this conclusion [that knowledge is recollection], I must, as I suppose, cease to argue or

allow others to argue that the soul is a harmony" (92). (2) The assumption that the soul is a harmony leads to the conclusions: (a) of degrees in the being of the soul; (b) of a harmony within a harmony in case of the virtuous soul, and of an inharmony within a harmony in case of the vicious soul; (c) all souls must be equally good. The significance of these curious and paradoxical consequences in refutation of the idea that the soul is a harmony is explicitly noted: "And can all this be true, think you? he said; for *these* are the consequences which seem to follow from the assumption that the soul is a harmony?" (93-94.) (3) The assumption that the soul is a harmony involves the view that the soul cannot utter a note at variance with the tensions, relaxations, etc., of the strings of which it is composed. This is in contradiction with the known fact that the soul leads, opposes, and coerces the "elements" (94).

From three different points of view it has now been proved by positive analysis with negative outcome that the soul is not a harmony. But this is not the positive result desired, namely, that the soul is immortal and indestructible. This proof is led up to by a preliminary discussion which serves to secure a long series of accepted truths relative to the final argument. This series concludes with the deduction from the essentially opposite and mutually exclusive character of life and death that the soul, which is the life of the body, cannot participate in death. This outcome is then made more rigorous by a further analysis both on the positive and the negative side. "If the immortal is also imperishable, then the soul will be imperishable as well as immortal." But this positive analysis is not felt to be conclusive; for if the soul is not immortal, "some other proof of her imperishableness will have to be given." But if the argument is put in the form of negative analysis, "no other proof is needed; for if the immortal, being eternal, is liable to perish, then nothing is imperishable." This is contrary to fact in the case of God and the essential form of life. Therefore the soul is imperishable (100-107).

The Theætetus.—The movement of thought in this dialogue is, taken as a whole, positive analysis with negative outcome. There are minor and subsidiary movements which might be otherwise classified as, *e. g.*, a recognition of alternatives in some places (164, 188, etc.), also at least one important doctrine developed by direct procedure (184-86). The argument of the dialogue commences with an attempt on the part of Theætetus to define knowledge. In the course of the dialogue three such attempts are made and discussed.

1. Knowledge is sense-perception (152-86). This is identified with the doctrine of relativity of the Protagorean school, and it is discussed largely from that point of view. But first of all the doctrine of relativity is itself developed so as to show what it involves in its relation to this problem. It is interesting to note that in this ancillary portion of the task of refuting the definition the method of analysis is employed. Perception may be relative (*a*) to the subject and the object, the percipient and the perceived; (*b*) to the subject and an object which is itself relative; (*c*) to a relative subject and a relative object, each having but a momentary existence. Each of these possible meanings of relativity is taken up in order and found to involve contradictions. The tacit assumption is, in each case, that when the cruder form of the doctrine of relativity breaks down through contradictions inherent in it there is an alternative, a way of escape, through taking refuge in a more refined form of the doctrine. In this way the doctrine is developed to its utmost logical limit. When this is done, it is found to involve difficulties still. But waiving these aside for the time being, a return is made to the definition itself. The fundamental assumption of this definition is the identity of knowledge and sense-perception (163 ff.). Analyzing this assumption, it is found to involve verbal contradiction (163-65). In connection with the assumption, "What seems to a man is to him," the doctrine of identity breaks down again (170-84), through analysis of it to a conflict with common-sense and other conflicts (170-71) and the destruction of any possibility of judgments involving futurity (177-79). A return is then made to the doctrine of "universal flux," and it is found also to involve irreconcilable conclusions. An examination is now made as to the sources of those elements of conscious experience which we are most ready to admit as knowledge, and it is found that they do not come through the sense-organs (184-86). This reconstruction and the negative outcome of the positive analysis both coincide in proving the falsity of the definition of knowledge as perception.

2. Knowledge is true opinion (187 ff.). This definition is taken up and analyzed, the first thing being noted that the specification "true" opinion would seem to imply the existence of false opinion. When this assumption is examined, it is found that in the sphere of knowledge false opinion is impossible (187-88), and likewise in the sphere of being (188-89); hence it must be sought elsewhere, if at all. There seems to be one other alternative — that false opinion is

a sort of heterodoxy, a confusion of one thing for another (189). A list of cases is drawn up where such confusion is impossible, and these are then excluded from consideration (192). The only remaining possibility is the confusion of thought and sense (193). Is it true? A serious difficulty arises from its failure to explain mistakes about pure conceptions of thought, like numbers (196). The outcome is that "we are obliged to say, either that false opinion does not exist, or that a man may not know that which he knows." The former alternative seems to be the only one possible. A further analysis of knowledge reveals the fact that the accounting for false opinion is bound up with the problem of defining knowledge. Hence a return is made to the original question, and the examination is resumed of the definition of knowledge as true opinion (200). But in the law court the lawyer may judge rightly on the basis of true opinion without knowledge. Now, "if true opinion in law courts and knowledge are the same, the perfect judge could not have judged rightly without knowledge;" for knowledge and true opinion are by hypothesis identical. But he did give the right judgment without knowledge, and "therefore I must infer that they are not the same" (201). This final argument is almost a perfect *reductio ad absurdum* of the identity of knowledge and perception. A new attempt has to be made.

3. Knowledge is true opinion combined with reason or explanation (201-10). This definition is attacked in the same way. If explanation means pointing out the elements of a compound, no gain is made by the addition of the term to the definition of knowledge; for analysis reveals insuperable difficulties. Giving a reason may mean reflection of thought in speech, enumeration of the parts of a thing, or a true opinion about a thing with the addition of a mark or sign of difference. In either of the first two senses, contradictions are deducible; and in the third sense you finally get knowledge defined in terms of itself, which is not a definition at all. The third definition of knowledge has then failed like the other two.

The final outcome of the *Theætetus* is negative. It could not well be otherwise when cast in the form of positive analysis. The definitions discussed are not related in an alternative or mutually exclusive way; hence there is no opportunity to infer from the proved falsity of two of them to the truth of the third. Yet this negative outcome has some positive significance in the mind of Plato. In the *Parmenides* the problem of being and not-being is

discussed at some length, and the difficulties of both conceptions are exploited. In the *Sophist* this problem of not-being is conceived of as at bottom one with the problem of false opinion. Without going into the discussion in detail, it might be well to point out the conclusion reached there.

If not-being has no part in the proposition, then all things must be true; but if not-being has a part, then false opinion and false speech are possible, for to think or to say what is not — is falsehood, which thus arises in the region of thought and in speech.⁴³

In undermining a theory of generalization like that of the modern associational school of Locke and Mill,⁴⁴ and like it based on an associational psychology, and in showing the inadequacy of the existential conception of judgment, Plato prepared the way for the further analysis and reconstruction of the function of judgment and of the negative which is worked out in the *Parmenides* and the *Sophist*. I hold that it is a mistake to suppose that Plato was necessarily ignorant of the bearing of the negative outcome of the *Theætetus*, or of any other dialogue, merely because he defers discussion of the problem till some other time. It certainly is a remarkable fact how he makes use of such negative outcomes in further reconstructions along positive lines.

The Parmenides.—It has already been noted that this dialogue may be regarded from one point of view as a long and thorough exposition of the method of analysis (see p. 50). Here first the positive and negative phases of analysis receive explicit and specific recognition as necessary parts of one complete method of investigation. This statement is worthy of quotation.

But I think that you should go a step further, and consider not only the consequences which flow from a given hypothesis, but also the consequences which flow from denying the hypothesis. (136.)

As an illustration of what is meant by this procedure, the Parmenidean hypothesis of the one is taken up and examined from every point of view on both the positive assumption and on the negative. The larger part of the dialogue is taken up with this analysis. It is preceded, however, by a critique of the Platonic Ideas.

The most apparent division of the dialogue is into two parts: (1) a criticism of Platonic Ideas, (2) a criticism of the Eleatic doctrine of Being. I think, however, that the real function of the

⁴³ *Sophist*, 260; cf. 261, beginning.

⁴⁴ *Theæt.*, 201 ff.

dialogue is somewhat different from that which appears on the surface from an observation of subject-matter. The result of the first investigation seems at first to be a proof of the untenability of the Ideas. The hypothesis is shown to involve great difficulties. There is the problem of the relation of individuals to the Ideas. Is it one of participation or of resemblance? Then, too, the process of referring back to an Idea, when once started, would seem to have to go on to infinity. And, thirdly, there is the difficulty of the relation of the ideas within us to absolute Ideas. Yet, in spite of these difficulties, Plato feels that the doctrine of Ideas is not to be abandoned. There is an alternative, the consequences of which are far more disastrous than those deducible from the doctrine of Ideas. That alternative is the *non-existence* of these Ideas. He feels that there are difficulties in the other position, but that this is wholly untenable, necessitating the acceptance of the other in spite of its difficulties. This is wholly in keeping with the movement of Plato's thought and his method of procedure. The way it is put in the *Parmenides* is as follows:

And yet . . . if a man, fixing his attention on these and the like difficulties, does away with Ideas of things and will not admit that every individual thing has its own determinate Idea which is always one and the same, he will have nothing on which his mind can rest; *and so he will utterly destroy the power of reasoning.*⁴⁵

The criticism of the Eleatic doctrine of Being seems not to have its greatest significance in the outcome with reference to that problem, but in its bearing upon the function of the copula and the negative in judgment. The eristics had made predication impossible,⁴⁶ through their treatment of the judgment as existential. Also the negative "is not" was given the existential force and made to signify absolute non-existence.⁴⁷ The judgment, then, if positive, could be nothing but an identical proposition, and hence valueless; if negative, was an absurdity and impossibility.

There is both in Greek and in English an ambiguity in the meaning of "is." The eristics played upon this ambiguity in such a way as to throw the emphasis wholly upon the existential force of the word, and thus brought out their contradictions of ordinary common-sense. Plato out-eristics the eristics in weaving to and fro between

⁴⁵ *Parmen.*, 135; cf. *Soph.*, 259-260, 249.

⁴⁶ *Soph.*, 251 E, 259 E, 251 C; *Theat.*, 201 E-202 A. These references from SHOREY'S *Unity of Plato's Thought*, p. 58, footnote 433.

⁴⁷ *Soph.*, 238 C-241 A; SHOREY, *op. cit.*, footnote 434.

the two meanings of the copula. He shows himself by his analysis a master not only of their game of producing contradictions, but also goes them one better by analyzing their own position to contradictory conclusions. All this, it seems to me, is something more than a play or a satire. It is a bringing to clear consciousness the fact that there *is an* ambiguity in the use of the copula and of the negative. When this is seen, the judgment can become a vital knowledge-process, having a function denied to it when viewed solely in the existential sense. In the negative judgment also you have not merely an assertion of not-Being. In the very denying of one thing to the subject you virtually assert otherness of the subject; in saying that a thing is not this, you are not saying that it is not anything, but that it is other than this. The only not-Being that is intelligible to thought is such not-Being as is implied in otherness.⁴⁸

The analysis both positive and negative of the *Parmenides*, though it results in both cases in a negative outcome through the contradictions which are reached, is a preparation for a reconstruction of the signification of predication and of negation. The negative outcome may be explained from the fact that, though we have the two phases of analysis here, yet they are not made to work through wholly unambiguous terms. It is another point of significance to this dialogue that it shows so clearly the necessity of viewing language as something other than a static thing, and hence in arguing it is necessary to use the terms employed always in the same sense. The abstract and highly rational use of such terms as "one," "being," "other," "like," "same," "whole," and their opposites is a different thing from their concrete use.⁴⁹ As concrete terms any sort of conclusion can be deduced from them through playing upon variations in their meaning.⁵⁰

The Sophist.—The argument of the *Sophist* is in large part in the form of the method of analysis. The problem of not-Being, where the term is used in the sense of absolute denial of existence and absolute separation from Being, is taken up in this way. The contradictions involved in predication, and even in the mere use of the word itself, are pointed out. The inference from this is made that the assumption is false, and that Parmenides's philosophy must be put to the test. Plato undertakes to show that such a separation between Being and not-Being must be abandoned; and he explicitly

⁴⁸ See SHOREY, *op. cit.*, pp. 58, 59.

⁴⁹ Cf. *Phileb.*, 14-15.

⁵⁰ *Parmen.*, 135; cf. *Soph.*, 259.

points out his reason for thinking so—the unavoidable contradictions which result from the Parmenidean position (237–41). His rejection of the various forms of philosophy, which he examines at some length (242–51) with reference to this problem, is on the ground of the contradictions into which they fall when analyzed out. The inference from these negative results of a supposed separation of Being and not-Being is that they ought not to be separated absolutely. But this does not mean that they necessarily mingle absolutely. And here comes in one of the best illustrations of analysis through alternatives. There are three possible alternatives: (1) no participation, (2) indiscriminate participation, (3) participation or intercommunion of some ideas with some. Each of these is taken up in turn. The first two are rejected on the ground of their contradictory consequences; and the third is accepted as the only remaining alternative. The whole argument is followed by a careful summary so that the full positive force of the *reductio ad absurdum* is brought out (251–52). Having established this doctrine of the intercommunion of ideas, he proceeds to develop it and to apply it to the reconciliation of the contradictions previously deduced in the *Sophist* and also in the *Parmenides* (253–58), indicating explicitly that one source of such contradictions, as was pointed out in our discussion of the *Parmenides*, is the verbal shifting of words and meanings (259). He concludes his argument on this point of the separation of Being and not-Being by an argument against the universal separation of classes that is very characteristic of the way in which he is always going back to the principle of contradiction and making it yield positive results rather than merely negative ones. This is his statement:

The attempt at universal separation is the final annihilation of all reasoning; for only by the union of conceptions with one another do we attain to discourse of reason.⁶¹

Any proposition that leads to the annihilation of reasoning or the impossibility of knowledge has been reduced to an absurdity and has to be abandoned. Having disposed of the absolute separation of Being and not-Being by a general argument, and thus made possible the reconciliation of the contradictions of the *Parmenides*, he proceeds, as has already been shown (see p. 72), to apply the conception of the nature of not-Being just reached to the solution of the problem of false opinion in the *Theætetus*. Thus the *Sophist* is the develop-

⁶¹ *Soph.*, 259 end to 260 beginning; cf. *Parmen.*, 135; *Soph.*, 249.

ment of the positive significance of the negative outcome of both the *Theatetus* and the *Parmenides*. Predication is again made possible and significant. The copula and the negative in the judgment have significance in the knowledge process.

The Statesman and the Sophist.—We have taken up but one phase of the *Sophist*. The other phase can be discussed in connection with the *Statesman*. Both these dialogues aim to get at definitions through the process of logical division. The definitions of the *Sophist* and of the *Statesman* come at the end of a long process of dividing species with the greatest care in the matter of getting classes that are mutually exclusive, until at last the thing sought to be defined is caught in a final class in such a way as to be distinguished from all other things and at the same time to have its own essential nature indicated. The Ideas, as was shown in the *Sophist*, have intercommunion some with others. Hence the problem of definition is the problem of dividing them off properly, while at the same time preserving their integrity as to the principle that runs through the whole.

He who can divide rightly is able to use clearly one form pervading a scattered multitude, and many different forms contained under one higher form; and again one form knit together into a single whole and pervading many such wholes, and many forms existing only in separation and isolation.⁵²

Summary.—The study of the foregoing dialogues is a revelation of the fact that Plato was familiar with and used the method of analysis in all its phases. In one dialogue one phase may be predominant, in another another, according to the purpose to be conserved. In some places the main object is the destructive one of clearing away obstacles to the position that he wishes to maintain. No positive conclusion is cared for; the main thing is refutation. Here positive analysis, with its negative outcome, is wholly adequate; and it is not necessary to suppose that this negative outcome has, in the mind of Plato, no positive significance. Positive analysis is also adequate when the main object is to satirize the position of his opponents or contemporaries, or when he skilfully stimulates the curiosity and awakens the interest of his hearers by leading them into a tangle of contradictions with reference to things which they thought that they understood perfectly. But when he wishes to secure positive results, he also knows how to set off alternative propo-

⁵² *Soph.*, 253.

sitions against each other, either of which excludes or negatives the other, so that by proving one of them false he has the right to infer from this negative outcome positively to the truth of the other proposition. The advance over what I have called Socratic analysis, whether that really represents Socrates's method, or whether it is employed purposely by Plato himself in that group of dialogues merely because it was adequate to the purpose in mind,⁵³ is in the use of negative analysis, especially in that form in which alternatives are either clearly stated or are clearly in mind. The *Theætetus* is a good illustration of positive analysis taken by itself; the *Parmenides*, of both positive and of negative analysis in more or less isolation from each other, so far as the inference to new truth is concerned; the *Sophist* and the *Statesman* exhibit the method whereby mutually exclusive alternatives may be derived; the *Sophist* furnishes a good illustration of the power of analysis when conscious use is made of the leverage which is given by mutually exclusive alternatives. Such instances may be found elsewhere, with a greater or less degree of perfection. So markedly do the phases, and the results of the different phases, of analysis stand out in the *Theætetus*, the *Parmenides*, and the *Sophist* and *Statesman* taken together, that one might with some reason argue that they were written with the pedagogic purpose in mind of exhibiting the method of analysis in detail.

THE RELATION OF MATHEMATICS TO PLATONIC ANALYSIS.

The clearest positive intimation of the influence of mathematics in the determination of Plato to the use of the method of analysis is to be found in the *Meno*. There the suggestion is made to discuss the question of whether virtue can be taught by assuming a hypothesis and deducing consequences, *as in geometry*.⁵⁴ Now, this argument from hypothesis, as we have seen at some length, is very characteristic of Plato's procedure. This he himself recognizes explicitly in several places, aside from the internal evidence which we have given.⁵⁵

Philosophical problems usually involve great complexity. On this account, while we may admit that it is possible that all the logical steps involved in the method of analysis might have been discovered wholly within the field of philosophical discussion, yet this is improbable. Especially is this true when the same method is actually being

⁵³ See footnote, p. 59 of this book. ⁵⁴ *Meno*, 86-87; see pp. 84 ff. of this book.

⁵⁵ See *Phædo*, 99-100; *Parm.*, 136; *Gorg.*, 509; *Phædo*, 106; *Rep.*, 6: 510-11.

used in mathematics — a field of investigation where the intellectual control of problems can be made more perfect, where relations are more sharply defined, and where simplicity is attainable in the highest degree. Now, Plato was interested in pure mathematics, and he was especially interested in mathematics on the side of its qualities, characteristics, processes, methods, and, in general, everything that had any philosophical or logical significance. Whether Plato's interest in the method of analysis had its origin on the side of philosophy or of mathematics makes little difference. When once this interest had dawned, it would find its greatest opportunity of realizing itself in complete logical form within the field of mathematics. It is also characteristic of Plato to study method in easier and clearer cases first and then to apply it to the more difficult.⁵⁶ We might naturally expect that he would first come to clear consciousness of this method in mathematics. In doing so, as we seem justified in inferring that he did, and as tradition confirms, he at once made a distinguished contribution to the logic of mathematics and at the same time got the clue to the essential conditions that the method must fulfil in order to be of service as a rigorous instrument of investigation in philosophy. It was under the influence of the mathematical element that he got the stimulus which made him transcend Socratic and Zenonian analysis by the introduction of those phases of the method which make it complete.

⁵⁶ See *Rep.*, 2: 368-69 and *Soph.*, 218; discussion on p. 48 of this book.

CHAPTER IV.

RELATION OF MATHEMATICAL PROCEDURE TO DIALECTIC.

It would appear from the preceding discussion that mathematical procedure — at least the method of analysis in some form — is on the logical side the most fundamental feature of Plato's dialogues of investigation. The term "dialectic" is quite loosely used to signify in general any procedure which gets at a new truth or higher point of view through discussion and analysis. In this sense of the word it includes Socratic analysis and also mathematical analysis. In some places Plato's use of the term "dialectic" makes it a sort of poetry. The soul gazes directly upon the reality of the universe, beholds unfettered by sense the eternal being by the aid of pure intelligence alone,¹ and finds in so doing her true love; here dialectic is akin to love,² a feeling of affinity with the truth. In this sense of the word, dialectic would include the mathematical process in so far as direct intuition is involved. But there are many places where Plato uses the term "dialectic" in a more restricted and technical sense, and where he appears, at least, to make a distinction between dialectic and mathematical procedure. This makes necessary some discussion of the relation of mathematical procedure to dialectic proper.

As has been suggested before, Plato seems to have been subject to a twofold movement of thought, the activities of which ran parallel to each other, interacting upon and modifying each other. One phase of his thinking moved along the path of a logical *a priori* demand; the other was mathematical. The first movement was closely connected with a fundamental interest of his — namely, the practical, or ethical. When the validity of ethical standards was impeached by the Protagorean sensationalistic philosophy, Plato dreamed of a method which should secure results free from skeptical outcome by being empirical in none of its elements or processes. It should attain its conclusions solely through the exercise of the reason. Its data, its processes, its results, should all be rational. All the ethical concepts should be deducible from hypotheses, or principles, demanded by an active intelligence, not imposed by sense, and these in turn should be traced back step by step to one supreme teleological,

¹ *Phædrus*, 247.

² *Symp.*, 210 ff..

non-empirical principle—the Idea of the Good. Such a method Plato conceived would give us knowledge of true being—abiding, changeless, eternal. The problems which center in securing this are *par excellence* the problems which should engage the thought of the philosopher. The method which would thus work wholly in the realm of the rational and secure absolute knowledge is called by Plato dialectic. Dialectic is, then, in this more technical sense of the word, the ideal of philosophic investigation. It is the demand of the *a priori* logical movement of thought. But described in these terms it has as yet little specific content. This content will come out in further discussion.

With reference to the points just made—Plato's ethical problem, and his feeling of the need of finding a method of attack which should proceed along wholly rational lines—a passage in the *Phædo* (96–101) is very significant. Here Plato seems to have reached the point where he is unwilling to accept the statement of conditions as an explanation of any phenomenon of nature or fact in mathematics, but he demands an explanation in terms of final cause—a teleological explanation. How to give such an explanation is his problem. He feels that the key to its solution is to be found in rational rather than in natural process. He “has in mind,” he says “some confused notion of a new method.”³ so when he finds Anaxagoras saying that “mind is the disposer and cause of all,”⁴ he hails this notion with “delight,” thinking that at last he is going to have the solution of his problem. The ground of his hope in Anaxagoras was that he thought that when he spoke of mind as the disposer of all things, he would show how all things are as they are because this was best.⁵ He expected to see cause identified with the good. He then goes on to tell of his great disappointment in Anaxagoras, for he learned only of conditions and not at all of final causes. The futility of such explanations he illustrates by supposing that the reason why Socrates sits and awaits his execution instead of running away be given in terms of the structure and function of the various parts of the body, instead of in terms of his “choice of the better and nobler part.” In developing this point, he says:

There is surely a strange confusion of causes and conditions in all this. It may be said, indeed, that without bones and muscles and the other parts

³ *Phædo*, 97; ἀλλά τιν' ἄλλον τρόπον αὐτὸς εἰκῇ φύρω.

⁴ νοὺς ἐστὶν ὁ διακοσμῶν τε καὶ πάντων αἴτιος.

⁵ εὖ ἔχειν, βέλτιστος, and ἀμείνων are used.

of the body I cannot execute my purposes. But to say that I do as I do *because of them*, and that is the way in which *mind* acts, and not from the choice of the *best*, is a very careless and idle mode of speaking. I wonder that they cannot distinguish the cause from the condition.⁶

Again, in explaining the relations of the physical universe they make the same blunder of ignoring final cause.

Any power which in arranging them as they are arranges them for the best never enters into their minds, and instead of finding any superior strength in it, they rather expect to discover another Atlas of the world who is stronger and more everlasting and more containing than the *Good* (τὸ ἀγαθόν); *of the obligatory and containing power of the Good they think nothing, and yet this is the principle* (τῆς τοιαύτης ἀτίας) *I would fain learn, if anyone would teach me.*⁷

Thus both in the realm of conduct and in the realm of nature Plato is seeking for explanation in terms of final cause; and without question in the realm of ethics he identifies that cause with the principle of the Good. Such is the outcome of the ethical problem for Plato when he follows along the path of the logical *a priori* demand—a demand which, we have seen, itself sprang out of a reaction against a particular solution of the ethical problem. He has come to the distinction between sense and intellect; and, throwing stress upon rational process, this emphasis being in turn strengthened by the mathematical influence, he has exalted *mind* to the highest place. But mind, intelligence, presupposes purpose. Ethics demands that this purpose be in the direction of the Good. Thus he reaches the demand for the teleology of the Good. In the Idea of the Good we have united both the rational, which is necessary in order to transcend the doctrine of relativity, and also the ethical. Plato feels that this is the outcome that is required. But what the method, or technique, of obtaining it? Certainly not any that admits empirical elements at any stage.⁸ In the *Phædo* he “has in mind some confused notion of a new method.”

We have seen the conditions out of which the demand came for a new method, and also the conditions which this new method, dialectic, must fulfil—what its nature in general must be. How is such a method to be evolved? Dialectic in this technical sense must certainly be a long and tedious process—the elimination of the sense-elements well-nigh impossible. He himself indicates that only after the severest practice can dialectic be mastered. It involves the

⁶ *Phædo*, 99.

⁷ *Phædo*, 99.

⁸ Cf. *Phileb.*, 58, 59, 61.

severest abstraction and the most highly rational processes. Also it involves elements of direct intuition on the part of a highly developed and exceedingly active and keen mental sight. Its processes cannot be readily exhibited, any more than the process of seeing green with the physical eye can be exhibited and explained to one who has never seen. Hence Plato turns to mathematics — the second path along which his thought moved to the same goal. Here is a process, which, in one realm — that of a particular exact science — attains to ideas wholly rational and free from the sense-element. This process serves as the model — the ideal which should be attained in every realm of philosophic thought. Mathematical method gives the cue to Plato for working out the problem of the “new method” which his definition of the ethical problem demands. Whatever may have been the difficulties in the mathematical method of reaching the goal of rational conclusions, it had for Plato the very great advantage of being actually capable of having its processes exhibited. Furthermore, it had so much in common with the method of which he was in search that training in it served as direct mental preparation for the exercise of dialectic.

As dialectic came to the full and clear consciousness of Plato in its relation to mathematical procedure, the nature of dialectic as a process can be best explained by a more detailed discussion of the relation of mathematical procedure to dialectic. What Plato seized upon as most suggestive in mathematical procedure was the method of analysis with its hypothetical procedure.⁹ This has already been explained at length. This method served as the point of departure for him in the formulation of his dialectic method. Also there was a style of argumentation prevalent against which Plato reacted. This he called eristic. The nature of dialectic needs to be studied in relation to this as well as in relation to the mathematical method of analysis, both in order to understand dialectic better and also in order to understand the significance of the mathematical element in dialectic.

Eristic starts from premises, but differs radically from dialectic in spirit. In the first place, the eristic prefers to start from his own premises, which he considers true unless his opponent can refute them; the dialectician is willing to start from the premises of his opponent and analyze them out to their conclusion. If he starts with his own premises, it is always with the assent of his opponent.¹⁰

⁹ *Meno*, 86-87; *Rep.*, 6: 510-11.

¹⁰ *Meno*, 75, 79; *Soph.*, 259.

Now, what does this difference imply? It means that the dialectician is fully conscious of the hypothetical character of his inquiry, while the eristic is not. The dialectician is interested in the interrelation of premises and conclusions, realizing that a conclusion of a certain sort has as much significance with reference to the premises as that premises of a certain sort have with reference to the conclusion, and he wants to sift out the truth in so far as that is a matter of the relation of premises and conclusion. Plato is fully conscious of the hypothetical character of his method and explicitly recognizes it.¹¹ In the second place, as has just been intimated, the dialectician approaches a discussion in the spirit of one who is searching for truth; the eristic, as one who will maintain a point, more particularly as one who delights to puzzle and overwhelm his opponent with contradictions. This is brought out in several passages. I will give one or two citations.

The disputer (*χωρίς δὲ διαλεγόμενος*) may trip up his opponent as often as he likes, and make fun; but the dialectician will be in earnest, and only correct his adversary when necessary.¹²

He will imitate the dialectician who is seeking for truth, and not the eristic, who is contradicting for the sake of amusement.¹³

In the third place, the eristic (*ἀντιλογικός*) confuses the hypothesis and its consequences¹⁴ — a point closely related to the preceding, whereas the dialectician understands their true relation to each other. The eristic has a tendency to take as final his conclusions, or at least to leave the discussion without any help for those who have been following it with reference to a positive outcome. He rejoices in having left them in the midst of puzzles and contradictions which seem hopeless. Indeed, that is the aim of the whole argument; for it gives everyone the impression of superior argumentative power on his part. The dialectician, on the other hand, while he may lead up to just as absurd, paradoxical, and contradictory conclusions, yet does so with a consciousness of the fact that these conclusions are so bound up with the premises that in coming out as he has done he has a right to a further inference, namely, with reference to the truth or falsity of the premises. He uses negative outcomes, not as necessarily final, but as indices of the need of reconstruction or of further inference. Dialectic is more than an instrument of refutation; it is a process of investigation.

¹¹ *Phædo*, 106, 107.

¹³ *Rep.*, 7: 539; cf. 5: 454 A; 6: 499 A; *Soph.*, 259.

¹² *Theætetus*, 167 E.

¹⁴ *Phædo*, 101 E.

The nature of dialectic can be seen with still greater clearness and with more of technical force by studying the relation between mathematical procedure and dialectic. The position has already been taken in the discussion of Plato's divided line¹⁵ that, so far as their ideal character is concerned, there is no difference between mathematical concepts and the Ideas. The third and fourth divisions on the side of content were held to be not essentially different, but it was intimated that there was an essential difference on the side of process. The point which I wish to make here is that this difference on the side of process is precisely the difference between mathematical and dialectical method; and that such difference as may be felt on the side of content as the result of this is not so much one of essence as of cognitive aspect. While I would agree with Milhaud in part, yet it seems inconceivable that Plato should have dwelt so much on the difference between mathematics and dialectic, if there was no sense in which the distinction was in his mind significant. With this preliminary statement of position, I will now turn to the discussion in more detail, so as to make this point clearer.

However high a value Plato may set upon the work of the mathematician, it is still true "that the skilled mathematician is not a dialectician."¹⁶ In this same context he intimates the ground upon which he makes this difference. It is that the mathematician cannot, like the dialectician, give a reason (*λόγον*). In what sense this is true will be seen later. I give the statement here to bring out vividly the fact that Plato does not identify mathematical procedure and dialectic. He seems to feel that the mathematician is under more or less constraint from the sense-element. His initiative is not found in a free activity of thought. There is something given to him behind which he does not go. "Mathematicians do not make their diagrams, but only find out that which was previously contained in them."¹⁶ They start with data that are given, or at the best from some fixed point intellectually determined, and reason through an intuition of relations and logical connections to necessary conclusions. But dialectic examines the validity of the data themselves. Plato brings this point out very clearly in his discussion of the different use which mathematics and dialectic make of hypotheses. Before further discussion, it may be well to give some significant citations.

The inquiry [in the third division, where mathematics has been placed] can only be hypothetical, and instead of going upward to a principle (*ἐπ'*

¹⁵ *Rep.*, 7: 531 D, E.

¹⁶ *Euthydemus*, 290. *Συμμ* 229

ἀρχήν) descends to the other end; in [dialectic or διαλεκτική], the soul passes out of hypotheses (ὑποθέσεων) and goes up to a principle (ἀρχήν) which is above hypotheses (ἀνυπόθετον), making no use of images (εἰκόνων) as in the former case, but proceeding only in and through the ideas themselves (αὐτοῖς εἶδеси).¹⁷

You are aware that students of geometry, arithmetic, and the kindred sciences assume (ὑποθέμενοι) the odd and the even and the figures and three kinds of angles and the like and their several branches of science: these are their hypotheses (ὑποθέσεις) which they and everybody are supposed to know, and therefore they do not deign to give any account of them either to themselves or others; but they begin with them and go on until they arrive at last, and in a consistent manner at their conclusion.¹⁸

And when I speak of the other division of the intelligible (τοῦ νοητοῦ,) you will understand me to speak of that other sort of knowledge which reason (ὁ λόγος) herself attains by the power of dialectic (τοῦ διαλέγεσθαι δυνάμει), using the hypotheses (ὑποθέσεις), not as first principles (ἀρχάς), but only as hypotheses (τῷ ὄντι ὑποθέσεις), that is to say, as steps and points of departure into a world which is above hypotheses (ἀνυπόθετον), in order that she may soar beyond them to the first principles of the whole (τῇν τοῦ παντός ἀρχήν), and, clinging to this, and then to that which depends on this, by successive steps she descends again without the aid of any sensible object (αἰσθητῶ) from ideas, through ideas, and in ideas she ends (εἶδεν αὐτοῖς δι' αὐτῶν εἰς αὐτά, καὶ τελευτᾷ εἰς εἶδη).¹⁹

A careful study of these passages reveals both likenesses and differences between mathematical process (διάνοια) and dialectic (νόησις). Both make use of hypotheses. *Mathematical process* (διάνοια) does not discuss their validity; dialectic (νόησις) traces them back to their ground, not regarding them as self-sufficient. Both seek ideas free from sense. *Mathematical process* (διάνοια) uses visible symbols as a means; *dialectic* (νόησις) makes no use of sensuous symbols. But the significant thing is the difference in the use of hypotheses. In the one case of the use of the method of analysis the mathematicians seem to have come to a somewhat clear conception of the hypothetical character of their reasoning. They are not aware that their ordinary data are also hypothetical in character, that they rest back upon something else. They view them as ἀρχαί not as ὑποθέσεις.²⁰ The dialectician goes to the full length of seeing, not alone the data of the method of analysis as hypothetical, but also the whole body of mathematical data. It would seem that, starting from the one clear instance of the hypothetical method in

¹⁷ *Rep.*, 6: 510 B.

¹⁸ *Rep.*, 6: 510 C, D.

¹⁹ *Rep.*, 6: 511 B; cf. *Phaedo*, 101 D, E.

mathematics, Plato perfected the method of analysis in this realm, and, then using it as a model of investigation, there grew up the conception of dialectic, in which the same principle of investigation is carried over to *all* fundamental concepts. The method is universalized, and that, too, in two directions: (1) in the extension of its field of application beyond the realm of mathematics; (2) in its depth of application, in the demand that all the fundamental concepts, not alone of mathematics, but also of all subjects, be subject to examination with reference to the possibility of grounding them in some higher principle. When this principle has been reached, then a descent is possible through wholly intellectual and rational process. Dialectic involves both the ascending and the descending processes. Thus a system is formed within which the free activity of the mind may exercise itself without being under the constraint of sense-perception, yet in accordance with the principles and laws of reason. Such a system would meet the demands imposed by following out the logical *a priori* line of thought, and would at the same time involve all that Plato had found of value in the mathematical method of procedure.

This same conception of dialectic as the universalization of the hypothetical method comes out in the *Sophist* (253). All the characteristics of the method of analysis are there hinted at, only universalized. There is the process of division, whereby, as we have seen, mutually exclusive alternatives are secured. There is the search to see if any universal class is dependent upon some other which makes it possible (hypothetical procedure). There is the ascending process — the attempt to see many different forms contained under one higher form; and the descending process — the viewing of one form as knit together into a single whole and pervading many such wholes. This passage seems worthy of quotation in its entirety.

And will he [the dialectician] not ask if the connecting links are universal, and so capable of intermixture with all things; and again, in divisions, whether there are not other universal classes, which make them possible? . . . Should we not say that the division according to classes, which neither makes the same other, nor makes other the same, is the business of dialectical science? That is what we should say. Then, surely, he who can divide rightly is able to see clearly one form pervading a scattered multitude, and many different forms contained under one higher form; and again, one form knit together into a single whole and pervading many such wholes, and many forms, existing only in separation and isolation.²⁰

²⁰ *Soph.*, 253.

From this point of view, dialectic is not concerned with one art any more than with another. Its function is universal. All arts are on an equality before it — that of the general and that of the vermin destroyer.²¹ It is concerned only with their pretensions or claims as arts. Its “endeavor is to know what is and what is not kindred in all arts, with a view to the acquisition of intelligence.”²²

Now, we may say that it is because dialectic is this intelligent aspect of all the arts that it is able to look upon them in this impartial manner as to their degree of honor or dishonor. Dialectic sees them all in their relation to the whole, as dependent upon some one common fundamental principle, on their relation to which their function and value depends.

This brings us back to the problem of the relation of mathematical procedure to dialectic from a new point of view. Mathematical procedure has transcended the productive arts by bringing out from one point of view what is common to them all, common not merely as an element, but as a principle of control, something on which they all alike depend, namely, the principle of measure or quantity. But still, from Plato's point of view, the mathematical principle is not the highest point of view from which to see all things. Mathematics as such just misses possessing the characteristic of the highest kind of knowledge; it stops short with its fundamental concepts unexamined, and hence does not have the comprehensive, all-inclusive, view of the whole which dialectic has. It itself needs further interpretation in the light of some further and higher principle. From this point of view it belongs in a realm somewhere between opinion (*δόξα*) and reason (*νοῦς*). Plato places it in the realm of the understanding (*διάνοια*).²³

If we work this point out still further, I think that it will give further light upon the relation which Plato sets up between mathematical procedure and dialectic. In the *Theætetus* there is quite a discussion of opinion (*δόξα*). There a distinction seems to be set up between opinion (*δόξα*) and *true* opinion (*δόξα ἀληθής*).²⁴ It is not denied that the judge may decide rightly on the basis of true opinion (*δόξα ἀληθής*). In outcome the result is precisely the same as if he had knowledge and could give a reason. But yet his judgment is lacking in that cognitive aspect which entitles it to be called knowl-

²¹ *Soph.*, 227 B.

²³ *Rep.*, 6: 511 D.

²² *Soph.*, 227; cf.. *Parm.*, 130 D, E.

²⁴ See especially *Theaet.*, 201 B. C.

edge in the higher sense.²⁵ So it is with mathematics, Plato feels. The mathematician does not use his results, but turns them over to the dialectician.²⁶ If Plato had used the terminology of true opinion and of the higher knowledge, he would have placed mathematics under true opinion. He *does* place it *between* opinion and reason, and that is where true opinion would have to fall. This interpretation would reconcile the apparent ambiguity, if not contradiction, in his stress upon the idealistic and absolute character of the results of mathematics, while at the same time making dialectic superior. In true opinion the content of mathematics, the essences, would be Ideas; but, viewed from the side of process, these mathematical Ideas would be lacking in the complete cognitive aspect of knowledge in the higher, more philosophical, sense. This interpretation, while making intelligible the difference which Plato makes between mathematics and dialectic, also makes intelligible the close relations which he is continually pointing out and insisting upon. In the *Meno* (98), while insisting in the most vigorous language that knowledge differs from true opinion, he yet shows that true opinion may pass over into knowledge "when fastened by the tie of cause." This fastening is there done by recollection (in the metaphysical sense). We have already pointed out that Plato in many places uses dialectic in a sort of poetical fashion. It is no stretch of the facts as we know them in the *Republic* and the *Phædo* to say that this fastening of true opinion down by a "chain" with "the tie of cause," which is the function of recollection, is identical with the process of dialectic in other places. The particular bearing of this point on the problem in hand is found in a statement in the *Republic* (511) in immediate connection with the discussion of the use of hypotheses in mathematics and in dialectic. There, in speaking of mathematical truths as inferior to those of dialectic, the very significant qualifying clause is added, "*although when a first principle is added to them they are cognizable by the higher reason.*" This statement, it will be observed, is in exact harmony with the one in the *Meno* regarding true opinion.

As the outcome of this interpretation we cannot say offhand and without qualification that Plato identifies, or that he distinguishes mathematics and dialectic in the knowledge-function. In content, viewed alone, they are alike; in attitude of mind they differ in cognitive aspect; in process they differ in degree and universality in the use of the hypothetical method. With reference specifically to

²⁵ Cf. *Gorg.*, 454-55.

²⁶ *Euthyd.*, 290.

the point of the influence of mathematical method upon Plato's conception of dialectic, the outcome of the discussion seems to show that there is good reason to believe that, while there were other factors at work to determine the movement of Plato's thought, yet the suggestion which he received from the application of the hypothetical method in mathematics was a very real factor in clarifying his thought and enabling him to formulate a method that should satisfy the logical demands under which he felt obliged to think his way through the ethical problem.

We have seen how pressure was continually brought to bear upon Plato to set up distinctions of value on the cognitive side. The solution of the ethical problem, from the logical *a priori* point of view, demanded the setting up of a distinction between the senses and the intellect. Mathematical procedure made the same demand. The distinction was set up. The analogy of the arts also conduced to the same end through its revelation of the principle of intellectual control involved in the mathematical element. But mathematics was not adequate to a knowledge of ends, nor was its method of procedure such as to be wholly adequate in giving knowledge which could be maintained from every point of view equally well—knowledge which could always be carried back to a further hypothesis, until it rested on that which was above hypothesis. So a distinction had to be set up between mathematics and dialectic, and dialectic had to be defined from this further point of view and to meet this further demand. Dialectic thus became the upper limit of knowledge. In this dialectic we have the height of the rational activity, the mind freed from the constraint of that necessity which is involved in the sense-perception type of experience. The ascent has been made to a principle intellectually determined, and the descent can be made by intellectual processes to all the particulars which fall under the control of that principle. The ideal is realized of procedure from ideas to ideas by means of ideas. Herein Plato finds the possibility of the solution of the epistemological problem, and together with it the solution of the ethical problem (stated on p. 79) out of the severe demands of which the cognitive problem grew. Ethical principles no longer need to rest upon any Protagorean doctrine of relativity. The senses may be inadequate; let that be granted. But when virtue is based on knowledge, and knowledge comes through the process of dialectic, then the fundamental ethical principles rest upon a secure basis—a basis not subject to the law of becoming and change.

Now that we have followed through the process of setting up distinctions, and have seen how Plato used these distinctions to state the epistemological problem in a way to ground ethics upon firm theoretic principles, it may be well to raise the further question as to the nature of these distinctions: Are they absolute or functional? When the highest distinction has been worked out, has it any relation of significance with reference to those which are lower? And have the lower any such relation of significance with reference to the higher? There certainly is much to be said in defense of the thesis that the distinctions which Plato sets up are functional rather than absolute.

The relation between the lower and the higher here is in many respects analogous to the relation between means and end. Mathematics came in at one level to mediate, to serve as the instrument of intellectual control, or organization of means with reference to results; dialectic performs the mediating function at a higher intellectual level still.

Even the senses have their value, but, from the full cognitive point of view, not as standing alone. They are *inadequate* rather than unnecessary. Their results have to be turned over to some other faculty to be judged, tested, and used. As is brought out in the *Laws*, the mind must mingle with the senses in order to secure the salvation of all. The safety of the ship is dependent, not on the pilot alone, nor on the sailors alone. It is when the *perceptions* of both pilot and sailors are *united* with the *piloting mind* that the ship is saved, together with those upon it.²⁷ In the city of the *Laws* the younger guardians have "their souls all full of eyes, with which they look about the whole city. They keep watch and hand over their perceptions to the memory, and inform the elders of all that happens in the city." These elders are wise in council, "and making use of the younger men as their ministers, and advising with them — in this way both together truly preserve the whole state."²⁸ In such imagery as this Plato indicates that there is a relation of connection between the distinctions which he has set up. From the side of *value* the higher is in a position of independence, while yet it is functionally related to the lower.

This point of the relation between the distinctions of higher and lower is well illustrated in the cases already cited (see p. 88) of true opinion becoming knowledge when "fastened by the tie of cause,"

²⁷ *Laws*, 12: 961.

²⁸ *Laws*, 12: 964-65.

and of mathematical truths "cognizable by the higher reason when a first principle is added to them." There is much also in the myth of the cave, or den, in the *Republic*, which would be in harmony with this same point of view.

This attitude of Plato with reference to the relation of the highest kind of knowledge, that of the philosopher, or wise man, to the lower and less critical forms of knowledge, is carried right over, as we might expect, into the realm of ethics. This is seen most clearly by starting from the side of the analogy of the arts. The individual arts have some legitimate place, function, and value in their isolation; but the knowledge they involve is not knowledge in the highest sense, because it is not exercised with reference to a further end or good than the mere making of some article as well as it can be made. The wise man, the philosopher, views the excellence of the art, not alone on the side of the adequacy of the technique to produce a certain product, but also with reference to the relation of that product to some ultimate and final end. He, then, has the higher knowledge with reference to this art. So it is with all the conduct of life. Habit, custom, routine in the realm of moral conduct, give virtues which are blind. These virtues become intelligent and truly ethical only when seen in relation to the Idea of the Good — the highest dialectical principle in the realm of ethical judgment.

With the attainment of dialectic we have seen that Plato was able to solve his epistemological, and particularly his ethical, problem. The ontological problem, already partly swallowed up in the idealistic point of view (see pp. 29, 43) received its final solution in the identification of the Ideas, realized in the procedure of dialectic, and the highest and most ultimate reality, or essence — immaterial and eternal — in relation to which the whole world of the things of experience is to be judged.

The cosmological problem also receives its solution. We have seen that mathematics came in to furnish the technique of production, and hence of the creation of the cosmos. But such technique implies intelligence, implies purpose, implies ultimate ends. These can be realized only through dialectic. Cause gets stated in teleological terms. As in the arts there was a workman who utilized the technique which rested on mathematical principles, so in the cosmological process we find the demiurge, the embodiment of this rational activity of mathematics. In the arts there are premonitions of ultimate goods, but in the cosmological process, from the point of view of dialectic,

there is clearly and intelligently grasped an ultimate end, the Idea of the Good, which the creator, seeks to realize.

While the main problem of this book has been the significance of the mathematical element in Plato's philosophy, the discussion of that element could not be adequate except in the setting and context of the problems with which it was connected. Hence it seemed necessary to give in a more or less dogmatic way a statement of the solution of these problems. However, the significance of the mathematical element does not come in so much in the solution of the problems as in the matter of its influence upon their formulation and the leverage which it gave to the problem of working out a method which Plato could regard as adequate to the process of investigation. The cap-sheaf of his method was dialectic, and for his conception of dialectic he owes very much to his interest in mathematical procedure.

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