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ON THE MOTION OF A SPHERE OF OIL THROUGH CARBON DIOXIDE AND AN EXACT DETERMINATION OF THE COEFFICIENT OF VISCOSITY OF THAT GAS BY THE OIL ~~DORP~~ Drop METHOD

BY

LEO JOSEPH LASSALLE

A DISSERTATION

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ON THE MOTION OF A SPHERE OF OIL THROUGH CARBON DIOXIDE AND A DETERMINATION OF THE COEFFICIENT OF VISCOSITY OF THAT GAS BY THE OIL DROP METHOD.

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ON THE MOTION OF A SPHERE OF OIL THROUGH CARBON DIOXIDE AND A DETERMINATION OF THE COEFFICIENT OF VISCOSITY OF THAT GAS BY THE OIL DROP METHOD.

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SYNOPSIS.

I. *Coefficient of Viscosity of CO_2 by the Oil-drop Method.*—The value of e being known, through work in air, to an accuracy of about .1 per cent., the oil-drop method as developed by Millikan has been used in CO_2 for the determination of the coefficient of vicsosity of that gas with the result at 23° C. $\eta = 1.490 \times 10^{-4}$.

II. *Coefficient of Slip between CO_2 and Oil.*—The value of the constant A in Millikan's equation $e_1 = e(1 + Al/a)^{3/2}$ is found in CO_2 to be 0.8249 as against Millikan's value in air 0.864.

III. *Variability of the Constant A for Values of l/a Greater than .5.*—Precisely as in Millikan's work in air, A was found to be constant only up to $l/a = .5$, beyond which it kept increasing as far as it was followed, viz., up to $l/a = 12$.

INTRODUCTION.

THE behavior of oil drops falling in air has been reported upon by R. A. Millikan.[1] Every precaution possible was taken to assure the highest degree of accuracy. The value of e obtained was shown to be accurate to within 0.2 of one per cent. The law of fall for oil drops in hydrogen was investigated by R. A. Millikan, W. H. Barber, and Y. Ishida.[2] The same precautions as those taken for air were observed.

The work here described was undertaken at the suggestion of Dr. Millikan in order to find the correction factor to Stoke's law for carbon dioxide, and the coefficient of viscosity of that gas by a new method.

The value of $e_1^{2/3}$ is directly proportional to the coefficient of viscosity, η, of the gas used. As is shown later, this relationship furnishes an elegant method of determining η for any gas, and is used in this paper to obtain the viscosity coefficient for CO_2 at 23° C.

The apparatus used was the same one used by R. A. Millikan.[3] The same precautions were taken to give as high a degree of accuracy as

[1] "On the Elementary Electrical Charge and The Avagadro Constant," PHYSICAL REVIEW, N.S., Vol. II., No. 2, Aug., 1913, p. 109.

[2] "The Law of Fall of a Droplet Through Hydrogen," PHYSICAL REVIEW, Series 2, 5, 1915, p. 334.

[3] PHYS. REV., N.S., Vol. II., No. 2, Aug., 1913, p. 109.

possible. All of the time observations were taken with a Gaertner recording chronograph. This instrument records time to one one-hundredth of a second. It was adjusted so as never to be in error by more than two hundredths of a second per minute.

The following method described by Langmuir was used in generating the CO_2. Using a Kipp generator, $CaCO_3 + 2HNO_3 = CO_2 + Ca(NO_3)_2 + H_2O + HNO_3$ vapor $+ H_2O$ vapor. The gas is bubbled through $NaHCO_3$, which takes up acid vapors but no CO_2; then it passes through P_2O_5 tubes which take up all H_2O vapor.

The HNO_3 was a 50 per cent. pure H_2O solution through which commercial CO_2 had been bubbled for 5 hours before being put into the Kipp generator. The $CaCO_3$ (marble) was broken and boiled 4 hours in a very dilute solution of HNO_3. The vessels into which the gas was introduced were evacuated to a pressure below 0.5 mm. before they were filled. They were then pumped down again to the same low pressure and refilled with CO_2. This process was repeated three times before any observations were taken, thus assuring that not more than one part of air in $(1{,}500)^3$ parts of CO_2 remained in the vessels.

For a complete discussion of the apparatus used and the precautions taken to assure accuracy in the determinations of the constants that enter into the following results see paper by R. A. Millikan.[1]

Part I.

Determination of Coefficient of Viscosity of CO_2.

Oil droplets were obtained for observation by aspirating oil with CO_2 gas under pressure. They were held for a sufficient length of time to observe the time of fall under gravity (t_g) on an average of eighteen times. The average number of changes of charge obtained for each drop was seven for the first thirty-six drops, which are the ones used in the determination of the coefficient of viscosity and the correction factors "b" and "A."

The results of the observations were such as to leave no uncertainty as to the greatest common divisor of $[(1/t_g) + (1/t_f)]$, which will be represented hereafter by $[(1/t_g) + (1/t_f)]_0$, and which is the variable factor upon which the value of $(e_1^{2/3}/\eta)$ and (a) are dependent.

It is significant that these thirty-six drops represent every drop observed where P.D. was constant to as much as 0.4 of one per cent. and where as many as three changes of charge were obtained. In the equation

$$e_n = \frac{4}{3} \cdot \pi \cdot \left(\frac{9}{2}\eta_1\right)^{3/2} \cdot \left(\frac{1}{g(\sigma - \rho)}\right)^{1/2} \cdot \frac{(v_1 + v_2)\cdot(v_1)^{1/2}}{F} \cdots, \qquad (1)$$

[1] R. A. Millikan, Phys. Rev., N.S., Vol. II., No. 2, Aug., 1913, pp. 109–143; also Phil. Mag., XXXIV., p. 13, 1917.

η_1 is the coefficient of viscosity of CO_2 at 23° C., g the acceleration due to gravity, σ the density of the oil, ρ the density of the CO_2, v_1 the speed of descent of the drop under gravity and v_2 the speed of ascent under the action of the electrical field of strength F.

If we now allow e_1 to be the greatest common divisor of all the various values of e_n found on a drop during the observations upon it, and if we let

$$v_1 = \frac{D}{t_g},$$

$$v_2 = \frac{D}{t_f},$$

and

$$F = \frac{\text{P.D.}}{d},$$

where D is the distance the drop falls in time t_g and the distance it rises in time t_f, and d is the distance between the two parallel, horizontal condenser plates, between which a difference of potential in volts, represented by P.D., is maintained by means of storage cells which do not change more than four parts in a thousand during a series of observations on a given drop, then equation (1) may be written in the form

$$e_1 = (300d)\cdot\frac{4}{3}\cdot\pi\cdot\left(\frac{9}{2}\eta_1\right)^{3/2}\cdot\left(\frac{1}{g(\sigma-\rho)}\right)^{1/2}\cdot\frac{\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0\cdot\left(\frac{1}{t_g}\right)^{1/2}\cdot D^{3/2}}{\text{P.D.}}, \quad (2)$$

where $[(1/t_g)+(1/t_f)]_0$ is the greatest common divisor of $[(1/t_g)+(1/t_f)]$ for the given drop.

Equation (2) may be written

$$\frac{e_1^{2/3}}{\eta_1} = \frac{C\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0^{2/3}\cdot\left(\frac{1}{t_g}\right)^{1/3}}{(\text{P.D.})^{2/3}}, \quad (3)$$

where

$$C = \left[\frac{9}{2}D\left(\frac{4\pi\times 300d}{3}\right)^{2/3}\cdot\left(\frac{1}{g(\sigma-\rho)}\right)^{1/3}\right]. \quad (4)$$

This gives a simple method of calculating $e_1^{2/3}/\eta_1$.

a, the radius of the falling drop, is calculated by means of the expression

$$a = \sqrt[3]{\frac{3Fe}{4\pi g(\sigma-\rho)}\cdot\frac{v_1}{(v_1+v_2)_0}}, \quad (5)$$

which may be written,

$$a = C'\left(\frac{\frac{1}{t_g}\times\text{P.D.}}{\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0}\right)^{1/3}. \quad (6)$$

The pressure of the gas p is observed directly.

Table I. gives the values of the factors which enter into the determination of $e_1^{2/3}/\eta_1$ and a, as well as into the values of $e_1^{2/3}$, e_1, and $e^{2/3}$. The last three values can only be obtained after η_1 and b have been calculated.

For the present let us assume that the table does not contain $e_1^{2/3}$, e_1, nor $e^{2/3}$, and plot $e_1^{2/3}/\eta_1$ against $1/pa$. Curve 1 shows this relationship.

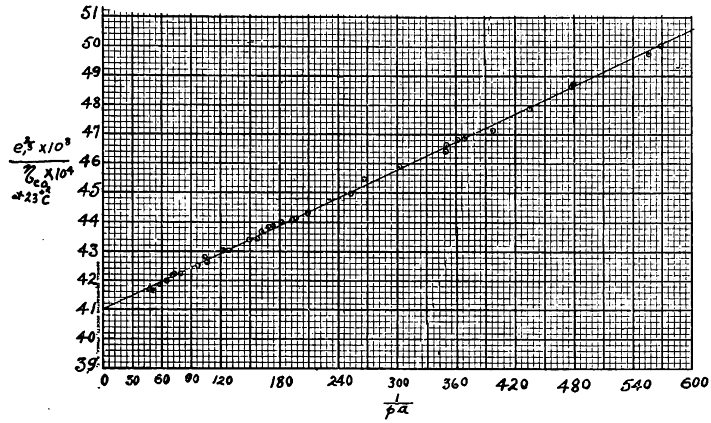

Fig. 1.

It is a straight line which crosses the $e_1^{2/3}/\eta_1$ axis at 41.00×10^{-4}.

If we write

$$e_1^{2/3} = e^{2/3}\left(1 + b\frac{1}{pa}\right),$$

then

$$\frac{e_1^{2/3}}{\eta_1} = \frac{e^{2/3}}{\eta_1}\left(1 + b\frac{1}{pa}\right).$$

It is evident that, for $1/pa = 0$,

$$\frac{e_1^{2/3}}{\eta_1} = \frac{e^{2/3}}{\eta_1}.$$

We are justified in saying that the value of the elementary electrical charge on an oil drop is independent of whether it be in CO_2 or in any other gas, say air. Since the value of $e^{2/3}$ in air is known to be 61.085×10^{-8} E. S. units, with a probable error of less than 0.1 of one per cent.,[1] then at $1/pa = 0$, $e_1^{2/3} = e^{2/3} = 61.085 \times 10^{-8}$;

$$\therefore \frac{e_1^{2/3}}{\eta_1} = 41.00 \times 10^{-4}$$

[1] This is the value obtained by reducing the number given by R. A. Millikan in Phil. Mag., XXXIV., p. 13, by .07 per cent. to allow for the change in the value of the coefficient viscosity of air from .0001824 to .00018227.

gives

$$\eta_1 = \frac{61.085 \times 10^{-8}}{41.00 \times 10^{-4}} = 1.490 \times 10^{-4},$$

which, because all the calculations were reduced to 23° C., gives the value of the coefficient of viscosity at 23° C.

Wherever the temperature differed from 23° C. during an observation the correction factor to be applied to the value of η_1 was introduced into the calculation of $(e_1^{2/3}/\eta_1)$. Sutherland's equation was used and the variation of log η_1 with temperature was obtained by plotting log η_1 against temperature. With the exception of drops 5 and 15, which differ by about 1° C., the variation from 23° C. was always less than 0.5° C.

By the oil drop method, then, the value of the coefficient of viscosity of CO_2 at 23° C. is found to be

$$\eta_{CO_2 \text{ at } 23° C} = 1.490 \times 10^{-4}.$$

The accuracy of this result is dependent upon the accuracy with which the value of $e^{2/3}$ is known and the accuracy with which the intercept on the $e_1^{2/3}/\eta_1$ axis for CO_2 is known.

$e^{2/3}$ is known with an accuracy of about 0.1 of one per cent. The $e_1^{2/3}/\eta_1$ intercept is known with a probable error of about 0.5 of one per cent. Therefore, the value of η_1 given by these observations should be accurate to about 0.5 of one per cent.

Therefore

$$\eta_{CO_2 \text{ at } 23° C} = (1.490 \pm 0.0080) \times 10^{-4}.$$

Breitenbach[1] obtained

$$\eta_{CO_2 \text{ at } 15° C} = 1.457 \times 10^{-4}.$$

Applying the Sutherland equation to his value there results

$$\eta_{CO_2 \text{ at } 23° C} = 1.494 \times 10^{-4}.$$

If we assume that this investigator's value is too high for CO_2 in the same ratio that his value for air is too high, we get

$$\eta_{CO_2 \text{ at } 23° C} = (1.494 \times 10^{-4}) \frac{1786}{1810} = 1.474 \times 10^{-4},$$

a value 1 per cent. lower than the one obtained in this paper.

Ernst Thomson,[2] using a vibrating disc, obtains a value of η for CO_2 which is 0.0000004 higher than Breitenbach's. However, the main object of his investigation was to find the relationship between the η for

[1] Breitenbach, Ann. d. Phys., 5, 1901.

[2] "Ueber die innere Aeibung von Gasgemischen," Inaugural Dissertation, der Konigl. Christian-Albrechts Univ. zu Kiel.

two gases and the η for various mixtures of these two gases. He does not claim any high degree of accuracy for his value of the coefficient of viscosity of CO_2.

P. Phillips[1] gives a value of η which reduced to 23° C. by Sutherland's equation, becomes

$$\eta_{CO_2 \text{ at } 23^\circ C} = 1.494 \times 10^{-4}.$$

He used the A. O. Rankin device,[2] which is a capillary tube method. The object of his investigation was to determine the variation of η with pressure, going to pressures of eighty atmospheres. No high degree of accuracy is claimed for the determination of the η at atmospheric pressure. The important point sought after was to obtain relative values as pressure varied.

If, however, we take the mean of the values obtained by Breitenbach, Thomsen and Phillips, a value of $\eta_{CO_2 \text{ at } 23^\circ C} = 1.489 \times 10^{-4}$ is obtained. This value is different by less than 0.1 of one per cent. from the one here obtained.

Part II.

The A and b Correction Terms in CO_2.

Table I. gives the values of $e_1^{2/3}/\eta_1$ corresponding to the various values of $1/pa$. $e_1^{2/3}$ is obtained by multiplying $e_1^{2/3}/\eta_1$ by the value of η_1 as found in part 1.

Fig. 2 gives the result of plotting $e_1^{2/3}$ against $1/pa$.

If, now, we write

$$e_1^{2/3} = e^{2/3}\left(1 + b\frac{1}{pa}\right), \tag{7}$$

and let $y = e_1^{2/3}$; $y_0 = e^{2/3}$, and $x = (1/pa)$, we have $y = y_0 + by_0x$. By differentiation, then, the above becomes

$$b = \left(\frac{dy}{dx} \div y\right) = \frac{\text{Slope of } (y, x) \text{ curve}}{y\text{-intercept}}. \tag{8}$$

This gives a simple method of determining the correction factor b.[3] The last column in Table I. gives the values of $e^{2/3}$ calculated by substituting in (7) and solving. The values of e follow directly from those of $e^{2/3}$. It will be noticed that no value of $e^{2/3}$ varies from 61.085 by as much as 0.5 of one per cent.

The value of b, obtained by substituting in (8) is

$$b = \frac{\text{Slope of } y, x \text{ curve}}{y\text{-intercept}} = \frac{(75.37 - 61.085)}{600 \times 61.085},$$

[1] P. Phillips, Roy. Soc. Proc., Ser. A, 87, pp. 48–61.

[2] Roy. Soc. Proc., 1910, A, Vol. 83, p. 265.

[3] R. A. Millikan, Phys. Rev., II., p. 118, 1913, and XXXII., p. 381, 1911.

TABLE I.

No.	Temp. °C.	P. D. Volts.	t_g (Sec.).	$\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0 \times \left(\frac{1}{\text{Sec.}}\right)$.	n	$a \times 10^5$ Cms.	p (Cms. Hg.).	$\frac{1}{pa}$.	$\frac{l}{a}$.	$e_1 \times 10^{10}$.	$e_1^{2/3} \times 10^8$.	$e^{2/3} \times 10^8$.
1	23.27	6036.4	8.09	0.007505	18–32	30.41	69.64	47.22	0.02231	4.895	62.11	60.99
2	23.11	5862.2	10.42	0.008259	12–26	26.81	72.67	51.33	0.02426	4.884	62.02	60.81
3	23.36	5856.1	12.51	0.009099	11–18	24.41	69.69	58.78	0.02770	4.922	62.34	60.94
4	23.05	5897.9	15.52	0.010269	8–18	21.87	71.33	64.09	0.03029	4.945	62.53	61.01
5	22.10	3767.0	19.90	0.007515	11–16	19.25	74.23	69.99	0.03070	4.984	62.86	61.19
6	23.05	5875.2	20.33	0.011833	5–9	19.04	72.47	72.46	0.03424	4.998	62.98	61.26
7	23.35	5864.3	24.27	0.012865	3–10	17.45	73.02	78.49	0.03709	4.989	62.91	61.05
8	23.42	5849.4	33.13	0.015142	2–7	14.88	69.79	96.27	0.04549	5.040	63.34	61.06
9	23.20	5183.0	43.74	0.015595	4–8	12.91	74.43	104.11	0.04920	5.094	63.78	61.30
10	23.13	5192.7	21.31	0.010837	5–11	18.52	50.91	106.04	0.05012	5.059	63.49	60.98
11	23.49	5852.2	58.04	0.020410	1–6	11.18	73.13	122.32	0.05780	5.132	64.10	61.19
12	23.08	5201.6	30.48	0.013180	3–8	15.41	50.48	128.55	0.06074	5.135	64.12	61.06
13	23.18	5189.1	41.71	0.015571	1–5	13.12	51.00	149.45	0.07062	5.200	64.67	61.12
14	23.00	3945.7	30.13	0.010080	4–13	15.43	41.38	156.65	0.07402	5.204	64.70	60.98
15	21.81	3948.4	36.55	0.011260	3–10	13.95	44.34	161.67	0.07640	5.247	65.06	61.20
16	23.21	3997.8	32.97	0.010820	2–5	14.69	40.60	168.50	0.07962	5.275	65.29	61.27
17	23.60	3253.7	54.71	0.011334	11–18	11.40	50.45	173.82	0.08213	5.282	65.35	61.20
18	23.20	3948.6	38.57	0.011630	3–7	13.55	40.92	180.36	0.08522	5.308	65.56	61.25
19	23.22	3926.1	43.90	0.012370	3–11	12.69	41.02	192.11	0.09078	5.322	65.67	61.10
20	23.06	3896.0	21.19	0.008537	6–17	18.29	27.89	196.26	0.09274	5.329	65.73	61.06
21	22.93	3910.5	20.774	0.008551	6–16	18.39	25.98	209.32	0.09892	5.362	66.00	61.03
22	23.05	3922.9	34.21	0.011250	3–7	14.23	27.81	252.76	0.1194	5.482	66.98	60.98
23	23.13	3947.5	37.41	0.012037	2–10	13.53	27.73	266.56	0.1260	5.576	67.75	61.37
24	23.13	3914.8	47.47	0.013643	2–8	11.96	27.70	301.97	0.1430	5.659	68.42	61.21
25	23.24	2603.0	23.599	0.006499	7–20	16.86	17.01	348.69	0.1648	5.752	69.16	60.88
26	22.67	3248.3	17.610	0.007081	10–24	19.45	14.71	349.55	0.1652	5.797	69.52	61.18

Table I (*Continued*).

No.	Temp. °C.	P. D. Volts.	t_g (Sec.).	$\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0 \times \left(\frac{1}{\text{Sec.}}\right)$.	n	$a \times 10^5$ Cms.	p (Cms. Hg.).	$\frac{1}{pa}$.	$\frac{l}{a}$.	$e_1 \times 10^{10}$.	$e_1^{2/3} \times 10^8$.	$e^{2/3} \times 10^8$.
27	23.14	2606.2	23.077	0.006521	10–26	16.97	16.27	362.09	0.1711	5.826	69.75	61.12
28	23.01	3905.2	36.624	0.012158	3–11	13.66	19.85	368.94	0.1743	5.830	69.79	61.02
29	22.64	3249.0	21.004	0.007855	6–20	17.72	14.23	396.66	0.1874	5.886	70.23	60.83
30	23.22	2604.1	34.214	0.008206	6–25	13.78	16.70	434.40	0.2053	6.028	71.36	61.03
31	22.70	3252.3	28.61	0.009614	5–12	14.95	14.01	477.56	0.2257	6.168	72.46	61.09
32	23.18	2605.3	40.115	0.009111	5–15	12.63	16.55	478.58	0.2261	6.177	72.53	61.12
33	23.05	2603.4	28.175	0.007890	6–21	14.90	12.07	556.13	0.2628	6.383	74.13	60.93
34	23.04	2601.3	35.045	0.008876	4–13	13.32	13.21	568.49	0.2686	6.443	74.60	61.07

or

$$b = 0.0003898$$

(see Fig. 2).

It should be noticed that the value of b is independent of any theory and is determined by the use of values which are directly measurable.

If, however, it is desired to write (7) in the form

$$e_1^{2/3} = e^{2/3}\left(1 + A\frac{l}{a}\right), \tag{9}$$

then A can be calculated in the same way that b was, provided the values of l/a are known.

Using $\eta = 0.3502mnl\bar{C}$, where the Boltzmann value of K is used so

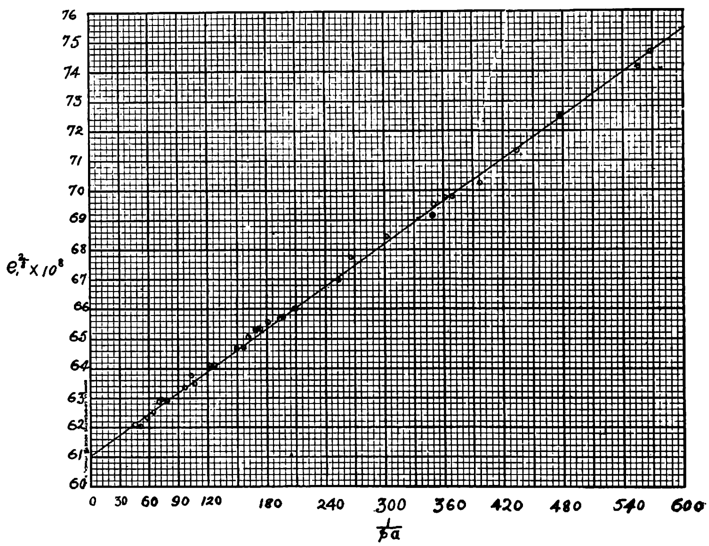

Fig. 2.

as to give an A that can be compared with that for air obtained by R. A. Millikan in the paper previously referred to, the values of l/a can be calculated.

From equations (7) and (9) it is evident that

$$A\frac{l}{a} = b\frac{1}{pa}. \quad \therefore A = b\frac{1}{pl}\cdots. \tag{10}$$

At a constant temperature $1/pl$ is a constant. Therefore, $A = bB$, which means that the $(e_1^{2/3}, 1/pa)$ curve becomes the $(e_1^{2/3}, l/a)$ curve simply by changing the scale of the abscissæ.

By calculating for a given drop and substituting in equation (10) the value of $A/b = B$ is obtained. This is checked by making the calcuation for several drops. In this way B was found to be equal to 2116. This gives for CO

$$A = 0.8249.$$

The value obtained by R. A. Millikan in his work in air was $A = .864$. The difference would seem to be slightly more than the experimental error involved in the determination of the slope of the line from which A is obtained. This error should scarcely exceed two or three per cent. There is then here a somewhat uncertain indication that the coefficient of slip between CO_2 and oil is a trifle less than that between air and oil.

TABLE II.

No.	Temp. °C.	P. D. Volts.	t_g (Sec.)	$\left(\frac{1}{t_g}+\frac{1}{t_f}\right)_0 \times \left(\frac{1}{\text{Sec.}}\right)$.	n	$a\times 10^5$ Cms.	p Cms. Hg.	$\frac{1}{pa}$.	$\frac{l}{a}$.	$e_1^{2/3}\times 10^8$.
35	22.70	3245.0	53.833	0.014186	3–9	10.63	16.70	631.6	0.2985	76.19
36	23.04	2604.8	38.036	0.00971	3–13	12.58	12.02	661.3	0.3125	77.01
37	23.15	2583.0	22.63	0.00771	8–19	16.11	8.160	760.83	0.3595	79.33
38	23.15	2589.5	27.67	0.00920	5–13	14.22	7.638	921.09	0.4352	82.94
39	22.90	1288.5	9.32	0.00274	67–168	24.24	4.210	980.14	0.4632	84.63
40	22.87	1288.4	14.83	0.003817	18–66	18.59	4.588	1172.5	0.5540	90.38
41	23.38	1938.6	24.02	0.00735	7–13	14.57	5.566	1233.3	0.5828	91.05
42	22.85	1286.5	20.81	0.00460	8–56	15.60	5.279	1214.6	0.5739	91.50
43	22.87	1286.0	26.67	0.00560	7–87	13.45	5.365	1386.1	0.6550	96.06
44	22.90	1288.2	20.36	0.00514	10–44	15.15	4.410	1497.1	0.7074	99.18
45	23.35	1933.3	38.55	0.01073	3–12	10.97	5.910	1542.7	0.7290	100.05
46	22.86	1288.8	19.20	0.00522	11–35	15.37	4.110	1583.1	0.7480	102.14
47	22.80	650.3	7.01	0.00169	86–137	24.94	2.450	1636.3	0.7732	106.2
48	22.85	648.5	16.91	0.00298	26–99	15.37	3.356	1938.5	0.9160	115.9
49	22.74	1289.5	27.39	0.00772	5–33	11.99	3.860	2161.6	1.021	117.8
50	22.79	651.4	10.19	0.00254	29–153	19.22	2.290	2271.5	1.073	123.0
51	22.90	653.5	9.58	0.00267	38–129	19.32	2.055	2518.7	1.190	129.5
52	23.20	92.70	9.90	0.000391	240–376	18.91	1.970	2684.7	1.269	131.0
53	22.83	649.5	23.23	0.00530	12–60	11.42	2.638	3319.6	1.569	152.9
54	22.95	656.7	16.06	0.00550	12–44	12.80	1.990	3925.1	1.855	176.0
55	22.79	651.1	42.68	0.01510	2–6	6.58	2.390	6356.7	3.004	250.4
56	23.55	649.8	9.96	0.00809	12–31	13.15	1.060	7171.5	3.389	269.3
57	22.08	156.0	31.26	0.00364	26–77	7.29	1.840	7457.8	3.524	278.4
58	23.14	155.72	13.45	0.00275	37–159	10.59	1.100	8582.6	4.056	307.2
59	22.83	192.50	7.18	0.00117	122–225	14.60	0.798	8584.2	4.056	302.7
60	22.90	155.21	6.87	0.00241	64–122	13.84	0.719	10051.	4.750	352.2
61	24.46	157.00	28.32	0.00507	3–85	6.76	1.450	10205.	4.822	359.9
62	23.22	156.8	13.23	0.00469	17–27	8.94	0.736	12076.	5.706	438.8
63	22.82	652.2	82.18	0.05502	1–2	3.44	2.211	13147.	6.212	476.1
64	22.86	156.72	13.03	0.00571	14–18	8.41	0.841	14142.	6.683	502.7
65	22.80	158.22	36.08	0.01945	3–6	3.99	1.112	22524.	10.64	805.2
66	23.02	92.56	9.87	0.00708	14–27	7.21	0.540	25702.	12.14	904.4
67	22.93	89.97	23.59	0.01103	2–8	4.60	0.824	26552.	12.55	926.4

PART III.

Limits to the Validity of Millikan's Equation in CO_2.

Observations were taken on drops at as large values of l/a as practicable with the method of obtaining drops that was used throughout these observations. It was found very difficult to get drops at pressure below one cm. of Hg. A more direct method of blowing the drops and one which will introduce less gas into the system is desirable in order to go to low pressures.

At low pressures it is not possible to have such high potential differences; also it is not possible to observe on drops of the same size as those used at pressures above, say 2 cm. These two factors tend to counterbalance each other as far as the difficulty of observing on drops with a small number of charges is concerned. If the number is not small the greatest common divisor of the series of speeds begins to be uncertain. However, no drop was used in these calculations where there was doubt as to this greatest common divisor.

Table II. gives the values of the various factors entering into the

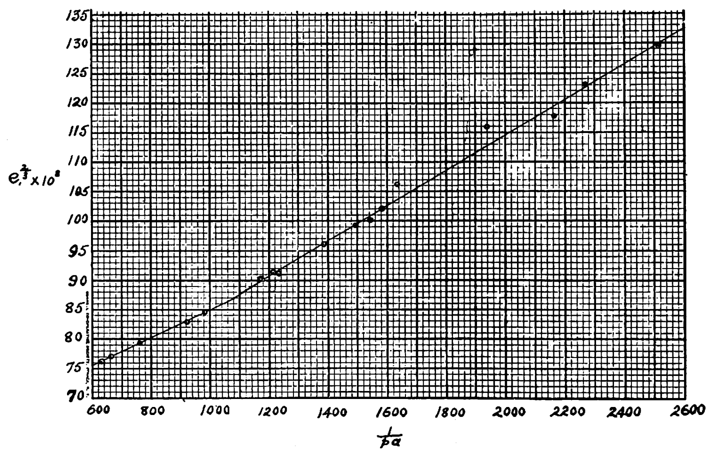

Fig. 3.

determinations of $e_1^{2/3}$, $1/pa$ and l/a for drops where the values of $1/pa$ are greater than 600.

Fig. 3 gives the relationship between $e_1^{2/3}$ and $1/pa$, which is also that between $e_1^{2/3}$ and l/a. It gives this relationship from $1/pa = 600$ to $1/pa = 2{,}500$. It will be seen that from 600 to about 1,100 the slope is the same as that for $1/pa = 0$ to $1/pa = 600$. But at about the value of 1,100 the slope clearly begins to change. This behavior is quite like that found by R. A. Millikan in his work with air. He found (PHYS.

REV., II., p. 138) that the linear relation between $e_1^{2/3}$ and $1/pa$ began to break down at about $1/pa = 650$, which corresponds to a value of l/a of about .5. Here the break comes at about $1/pa = 1,100$ which will be seen from Table II. to also correspond to a value of $l/a = .5$.

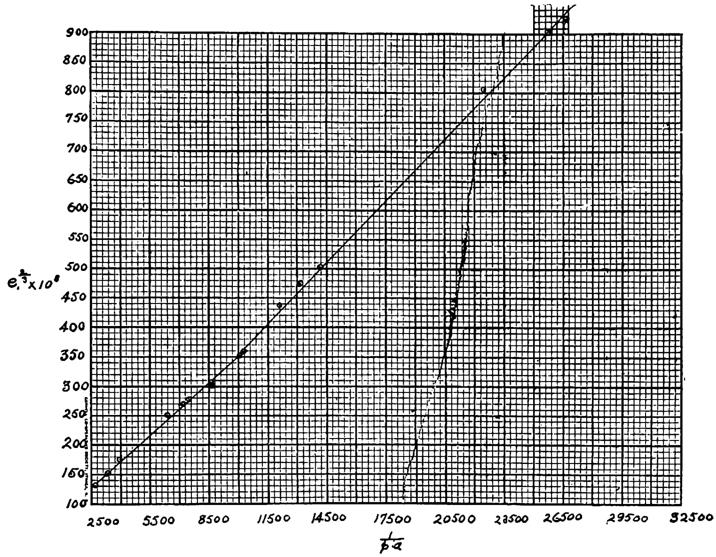

Fig. 4.

Fig. 4 shows the graph of the relations contained in Table II. in the range $1/pa = 2,500$ up to $1/pa = 26,500$, *i.e.*, in the range $l/a = 1$ to $l/a = 12$.

SUMMARY.

I. Using the oil-drop method, the coefficient of viscosity, η, of CO_2 at 23° C. was found to be 1.490×10^{-4}.

II. The correction factors b and A as given in the equations

$$e_1^{2/3} = e^{2/3}\left(1 + b\frac{1}{pa}\right),$$

and

$$e_1^{2/3} = e^{2/3}\left(1 + A\frac{l}{a}\right)$$

were determined for CO_2 and found to be

$$b = 0.0003898,$$

$$A = 0.8249.$$

Applying the correction term to the various drops, 36 in number, values of $e^{2/3}$ were obtained, no one of which varies from 61.085×10^{-8} by as much as 0.5 of one per cent.

III. The relationship between $e_1^{2/3}$ and $1/pa$ was found for values up to $1/pa = 26,500$. The value of $A = 0.8249$ holds until $l/a = 0.50$, approximately. The slope of the curve then increases.

The values of l/a for CO_2, at which the change in slope begins is the same as that found by Millikan in the case of air.

It gives me pleasure to acknowledge the cheerful and able assistance of Dr. Y. Ishida and Mr. B. L. Steele. I am especially indebted to Dr. R. A. Millikan, who suggested the problem and who advised me throughout the course of the investigation.

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO,
August 10, 1917.