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FOUNDED BY JOHN D. ROCKEFELLER

THE EFFECT OF TENSION ON THERMAL AND ELECTRICAL CONDUCTIVITY

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BY
NEWLAND F. SMITH

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THE EFFECT OF TENSION ON THERMAL AND ELECTRICAL CONDUCTIVITY.

BY N. F. SMITH.

THE development of the electron theory of metallic conduction has given new interest to measurements of thermal and electrical conductivity. Since it has been shown that the ratio of the two conductivities is approximately constant for all metals it is natural to expect that anything which produces a change in the one would cause a corresponding change in the other. The effect of tension on the electrical conductivity of wires has been measured by Cantone, Williams,[1] Ercolini,[2] Gray[3] and others. These measurements have been made on small wires stretched by correspondingly small weights. It has been shown that, in general, the resistance of such wires increases when under tension; that up to the elastic limit, the increase in resistance is nearly proportional to the stretching force; that beyond this state, the change is roughly proportional to the elongation. Bismuth shows a change in the opposite direction. So far as the writer knows, no measurements have been made upon the corresponding thermal problem. In 1888 F. Kohlrausch[4] compared the thermal and electrical conductivities of various specimens of steel when tempered to different degrees of hardness. He found that while the conductivities changed by as much as 50 per cent., their ratio remained practically unchanged.

The present investigation was undertaken to determine what changes, if any, are produced in the thermal conductivity of metal bars when they are subjected to a stretching force, and to compare these changes with those which are produced in their electrical conductivity. Two distinct problems were therefore presented, the one involving thermal measurements, the other, electrical. In neither

[1] W. Ellis Williams, Phil. Mag., 13, p. 635, 1907.

[2] G. Ercolini, N. Cimento, 14, p. 537, 1907.

[3] T. Gray, Trans. Roy. Soc. Edin., 1880.

[4] F. Kohlrausch, Wied. Ann., 33, p. 678, 1888.

case was any attempt made to determine the absolute value of the conductivity, but merely the per cent. of change produced by the stretching force.

THE METHOD.

The method adopted for the thermal problem was substantially that of Wiedemann and Franz. Two bars *A* and *B*, of the same size and material and as nearly alike as possible in every respect, were heated at one end to a constant temperature. When the steady state was reached, a point was determined on *B* whose temperature was the same as that of a fixed point on *A*. *B* was then subjected to a stretching force while the condition of *A* remained unchanged. When the steady state was again reached, a new point was determined on *B* having the same temperature as the fixed point on *A*. Assuming that the conductivity is proportional to the square of the length from the hot end to this point of constant temperature, we have a means of measuring the change produced by the force applied. While this assumption is not rigorously true, yet, since the changes in conductivity are relatively small, the error introduced by its adoption is less than the experimental errors.

It seemed desirable to study the changes in the electrical conductivity of the same bars used in the thermal problem and under the same conditions. Accordingly the electrical resistance of a measured length of the bar *B* was compared with that of approximately the same length of bar *A* as each successive tension was applied to *B*. The method here used was a modification of the Kelvin "double-bridge method" due to J. H. Reeves.

THE APPARATUS.

After many modifications the apparatus finally used was constructed as shown in plan in Fig. 1, and in elevation in Fig. 2. Round bars were employed $\frac{5}{16}$ inch in diameter and about $4\frac{1}{2}$ feet long. The bars *A* and *B* were mounted horizontally side by side about 3 inches apart. At the left-hand end they passed completely through a cast-iron box *C*, each edge of which was 8 inches and whose sides were 1 inch thick. This rested on a tripod stand over a Bunsen burner. The box was provided with a cast-iron lid and was thickly wrapped in asbestos. On the inside of the box the bars were clamped between two heavy pieces of copper *D*, one above the other, each 5 inches long, $1\frac{1}{2}$ inches wide and 1 inch thick. These were bored out to receive the bars and held together by bolts. The bars were carefully insulated from the box and from the copper clutch by thin sheets of mica. A hole in the center of the copper

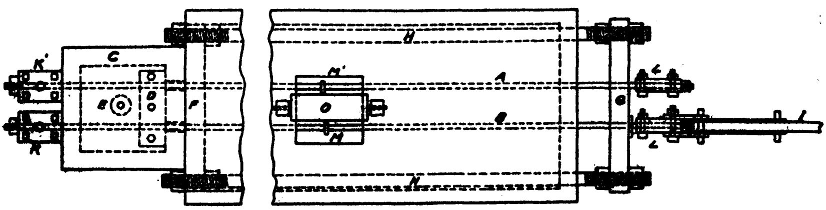

Fig. 1.

clutch received the bulb of a thermometer whose stem projected through the cover of the box. The copper clutch was maintained at a constant temperature by a thermostat. This consisted of a mercury bulb *E*, placed at the bottom of the box directly over the flame, having a capillary stem projecting above the cover. A wire inserted in the top of the stem touched the mercury when a certain

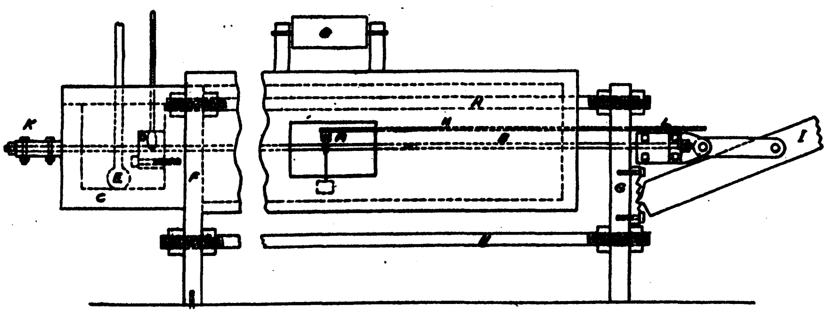

Fig. 2.

temperature was reached. This closed an electric circuit and shut off the gas from the burner. As soon as the temperature commenced to fall the circuit was opened and the gas turned on and lighted by a small pilot light which burned continuously. Some difficulty was experienced in making the mercury bulb respond quickly enough to the changes of temperature. It was at first submerged in a bath of paraffin, but this was given up because of the smoke given off when somewhat high temperatures were employed. A bath of mercury was next used, but this partly vaporized and then condensed in the cooler parts of the box, destroying the insulation of the bars. Finally, the alloy used in making fuse wire was employed. This melted at the temperatures used, but did not vaporize and secured intimate thermal connection between the bulb and the bottom of the iron box. With this arrangement the thermometer, graduated to degrees, showed no measurable variation. If there was a slight variation in the temperature of the copper clutch it affected both bars alike and hence introduced at most a very slight error in the results.

In order to apply tensions equal to the breaking strength of the bars it was necessary that they be mounted in an extremely rigid frame. This was constructed as follows: A cast-iron plate *F*, 11 inches wide, 1¾ inches thick and 13 inches high, was mounted upright on a table immediately in front of the iron box. A similar plate *G* was mounted parallel to the first and about 36 inches in front of it. The bars under observation passed through suitable holes in these plates, being insulated from them by fiber tubing. Four ¾-inch iron rods *H*, connected the two plates, and double nuts at each end of each rod held the plates securely in position. Two ⅜-inch iron rods were screwed into the bottom of the plate *F* and made fast to the floor joists below. Two other rods ½ inch in diameter passed through *F* horizontally, then through a 12-inch brick wall of the building, and were made fast to a large iron plate on the opposite side.

The stretching force was applied by means of a lever *I*. This consisted of a bar of the best tool steel, 4 feet long, ¾ inch thick and 2¼ inches wide. A steel knife-edge 2½ inches long was attached at right angles to the lever at its lower end. To the plate *G* was screwed another iron plate *J*, insulated from *G* by sheets of mica. The plate *J* had several horizontal notches cut in it, in which the knife edge on the lever could rest, serving as a fulcrum.

To hold the bar *B* from slipping and to afford a means of attachment to the lever, a friction clutch *K*, *L* was provided for each end. Each of these was constructed from two pieces of iron ½ inch thick, 2 inches wide and 3 inches long. These two iron blocks, with a thin sheet of metal between them, were first securely bolted together by four $\frac{3}{8}$-inch steel bolts. A $\frac{5}{16}$-inch hole was then drilled lengthwise through the center. The sheet metal was removed and the bars ground with emery between the parts of the clutch. When tightly bolted together these clutches would not slip even when a tension was applied sufficient to break rods of steel. However, as a further precaution threads were cut for a half inch at each end of the bar and steel nuts screwed on outside the friction clutches. The clutch *L* was connected to the lever *I* by a link made of two pieces of strap iron 1 inch wide and $\frac{3}{16}$ inch thick, one piece being placed on each side of the lever. Steel pins connected this link with the clutch and the lever. By means of the nuts on the rods *H*, the plate *G* could be moved parallel to itself and so adjusted that the lever always exerted a horizontal pull upon the bar. A box for carrying weights hung from the end of the lever and could be gradually lowered or raised by means of a jack-screw.

In order that the bar *A* might be under conditions as nearly as possible identical with those of *B*, its two ends were provided with a similar pair of friction clutches and lock-nuts. This was done in order that any changes in room temperature might produce the same effect upon the outer ends of the two bars.

To protect the bars from air currents, they were enclosed by a tight wooden box 12 inches wide, 9 inches high and 34 inches long, extending from the iron plate *F* to within two or three inches of plate *G*. The top of this box was hinged and could be lifted to afford access to the bars and glass windows in the top and side afforded means of observation.

Observations of temperature were made by means of thermo-electric couples. According to the method employed, it was not necessary to make measurements of temperature upon either bar, but merely to determine two points, one on each bar, which had the same temperature. Two thermo-electric couples made of No. 40 iron and nickel wire were mounted as shown in Fig. 3. *M* and *M′* are pieces of hard rubber from which are hung equal weights, *W* and *W′*. These could slide along on the tops of the bars *A* and *B*. A piece of nickel wire connected *M* and *M′*, passing through a small hole in each and ending in a loop. A piece of iron wire was similarly mounted on each side, the junction of iron and nickel coming directly on top of the bar. The iron wires passed out through holes in the bottom of the wooden box and were soldered to copper wires which ran to the galvanometer. The two junctions of iron and copper were immersed in a bottle nearly filled with kerosene. To avoid the possibility of thermo-electromotive forces being introduced at the connection with the galvanometer, the instrument was enclosed in a wooden box and the mirror observed through a small opening in the front. All parts of this enclosure were doubtless at substantially the same temperature Before each observation, the bottle of kerosene was thoroughly shaken to insure equality of temperature. It is therefore evident that if the two iron-nickel junctions are at the same temperature, the galvanometer will show no deflection. The couple on the bar *B* could be moved along in either direction by means of two prongs projecting at right angles to a light brass rod *N* which passed through the end of the box and the iron plate *G*. The galvanometer used was a four-coil Kelvin having the coils in parallel so that its resistance was about 6 or 7 ohms. The sensitiveness was such that with the two couples at about 50 cm. from the copper clutch settings could be made to within about .2 mm.

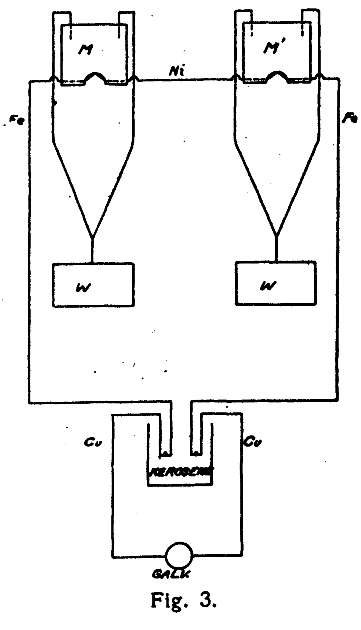

Fig. 3.

To observe the relative position of the two couples, a white paper index was pasted to the top of each piece of hard rubber. A mirror *O* was mounted on a horizontal axis parallel to the bars above the window in the top of the wooden box. This served to throw the image of either index into the field of the telescope of a cathetometer mounted horizontally parallel to the bars. The windows in the box were kept covered except when readings with the cathetometer were being made.

To measure the elongation of the bar under tension, a telescope was mounted which could be moved horizontally by a micrometer screw reading to hundredths of a millimeter. This was focused simultaneously upon a scratch on the bar *B* and a brass index securely clamped to *A*. Since the whole frame yielded somewhat under stress the elongation was determined from the change in the distance between these two index marks. The probable error of setting on each mark was about .01 mm.

Heavy copper lead wires were soldered to the friction clutches *L* and *L'*, while holes drilled in the tops of *K* and *K'* served as mercury cups through which the two bars *A* and *B* could be electrically connected by a piece of copper rod bent twice at right angles.

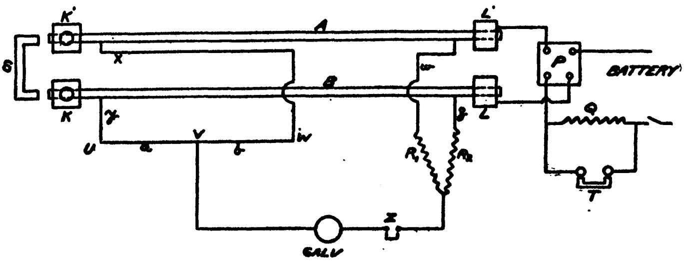

Fig. 4.

The arrangement of apparatus for the comparison of resistances is shown diagrammatically in Fig. 4. A number of storage cells could be connected with the two bars *A* and *B* through the commutator *P*. A resistance *Q* (consisting of two incandescent lamps in parallel) was introduced into the circuit. This resistance could be completely cut out by inserting the copper connector *T* in its mercury cups. After trying various forms of movable contacts, the fine lead wires *x*, *y*, *z*, *w* were finally soldered directly to the bars. Great difficulty was experienced in eliminating the effects of thermo-electromotive forces developed at various contacts of dissimilar metals. For this reason the lead wires were wherever possible made of the same material as the bars to which they were attached. The other ends of *x* and *y* were joined to the terminals of a slide-wire bridge whose resistance wire *a*, *b* was wound on a revolving cylinder and the whole enclosed in a wooden box so that the three junctions U, V, W should be at the same temperature. The free ends of z and w were joined to the terminals of two standard resistance boxes R_1 and R_2. R_1 was keep fixed at 1,000 or 2,000 ohms while R_2 was adjusted. Since it was found that small thermo-electromotive forces were developed by the pressure of an ordinary contact key, a key was constructed at Z consisting simply of a bent copper wire mounted in a piece of fiber on a spring and bridging the gap between two mercury cups. This insured always a certain and uniform contact and one free from any E.M.F. Adjustments of the sliding contact V and of the resistance R_2 were made until the galvanometer showed no deflection whether the connector S was in the mercury cups or removed. When S was removed T was also removed and a relatively small current employed. When S was inserted T was also inserted and a current of from 10 to 20 amperes sent through the bars. In this way the resistance of the portion of B between y and z was compared with that of A between x and w. The direction of the current was then reversed by the commutator P and the measurements repeated. The results obtained by reversing the current seldom differed by more than one part in two thousand and in many cases no measurable difference could be observed.

The Observations.

The procedure in making a set of observations was as follows: An electro-magnet connected with the program clock served to turn on the gas at any hour desired. Since four or five hours was required before the steady state was reached, the mechanism was usually set for three o'clock in the morning. The thermostat was adjusted to maintain the temperature desired in the experiment by setting the contact wire at the proper height in the tube. At eight or nine o'clock in the morning observations were usually begun. The thermo-electric couple on B was adjusted till the galvanometer showed no deflection and the position of the two couples read by the cathetometer. The relative position of the index marks on the two bars was read by the micrometer telescope. The electrical resistances of fixed lengths of the two bars were then compared. A suitable weight was then hung from the end of the lever and the apparatus left for at least an hour, or until observation showed that the steady state was again reached. The series of observations was then repeated. This was continued till the maximum tension which the bar could safely stand was reached when the weights were gradually removed and the set of observations made with decreasing tension.

Results.

Measurements have been made on bars of iron, steel, copper, brass and a few on aluminum and zinc. One typical set of measurements is given in full in Table I. In this and subsequent tables T

Table I.

Iron Bar B_{11}. *July* 10, 1908. *Temperature* 200°.

T	Micrometer Readings.		ΔL	Resistance.		
Kg.	A	B	$= B - A$	Direct.	Reversed.	Ave.
	mm.	mm.	mm.			
0	2.50	2.50	0	1999.0	1999.0	1999.0
40	3.95	4.90	.95	2005.0	2005.2	2005.1
80	4.91	7.34	2.43	2017.0	2018.0	2017.5
40	4.45	6.54	2.09	2012.0	2013.0	2012.5
0	3.33	4.63	1.30	2003.0	2003.4	2003.2

Couples.			l	l^2	K	Δs	σ
A	B	$B - A$	$= 50$ cm. $- [B - A]$				
cm.	cm.	cm.					
3.19	3.54	.35	49.65	2465	1.000	0	1.0000
3.04	2.96	−.08	50.08	2508	1.017	.000235	1.0020
2.99	2.53	−.46	50.46	2546	1.033	.000601	1.0065
3.03	2.97	−.06	50.06	2506	1.016	.000541	1.0044
3.03	3.35	.32	49.68	2468	1.001	.000321	1.0070

Length of bar between points from which ΔL was measured, 111.5 cm.
Length of bar between points from which resistance was measured, 85.4 cm.
Value of Poisson's ratio employed in computing Δs, .277.

is the load in kilograms hung from the lever. Where the load is marked 0, the bars were stretched by a force due to the weight of the lever. This was equivalent to a load of about 5 kg. To express the tension in kilograms per square centimeter, T must be multiplied by 29.3 ΔL is the elongation in centimeters; l, the distance from the edge of the copper clutch to the position of the thermo-electric couple on bar B; K, the ratio of the thermal conductivity of the bar to its conductivity before any tension was applied; σ, the corresponding ratio of specific resistances; Δs, the relative change in cross-section of the bar.

TABLE II.

Further Measurements on B_{11}.

T	ΔL		σ	T	ΔL	K	σ
July 11, Temp. 195°.				July 14, Temp. 196°.			
Kg.	mm.			Kg.	mm.		
0	1.14	1.002	1.0009	0	1.68	.9952	1.0019
40	2.00	1.018	1.0019	40	2.32	1.019	1.0030
80	2.85	1.022	1.0053	80	2.90	1.024	1.0047
40	2.40	1.007	1.0042	40	2.35	1.019	1.0031
0	1.61	1.000	1.0019	0	1.70	1.003	1.0017
July 13, Temp. 196.5.				July 15, Temp. 196°.			
0	1.62	1.006	1.0019	0	1.70	1.003	1.0018
40	2.42	1.016	1.0042	40	2.36	1.022	1.0037
80	2.88	1.019	1.0044	80	2.95	1.030	1.0047
40	2.31	1.010	1.0031	40	2.35	1.022	1.0035
0	1.76	.9983	1.0020	0	1.79	.9972	1.0016

In computing the relative thermal conductivity no correction has been made for the decrease in cross-section of the bar as it is stretched. This change in cross-section may be computed from the value of Poisson's ratio for the given material and the correction applied. It is found, however, that it affects only the fourth decimal figure and hence is beyond the limits of accuracy of the experiment. Since the value of σ is computed to four decimal figures, the correction is there computed and applied.

In Table II. are given in condensed form the results of four other sets of measurements on the same iron bar B_{11}, made after those given in Table I. The results for other bars of different metals are given in the other tables. Some of the results are plotted in the accompanying curves.

Many other measurements were made on other bars of the same metals, a total of eighteen different bars being employed. The results obtained for the other bars were substantially the same as those recorded. The difference in the behavior of different bars of the same metal was no greater than the difference in successive

Table III.

Steel Bar, B_{10}.

T	ΔL	K	σ	T	ΔL	K	σ
June 25, Temp. 201°.				June 30, Temp. 205°.			
0	0	1.000	1.0000	0	.59	1.051	1.0018
40	.43	1.034	1.0009	40	1.30	1.070	1.0018
80	.88	1.050	1.0008	80	1.79	1.079	1.0021
120		1.047		120	2.19	1.080	1.0022
Weigh hung on over night.				175	2.85	1.076	1.0039
June 26, Temp. 204°.				July 1, wt. hung on over night. Temp. 203°.			
175	2.42	1.050	1.0017	175	2.91	1.076	1.0028
120	1.87	1.059	1.0023	120	2.11	1.064	1.0023
80	1.35	1.057	1.0021	80	1.42	1.068	1.0018
40	.74	1.056	1.0014	40		1.058	
0	.33	1.041	1.0012	0	.59	1.042	1.0009
June 27, Temp. 204°.				July 2, Temp. 202°.			
0	.28	1.039	1.0004	0	.59	1.041	1.0014
40	1.22	1.057	1.0010	40	1.31	1.063	1.0002
80	1.69	1.071	1.0014	80	1.68	1.073	1.0012
120	2.16	1.072	1.0020	120	2.06	1.078	1.0017
175	2.69	1.069	1.0058	175	2.76	1.080	1.0028
une 29, weight hung on from Saturday until Monday. Temp. 201°.				July 3, weight hung on over night. Temp. 202°.			
17	2.75	1.070	1.0056	175	2.76	1.085	1.0037
120	2.01	1.062	1.0053	120	2.07	1.079	1.0034
80	1.24	1.057	1.0050	80	1.43	1.071	1.0035
40	.91	1.067	1.0044	40	1.04	1.070	1.0022
0	.59	1.052	1.0027	0	.59	1.049	1.0013

measurements on the same bar. The experiment has been set up twice with different apparatus and in all cases has yielded results of the same order of magnitude.

Conclusions.

It is probable that the apparent change in the thermal conductivity of the same bar under the same tension from day to day is due in part to changes in the character of the surface of the bar. No attempt was made to prevent oxidation, which affected somewhat the surfaces of both the bars compared. If the radiating power of one surface changed more than that of the other an apparent change in conductivity would result. It is hardly probable that this change

TABLE IV.

Copper Bar, B_{14}.

T	ΔL	K	σ
July 27, Temp. 152°.			
0	0	1.000	1.0000
30	1.63	1.026	1.0001
55	3.40	1.030	1.0010
30	2.66	1.028	.9993
0	1.74	1.020	.9985
July 28, Temp. 152°.			
0	1.67	1.024	.9985
30	2.52	1.036	1.0000
55	3.67	1.030	1.0016
30	2.97	1.037	.9999
0	1.90	1.023	.9982
July 31, Temp. 150°.			
0	1.92	1.021	Resistance not measured.
30	2.75	1.032	
55	3.14	1.024	
30	2.52	1.028	
0	1.83	1.018	
30	2.68	1.029	
55	3.52	1.020	
30	2.66	1.021	
0	1.86	1.010	

T	ΔL	K	σ
August 4, Temp. 150°.			
0	1.83	1.015	Resistance not measured.
20	2.51	1.019	
30	2.74		
40	3.08	1.028	
55	3.40	1.016	
40	2.98	1.026	
30	2.76	1.018	
0	1.98	1.013	
August 5, Temp. 151°.			
0	1.96	1.010	Resistance not measured.
20	2.45	1.018	
30	2.60	1.015	
40	2.96	1.014	
55	3.65	1.007	
40	3.02	1.018	
30	2.97	1.010	
0	2.05	1.010	

during the progress of a single experiment was sufficient to affect the comparison of these results among themselves. An inspection of the tables and the curves shows that in every case the thermal conductivity of the bars increases when a moderate tension is applied. As the limit of elasticity is reached this increase approaches a maximum. As the tension exceeds this limit, the conductivity remains practically constant in the case of the more elastic metals, such as steel and brass, or begins to diminish in the case of the softer metals. This is shown in some of the tests on iron bars and particularly in the case of copper. After a bar has been stretched its conductivity does not immediately return to its former value. This is most noticeable in steel and brass. In the softer metals the return to the original value is more perfect. It is probable that steel and brass would regain their original conductivity if given

Table V.

Brass Bar, B_{18}.

T	ΔL	K	σ
August 12, Temp. 156°.			
	mm.		
0	0	1.000	1.0000
30	1.46	1.027	1.0000
60	2.19	1.030	1.0023
90	3.21	1.033	1.0047
60	2.78	1.035	1.0012
30	1.94	1.025	
0	.92	1.015	1.0000
August 13, Temp. 154°.			
0	.81	1.021	1.0086
30	1.88	1.037	1.0088
60	2.60	1.041	1.0099
90	3.22	1.042	1.0102
August 13, Temp. 154°.			
	mm.		
60	2.50	1.041	1.0099
30	1.53	1.037	1.0077
0	.83	1.024	1.0080
August 14, Temp. 154°.			
	mm.		
0	.74	1.020	1.0082
30	1.71	1.039	1.0094
60	2.50	1.041	1.0100
90	3.19	1.038	1.0106
60	2.38	1.036	1.0096
30	1.54	1.034	.0097
0	.71	1.018	1.0092

sufficient time, but this has not been tested by experiment. The total change in the conductivity of steel may amount to 7 or 8 per cent.; that of iron to 4 or 5 per cent.; brass, about 4 per cent. and copper 2 or 3 per cent. A few measurements were made on aluminum before the apparatus was put in its final form. These showed an increase in thermal conductivity of about one half per cent. under the maximum tension. Measurements were attempted upon bars of zinc but these proved unsatisfactory. The bars were not of uniform structure or cross-section, and stretched continuously and slowly under any weight applied, not being able to support even the weight of the lever. At the same time they showed apparently a larger increase in conductivity than bars of any other material tested. Since constant conditions were never attained, no great confidence can be placed in the results.

In every case the electrical resistance increased with increasing tension ; that is, the conductivity diminished. These results are in substantial agreement with those obtained by other experimenters on the resistance of wires. Of course the conditions of this experiment were not suited to a highly accurate determination of this effect. It is, however, of interest to note that the changes in the thermal conductivity produced by moderate tensions are opposite in direction to those in the electrical and are of an order of magnitude about ten times greater. Some rather rough experiments upon the effect of

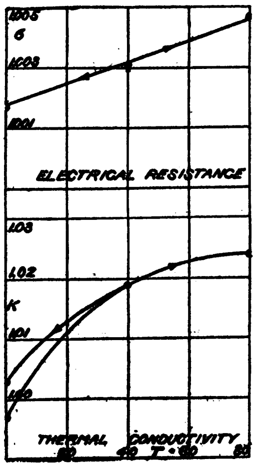

Fig. 5. Iron Bar B_{11} (July 14).

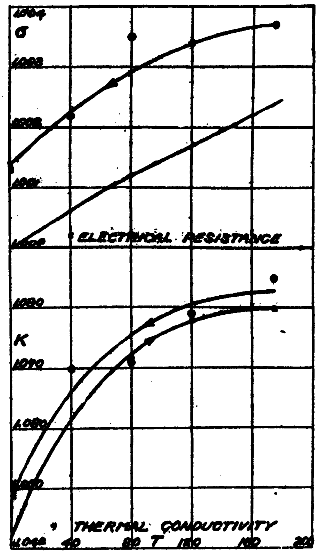

Fig. 6. Steel Bar B_{10} (July 2 and 3).

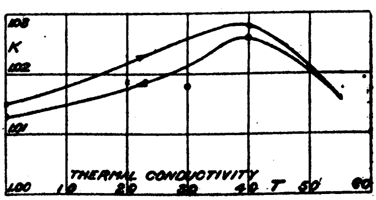

Fig. 7. Copper Bar B_{14} (Aug. 4).

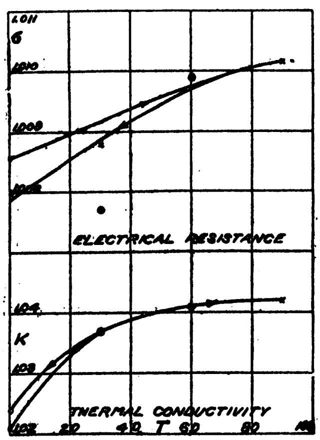

Fig. 8. Brass Bar B_{18} (Aug. 13).

bending bars of iron and copper so that a large permanent deformation was produced seemed to show a marked decrease in thermal conductivity of from five to eight per cent. This is to be expected from the behavior of copper bars when stretched beyond their elastic limit. It is hoped at some future time to make a further study of this effect and also to investigate the changes produced by torsion.

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Olivet College,
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