The University of Chicago

ON AN APPARATUS AND METHOD FOR THERMO-ELECTRIC MEASUREMENTS FOR PHOTOGRAPHIC PHOTOMETRY

A DISSERTATION

SUBMITTED TO THE FACULTY OF THE OGDEN GRADUATE SCHOOL OF SCIENCE IN CANDIDACY FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

(DEPARTMENT OF ASTRONOMY)

BY

HARLAN TRUE STETSON

A Private Edition
Distributed by
The University of Chicago Libraries

Reprinted from
THE ASTROPHYSICAL JOURNAL, Vol. XLIII, Nos. 4 and 5
Chicago, 1916

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XLIII | MAY 1916 | NUMBER 4

ON AN APPARATUS AND METHOD FOR THERMO-ELECTRIC MEASUREMENTS IN PHOTOGRAPHIC PHOTOMETRY. I

By HARLAN TRUE STETSON

The desirability of purely physical methods for the determination of the magnitudes of stars has long been felt among astronomical workers in the field of photometry. Although the introduction of polarizing and wedge photometers has greatly increased the precision of visual work, systematic errors of considerable proportions may nevertheless escape detection where in the end the eye is the sole interpreter of stellar brightness. Aside from the unavoidable differences in spectral sensitiveness of individual eyes, the errors introduced in consequence of the well-known "Purkinje effect" involve considerable uncertainty in the photometric observations over wide ranges of magnitudes.

From the very introduction of the dry plate into astronomy it was seen that the size of the stellar image upon the plate might be taken as an index of the star's magnitude. As early as 1857 Bond[1] made use of the parabolic formula

$$Pt+Q=y^2$$

[1] *Astronomische Nachrichten*, **49**, 81, 1857.

to demonstrate the relation between exposure time t and the diameter y of the photographic image, P and Q being constants of the plate used. Later investigations of Charlier[1] showed close agreement between magnitudes and measured diameters when the relation was logarithmically expressed:

$$m=a-b \log D.$$

Investigations at Greenwich and elsewhere have seemed to indicate that a square-root relation was applicable to a wider range of conditions as regards plates and instruments than the logarithmic expression would satisfy, and accordingly the well-known form has found wide acceptance:

$$m=a-b\sqrt{D}.$$

Given the relation above, the precision of results depends upon the accuracy with which D may be measured. Since the images at best show no well-defined periphery, the principal source of error in measuring is the uncertainty of locating the extremities of the diameter to be measured. The difficulty is augmented if the images are elongated or poorly defined. Again, the same eye may pass different judgments upon large and small images in the same field. The amount of agreement obtained by different observers using the same method is commendable and indicates the degree of reliability of the results.

Other methods for the reduction of magnitudes include the scale method of estimation employed extensively at the Harvard College Observatory,[2] and the method of extra-focal images.[3] In the latter case, the plates being taken at a considerable distance from the focus, the star-disks are appreciably all of the same size and the difference of magnitude is determined by comparison of the opacity of the images with a calibrated photographic wedge, by means of a Hartmann micro-photometer or other similar device.

The encouraging results obtained in the use of the selenium cell for the direct measurements at the telescope as developed by

[1] *Publikationen der Astronomischen Gesellschaft*, No. 19.

[2] *Harvard Annals*, 18, 120; 71, 4.

[3] *Astrophysical Journal*, 26, 244, 1907.

Minchin,[1] and by Stebbins,[2] and the still more recent investigations with the photo-electric cell, particularly those of Elster and Geitel,[3] Guthnick,[4] and others, indicate a far-reaching advance in the sensitiveness of photometric methods. At present, however, such methods are confined chiefly to stars of the first few magnitudes and lose advantages otherwise gained by photography.

The possibility of the application of some physical method for the measurements of photographic magnitudes from the rapidly accumulating collection of astronomical negatives has often been suggested, and emphasized particularly on various occasions by E. C. Pickering[5] in connection with the photometric researches of the Harvard Observatory. Accordingly, a series of experiments was begun by the writer in 1911 at the Wilder Laboratory of Dartmouth College, in search of suitable apparatus for the problem in hand. It is with the results obtained from this series of experiments and their application to the determination of stellar magnitudes that the present paper has to deal.

DESCRIPTION OF APPARATUS

The general principle involved is to measure the energy absorbed from a beam of light by the silver grains in the stellar image on a photographic plate, and to interpret such absorption in terms of stellar magnitude. The thermopile, bolometer, radio-micrometer, and radiometer as "detectors" were respectively considered. After some preliminary experiments and deliberation the thermopile was selected for the work as the instrument best combining simplicity and sensitiveness with convenience of manipulation.

It was seen from the outset that the apparatus in question would be more serviceable if adapted to the measurement of original negatives, rather than made dependent upon positive copies. Accordingly the scheme adopted was to restrict the region of the negative in the immediate vicinity of the star to be measured by a very small circular diaphragm, allowing the unobstructed

[1] *Proc. Roy. Soc.*, **58**, 142, 1895.

[2] *Astrophysical Journal*, **32**, 185, 1910.

[3] *Physikalische Zeitschrift*, **12**, 609, 1911; **14**, 1287, 1913.

[4] *Astronomische Nachrichten*, **196**, 357, 1913.

[5] *Harvard Annals*, **71**, 5.

light-beam to register its energy in a corresponding manner. When a successful combination of light-source, diaphragm, projecting lenses, and thermopile had been obtained, the whole apparatus was assembled, and placed in a convenient portable form. The diagram (Fig. 1) will serve to make clear the arrangement, and is reproduced as a suitable design for a more permanent and elegant form of the apparatus herein described.

Light from a source *L* passes through the condenser *C* and is brought to a focus on the surface of a metallic plate *S*, forming the stage of the instrument, upon which the photographic plate is laid. In the center of the stage is placed a small pinhole-diaphragm *d*, which serves the purpose of isolating a small portion of the plate in the immediate neighborhood of the stellar image to be measured. A projection lens *P* of short focal length is then used to project an image of *d*, and the stellar image with which it is in contact, upon the surface of a thermopile *T* connected to a galvanometer. Stellar images of different sizes corresponding to known magnitudes are placed in turn upon the diaphragm and the corresponding change in the galvanometer deflection is noted. This affords means for the calibration of the instrument. A detailed description of the method of reduction will be presented shortly. As considerable difficulty was encountered before the instrument could be relied upon for consistent readings with the degree of accuracy sought, the essential features in the design of the apparatus in its final form will be mentioned.

In regard to the thermo-element, it was seen at once that a surface-element would be necessary, of small heat-capacity, hence short period, and of the maximum sensitiveness. After futile efforts to obtain a suitable element in the market, the requirements were met by a thermopile specially designed and constructed for the purpose by W. W. Coblentz, of the Bureau of Standards, Washington. The element is a bismuth-silver combination of symmetrical design, having a circular receiving surface 5 mm in diameter, and a resistance of 2.3 ohms.[1] Tests reveal a sensitiveness

[1] For a detailed description of the construction of bismuth-silver thermo-couples, see "Instruments and Methods Used in Radiometry," by W. W. Coblentz, *Bulletin of the Bureau of Standards*, **9**, 15, 1912.

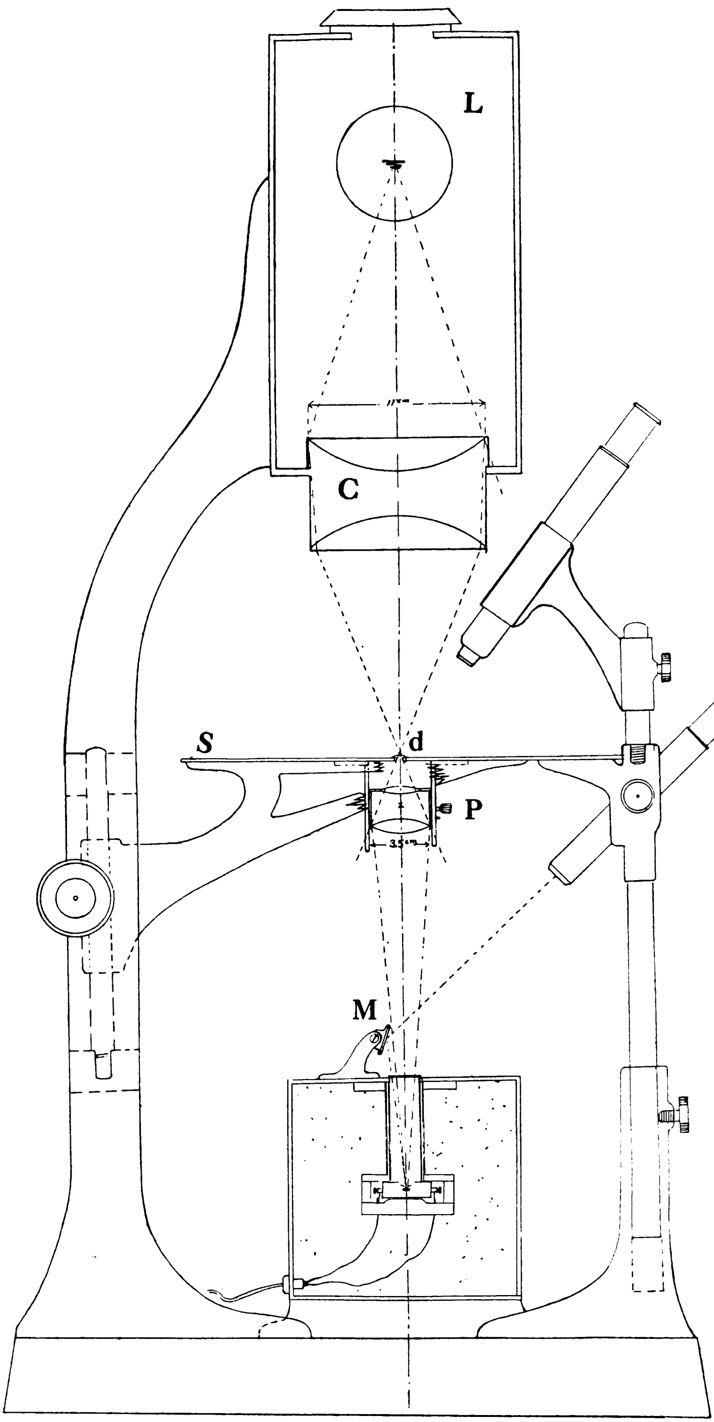

FIG. 1

of one microwatt per 0.18 microvolt, and a period of but a few seconds' duration. The element was incased in a brass receptacle and the whole mounted in the interior of a box of nonconducting material packed with cotton wool, the opening to the well through which the radiation passes being closed with a thin glass plate. Such insulation has proved wholly satisfactory under all ordinary conditions of temperature. With the seclusion of the thermopile, it became necessary to introduce a small mirror at *M* to reflect light from the element into a telescope of short focus at *E*, which serves to locate the stellar image upon the receiver.

Any error due to imperfect centering of the stellar image upon the thermopile is avoided by using the finding eyepiece and making several settings for each star. The agreement is generally better among the image-readings than among the plate-readings, as the latter are distributed more or less uniformly about the image in question, and represent the transparency for slightly different parts of the film.

As at best but a small part of the light from the source is transmitted through the restricting diaphragm *d* (Fig. 1), it is seen that the light-source used must combine intensity with smallness of dimension. The arc, though complying beautifully with these requirements, becomes entirely unsuited for the purpose, on account of its marked unsteadiness. Since it was hoped for the sake of compactness and convenience to use a direct-deflection method, an incandescent lamp operated from a storage-battery of 50 volts was sought. The most convenient luminant tested thus far has been found to be a 50 c.-p. stereopticon-bulb with a carbon filament spirally wound. Such a source when projected may be adjusted to give a satisfactory illumination over the diaphragm, uniformity of light being obtained by so orienting the bulb that projected images of the neighboring coils of the filament overlap slightly. The Nernst lamp at first appeared most promising, but in the final arrangement was abandoned in favor of the type described. Interviews with lamp-makers in consideration of filaments of special design have given little promise of improvement. The ideal source would be an incandescent lamp with a disk or spherical source, some two or three millimeters in diameter, of about the intrinsic brightness of the carbon lamp. The 50-volt stereopticon-bulb has the advantages of a filament of large radiating surface, and ease of duplication, and when operated from storage-batteries gives very satisfactory results.

In general any change in voltage is kept under control by connecting a precision voltmeter across the terminals of the lamp. An adjustable rheostat is provided for regulating the voltage when necessary. With storage-batteries of sufficient capacity, little error is introduced from this source, and any progressive or gradual fluctuations in voltage are duly corrected for in the method of reduction, as the plate-readings take into account both variation in the transparency of the film and any gradual change in the intensity of the source. As the plate-readings for each star are necessary in any instance, and form a convenient check on the readings, further precautions beyond the employment of storage-batteries and a sensitive voltmeter seem unnecessary as regards the source.

The essential feature of the diaphragm at *d* (Fig. 1) is the restriction of the cone of light to the approximate dimensions of the stellar image under examination. By the use of two such diaphragms used interchangeably, a wide range of magnitudes is covered for any given plate. In practice the diameters of the apertures most generally used were Nos. 60 and 75 on the standard drill gauge, corresponding to about 1.0 mm and 0.5 mm, respectively.

As originally designed, the diaphragms employed were drilled in a sliding strip of brass flush with the stage of the instrument. Since the ordinary commercial plates are slightly concave, it was found that a large and troublesome source of error was being introduced, the effect of the concavity providing a variable distance between the stellar images and the diaphragm. This varied seriously the dimensions of the cone of light transmitted to the thermopile for stars of equal magnitude when located on different parts of the concave surface. After several experiments, a conical design for the diaphragm was adopted, projecting slightly above the level of the stage of the instrument and thus insuring an invariable contact with the film under all cricumstances (Fig. 2). Any error due to the variation of the slight tilt of the plate thus made necessary was carefully looked for but found negligible.

In the design of the projecting lens two factors have been borne in mind: first, the requisite focal length to produce proper magnifying power within the limits of adjustment; and, secondly, utilization of all the principal rays of the cone emanating from *d*. Change of diaphragm in general will be accompanied by change in the projecting lens in order that the projecting image may cover but not overspread the receiving surface of the thermopile. Two projecting lenses are thus provided corresponding to the two diaphragms most generally employed.

The complete instrument and auxiliary apparatus consisting of galvanometer, resistance box, volt- and amperemeters are shown as in use in Fig. 3.

Adjustments are provided for the light-source in three coordinates. The stage and projecting lens are also adjustable for centering and focusing. A series of diaphragms are provided and may be placed interchangeably upon the stage.

FIG. 2

In the selection of a galvanometer, several factors enter. Theoretically the thermopile is working most efficiently when the resistance of the galvanometer equals its own. For this reason as well as for its high sensitivity, the Thompson type is that usually employed in thermo-electric measurements. On the other hand, convenience of manipulation and freedom from outside disturbances favor the D'Arsonval or moving-coil type whenever such can be adapted to the end sought. Most satisfactory results have been obtained in the present investigation with a Leeds and Northrup galvanometer, Type H, having a resistance of 50 ohms and a sensitivity of 0.75 microvolt. The critical damping resistance of such an instrument is 120 ohms, and is obtained by using external resistance in series with galvanometer and thermopile. With diaphragm No. 60 and a photographic plate of normal density, these conditions afford deflections conveniently large and within the limits of the scale. To obtain corresponding deflections with smaller diaphragms the external resistance is necessarily reduced, increasing the sensitivity of the galvanometer and lengthening its period.

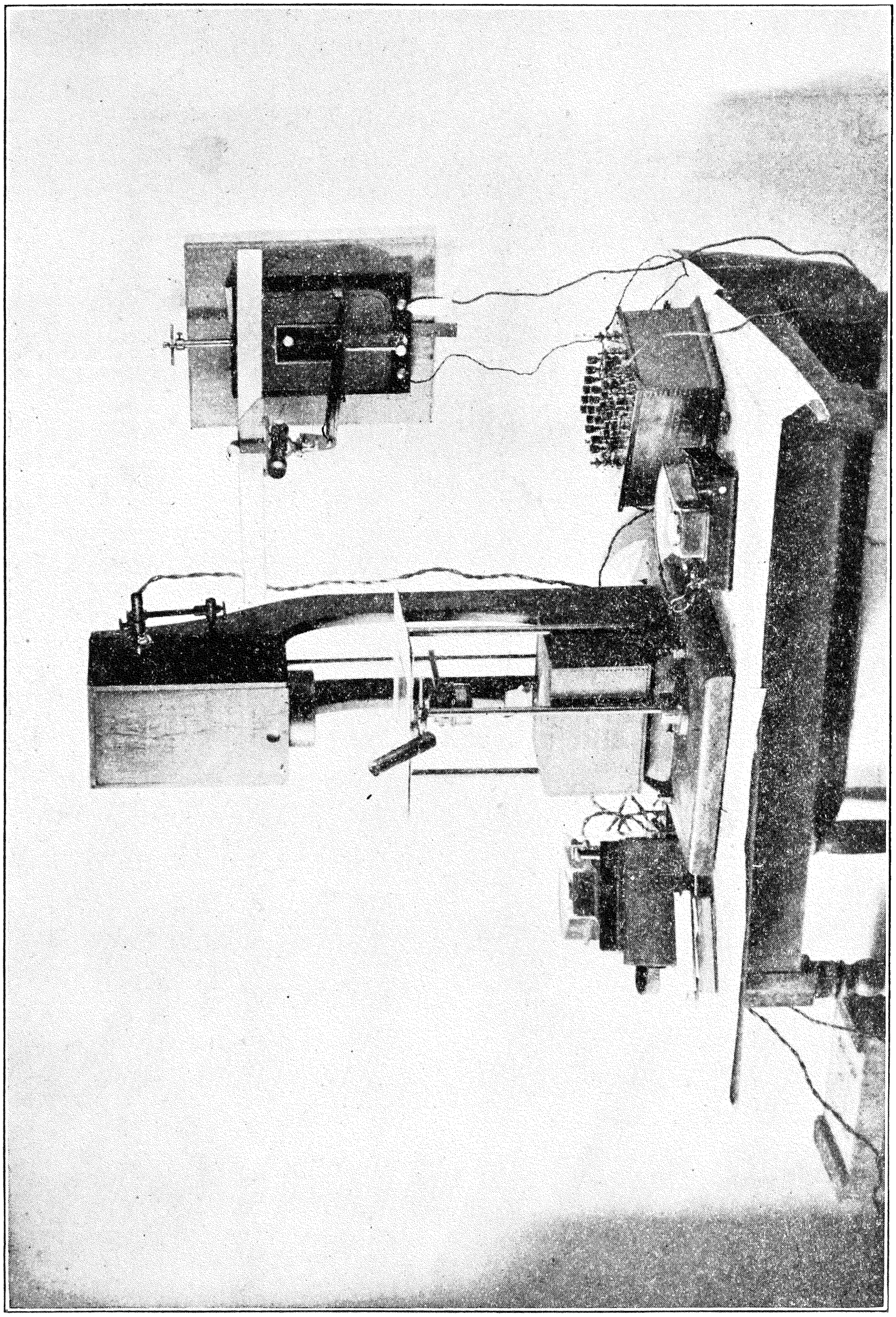

FIG. 3.—The thermo-electric photometer and auxiliary apparatus

As few connections as possible should be made between the galvanometer and thermopile, care being taken to avoid unlike metals, to use ordinary means for maintaining uniform temperature and freedom from draughts, and to protect all binding-posts and connectors with cotton wool.

METHODS OF MEASUREMENT AND REDUCTION

In measuring a plate for reduction, a diaphragm somewhat larger than the stellar images to be measured is selected and put in place upon the stage of the instrument. Since the construction of the diaphragm is such that it projects slightly above the level of the stage-platform to insure contact with the film, the plate is always placed film side down with care. The stellar image in question is placed over the diaphragm, and brought to the center of the latter by viewing its image upon the thermopile by means of the finding eyepiece. As soon as the galvanometer has come to a steady deflection the reading is noted. The stellar image is then moved just off the diaphragm and the reading of the galvanometer again noted as an index of the amount of energy transmitted through the unobstructed portion of the plate and film.

Since the deflections of the galvanometer are taken proportional to the difference of potential impressed, and the difference of potential maintained by the thermopile is directly proportional to the heating effect, then the amount of energy absorbed by the stellar image may be expressed as

$$q=K(D-D'),$$

where D and D' represent the two readings of the galvanometer and K is a constant of proportionality.

Again, since the size and density of the stellar image upon the plate increase with the brightness and hence with the magnitude of the star, which will be denoted by m, for any given exposure we may write:

$$m=f(D-D'),$$

and the problem of reduction is the problem of ascribing the form of the function existing for given conditions. Let A be the area of the free aperture of the diaphragm (Fig. 4). Consider the stellar image which, because of its size and blackness, absorbs energy, as having an effective diameter d equal to that of an opaque disk absorbing the same amount of energy. If ρ = the energy transmitted per unit-area, then we have the relation

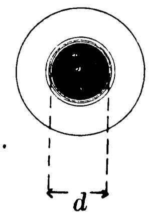

FIG.

$$\frac{\rho A}{\rho\left(A-\frac{\pi}{4}d^2\right)}=\frac{D}{D'} \tag{1}$$

where D and D' represent the galvanometer-deflections corresponding to the plate-readings taken with free aperture, and with a stellar image interposed, respectively. The above may be simplified and written

$$\frac{A}{\frac{\pi}{4}d^2}=\frac{D}{D-D'} \tag{2}$$

whence

$$d^2=\frac{4A}{\pi}\left(\frac{D-D'}{D}\right)=c\delta \tag{3}$$

where δ represents the difference in deflection between the free and image-readings for a given star per centimeter of deflection for free aperture, and c is the constant of proportionality.

If we assume the well-known relation between magnitude and measured diameters to hold in the present instance, we have

$$m=a-b\sqrt{d},$$

or

$$d=\left(\frac{m-a}{b}\right)^2.$$

Hence, substituting in (3), we obtain

$$\left(\frac{m-a}{b}\right)^4=c\delta$$

$$m-a=bc^{\frac{1}{4}}\delta^{\frac{1}{4}};$$

or, writing new constants,

$$m=a-\beta\delta^{\frac{1}{4}} . \tag{4}$$

This formula, though indirectly derived from the square-root formula, stands merely as an empirical expression in an attempt at the solution of $m=f(\delta)$. It will be observed that in defining δ as the ratio $\frac{D-D'}{D}$ no assumption as to the constancy of D is involved. The value of D will depend upon the transparency of the plate-film, thickness of the glass, voltage of the source, resistance in the galvanometer-circuit, etc.

Since the reduction involves only the ratio $\frac{D-D'}{D}$, it will be noted that a change in D due to a change in intensity will not affect the results, provided conditions remain constant throughout the measurement of D and D' for a given star. In like manner, variation in the transparency of glass and film over different parts of the plate will be largely if not wholly eliminated, since an increase in absorption though reducing the deflections does not alter this ratio. This was verified experimentally by taking readings on stellar images on a given plate in the usual manner, then remeasuring the same stars with absorbing media interposed in the path of light to the thermopile. Although D was reduced about 50 per cent, the ratio $\frac{D-D'}{D}$ remained the same for each star within the limits of accidental errors. Similar results were obtained from measurements of the same stellar images for different values of voltage.

The first plates reduced were those taken at the Dearborn Observatory, with the 18-inch refractor, a color-filter, and Cramer Isochromatic plates. It was known from careful measurements that the square-root formula gave good results for measured diameters on these plates.

The close approach to a straight-line relation between magnitudes of the Pleiades and the fourth root of the galvanometer-deflections obtained by measuring images on the thermopile is shown graphically in Fig. 5, and justifies the reduction-formula experimentally. Müller and Kempf's[1] values for the magnitudes

[1] *Astronomische Nachrichten*, 150, 203, 1899.

are laid off as abscissae, and the fourth root of the corresponding galvanometer-deflections as ordinates. The constants α and β of (4) may be taken at once from the graph, and used for the determination of any unknown magnitude on the plate.

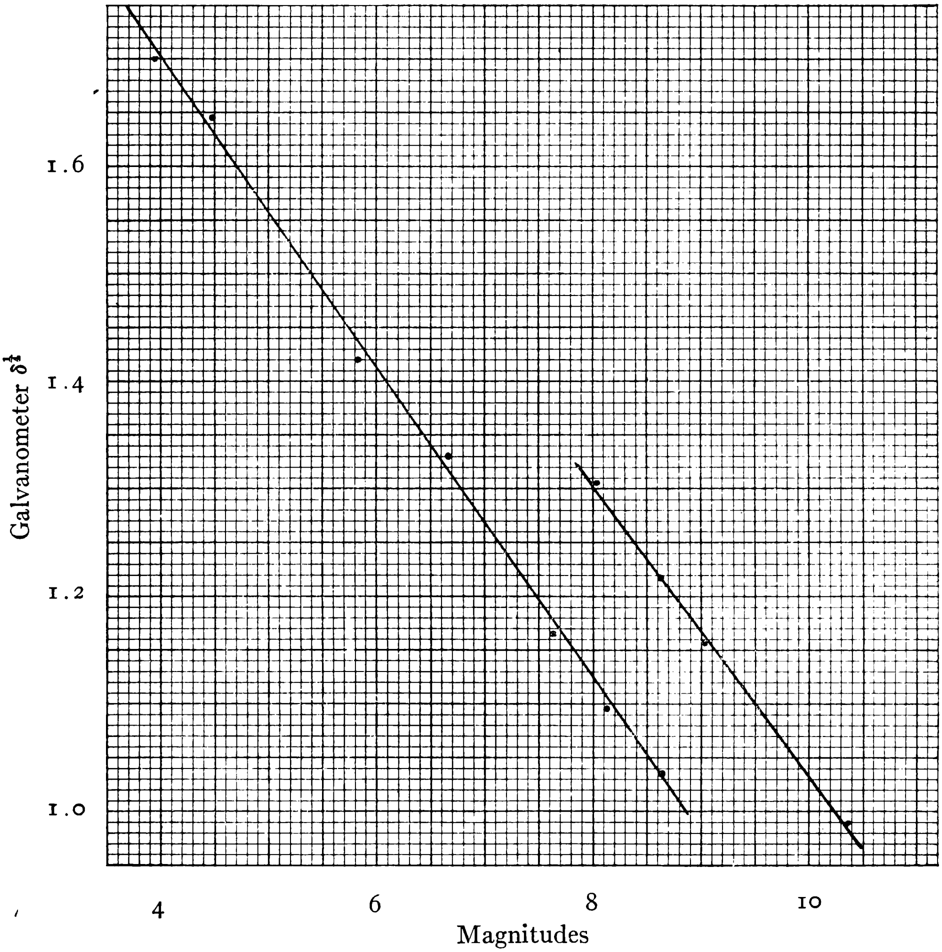

FIG. 5.—From Plate A 140 taken with 18-inch refractor and color-filter at Dearborn Observatory.

As the stellar image becomes smaller relative to any particular diaphragm, the proportional amount of energy absorbed becomes less and the consequent change in the galvanometer-deflection decreases, as is evident from the algebraic expression $m = \alpha - \beta\ \delta^{\frac{1}{4}}$. For small values of δ the change in $\delta^{\frac{1}{4}}$ becomes less appreciable and therefore this increase of error is compensated for by substituting a smaller diaphragm and proceeding as before.

If Δm represents a known difference in magnitude between two stars, and $\Delta\delta^{\frac{1}{4}}$ the corresponding difference in $\delta^{\frac{1}{4}}$ for the two readings, then, for any given diaphragm, the slope of the straight-line relation becomes

$$\frac{\Delta m}{\Delta\delta^{\frac{1}{4}}}=\beta .$$

For the same Δm measured with a second diaphragm we have similarly

$$\frac{\Delta m}{\Delta\delta'^{\frac{1}{4}}}=\beta' ,$$

whence the relation between the slopes of the two plots becomes

$$\frac{\beta}{\beta'}=\frac{\Delta\delta^{\frac{1}{4}}}{\Delta\delta'^{\frac{1}{4}}} , \tag{5}$$

and readings taken with any diaphragm may be reduced to the corresponding scale of any other when two stars are measured in common with the two diaphragms.

In Fig. 5 the graphs are drawn separately for each diaphragm on the same scale and indicate clearly the use of the smaller aperture for the fainter magnitudes. It should be added that with the substitution of the smaller diaphragm, the sensitivity of the galvanometer was increased by lowering the external resistance from 120 to 40 ohms.

In practice it is found that a single diaphragm may be relied upon for a range of three or four magnitudes in measurement, and that the use of two different diaphragms is usually sufficient to cover the entire range of measurable magnitudes on a single plate.

To understand more fully the rôle of the diaphragm and conditions of sensitivity in measurement, let R be the radius of any given aperture used for the diaphragm, and r the effective radius of the stellar image. From Fig. 4 and equations (1) and (2) we may write

$$\frac{\pi R^2}{\pi r^2}=\frac{D}{D-D'} ,$$

or, writing δ for $\frac{D-D'}{D}$ as before, we have

$$\delta=\frac{r^2}{R^2} ; \tag{6}$$

whence by differentiation

$$\frac{d\delta}{dr}=\frac{2r}{R^2}. \tag{7}$$

The sensitivity of the arrangement represented by the derivative $\frac{d\delta}{dr}$ is therefore directly proportional to r, since R is constant for any given diaphragm.

A varying degree of sensitivity dependent upon the size of the image might seem at first a serious and troublesome disadvantage. As will be shown, however, the difficulty is more apparent than real, and may be controlled within limits usually exceeded by accidental errors.

As the sensitiveness of the instrument falls as a result of small values of r relative to R, a new diaphragm of smaller dimensions is substituted. Reference to equation (7) will show that such a change causes an increase in the sensitivity, inasmuch as $\frac{d\delta}{dr}$ varies inversely with R^2, although the free deflection D will be less than before in consequence of the corresponding change in the amount of energy admitted to the thermopile. This change in D, however, may be conveniently compensated for by decreasing the resistance in the galvanometer-circuit until D regains its former value. Although theoretically $\frac{d\delta}{dr}$ depends only on r and R and is quite independent of the magnitude of D, nevertheless in practice it will be seen that the accuracy with which δ may be observed is greater for large values of D. For this reason the resistance in the galvanometer-circuit is always adjusted to maintain D conveniently large for a given set of readings.

Let us suppose for the sake of comparison that we were to measure transparent stellar images upon an opaque ground, such as would be the case in measuring a glass positive instead of the original negative. In this case the galvanometer-deflection D, due to the energy transmitted by the stellar image, would be directly proportional to the area of the image or to r^2, in the form

$$D=Kr^2,$$

the derivative of which would indicate the sensitiveness of the arrangement

$$\frac{dD}{dr}=2Kr\,.$$

Comparing this expression with equation (7) above, we see that as before the sensitiveness varies directly with the radius of the image without the advantage of the controlling factor R^2. Experimental tests with both positives and negatives revealed at once the increased sensitiveness to be obtained with the use of the interchangeable diaphragms on negative plates.

Aside from the advantage of using the original plate, moreover, the ease of manipulation in locating and setting upon the several stellar images is much facilitated by using the plates with dark images against a clear background, as is the case in the negative form. Again, since readings taken from positive plates would be subject to uncertain corrections for opacity of the background and for varying degrees of transparency of glass and film, the advantage lies in the clear glass background, where the condition of the film is constantly under observation and the galvanometer-deflections are large.

Since the sensitiveness of the thermo-electric photometer, if we may so term the apparatus, has been shown to be a function of r for any particular diaphragm, it becomes important to investigate the changing effect of error in δ upon the resulting magnitude.

Making use of the equation of reduction (4),

$$m=\alpha-\beta\delta^{\frac{1}{4}}\,,$$

and differentiating, we obtain

$$\frac{dm}{d\delta}=-\frac{\beta}{4\delta^{\frac{3}{4}}}\,.$$

If the galvanometer-deflections are recorded in centimeters, the value of β for the Dearborn plates is very nearly 6.5. The same is true for Cramer plates and color-filter used with the 6-inch Zeiss doublet of the Yerkes Observatory. If we assume for the error of the mean of several settings on a star an accidental error not greater than $\Delta\delta = 0.01$ cm, a very plausible assumption, we may express the resulting error in stellar magnitudes as

$$\Delta m = \frac{6.5 \times 0.01}{4\delta^{\frac{3}{4}}} = 0.0162\ \delta^{-\frac{3}{4}}.$$

A curve was therefore to be platted for $\beta = 6.5$ using values of δ as abscissae and the corresponding computed value of Δm as ordinates (Fig. 6). Similarly, a family of such curves could be constructed

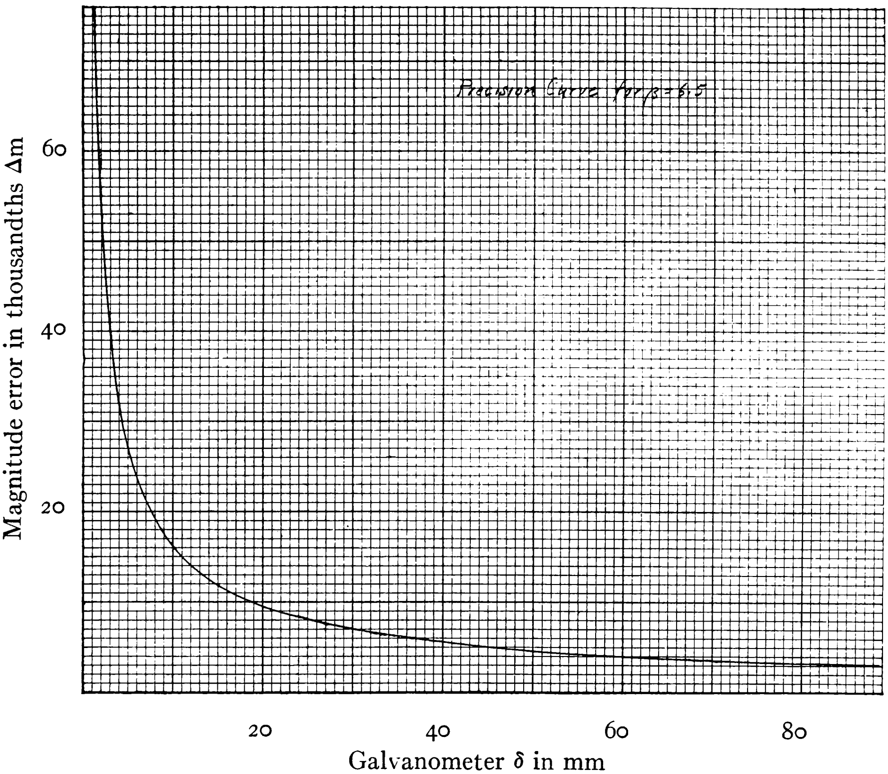

Fig. 6.—Precision curve for $\beta = 6.5$

with the parameter β. Such a curve serves at once to indicate the degree of precision attainable with given conditions of plate, galvanometer, and diaphragm. As will be seen, for example, when δ, the fall in galvanometer-deflection due to the interposition of the stellar image, is equal to 1 cm, the error in magnitude resulting from a reading-error of 0.01 cm, will be $0^{M}_{.}016$ and will remain less than $0^{M}_{.}01$ if δ does not fall under 1.6 cm. Unfortunately, systematic errors due to conditions of atmospheric transparency or irregularities in "seeing" and plate-sensitiveness seldom warrant higher precision in measurement, though such should be easily obtainable if the stellar images are not too minute to afford larger values of δ. Except in the case of the faintest images, readings are seldom made when $\delta < 1$ cm. When, therefore, in a series of measurements δ reaches the limiting value, a smaller diaphragm is substituted, and the sensitiveness of the galvanometer increased by lowering the variable resistance, with the resulting increase in δ for the same star.

In the actual reduction of plate-magnitudes and the platting against a known or assumed scale of magnitudes as is shown for the Dearborn plate in Fig. 5, care must be taken to differentiate between errors of measurement and systematic errors often neglected, such as correction to the center of the plate, color-curve of the lens, and the relation of color-equation to plate and filter used with a given outfit. As many of these quantities are at best uncertain, it is seen that we could hardly expect a given plate to show residuals not larger than those attributable to errors of measurement. In the interpretation of the curve in Fig. 5 it is to be noted that no correction to the center was attempted, and the approximation to the straight line indicates the degree of agreement between the color-filter and the Cramer plate combination, and the Müller and Kempf "eye."

MEASURES OF THE MAGNITUDES OF STARS OF THE PLEIADES

In investigations with the thermo-electric photometer at the Yerkes Observatory, plates have been measured, taken with each of the several instruments regularly in use, the 6-inch Zeiss camera, the 2-foot reflector, and the 40-inch refractor. Fig. 7 shows the results of platting measurements of the Pleiades in precisely the same manner as was done in Fig. 5. Inasmuch as the field of the Zeiss doublet shows no appreciable correction for distances less than 0.05 from the center,[1] this somewhat uncertain quantity may be considered as inappreciable in the case of the Pleiades plates where the group is centrally located.

[1] *Astrophysical Journal*, **36**, 179, 1912.

In this comparison the agreement with Müller and Kempf's values is at once apparent. The value of the constant β of the reduction-formula as determined by the graph is 6.49 against 6.50 for the Dearborn plates.

To indicate clearly the method of measurement and reduction as well as the amount of agreement among individual settings, a specimen reduction-sheet is shown for Plate UV 1261 (Table I).

The first column gives the Bessel designation of the star. The second column gives the galvanometer-readings in centimeters, when the stellar image is located centrally on the diaphragm, and

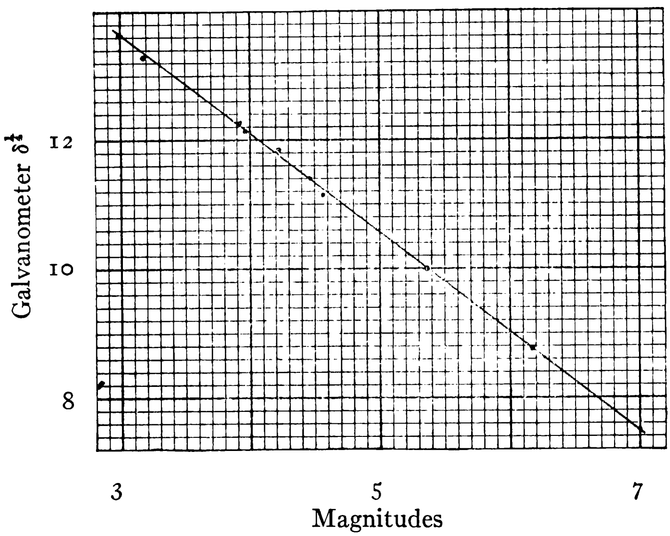

FIG. 7.—From Plate UV 1261, taken with Zeiss doublet and color-filter at Yerkes Observatory.

the third column shows the readings corresponding to the free film of the plate in the immediate neighborhood of the image. Three settings are usually taken as sufficient for each star, and the mean is used. The plate-readings serve in each instance to check any progressive errors in the transparency of the film or glass as well as the effect of possible change in the voltage of the light-source. Although a slight fogging of the background seems to have little effect upon the size or blackness of the stellar image, it enters of course with full value upon the transparency of the background with which each image is compared, and so needs to be eliminated. To

TABLE I

PLATE* UV 1261, PLEIADES. MEASURED DECEMBER 17, 1914

Volts, 51.5. Amperes, 3.8. Galvanometer Resistance, 40. Diaphragm No. 75

Star	Galvanometer Deflection			δ	$\delta^{\frac{1}{2}}$	Magnitude		
	D'	D	$D-D'$			Müller and Kempf	Computed	Residual
η..........	9.83 cm	14.20 cm						
	9.80	14.21						
	9.75	14.16						
	9.79	14.19	4.40 cm	3.10	1.326	3.19	3.23	+.04
f..........	10.88	14.10						
	10.94	14.08						
	10.90	14.08						
	10.91	14.09	3.18	2.26	1.225	3.92	3.90	−.02
b..........	10.83	13.80						
	10.83	13.88						
	10.88	13.81						
	10.85	13.83	2.98	2.16	1.212	3.96	3.98	+.02
c..........	11.40	14.18						
	11.38	14.19						
	11.39	14.18	2.79	1.97	1.185	4.21	4.16	−.05
d..........	11.56	13.89						
	11.58	13.99						
	11.69	14.04						
	11.61	13.97	2.36	1.69	1.139	4.48	4.46	−.02
e..........	11.02	13.10						
	11.00	12.95						
	11.00	12.91						
	11.00	12.98	1.98	1.53	1.114	4.57	4.62	+.05
h..........	12.69	14.12						
	12.70	14.10						
	12.70	14.11	1.41	1.00	1.000	5.38	5.36	−.02
k..........	12.78	13.53						
	12.79	13.59						
	12.78	13.56	0.78	0.58	0.874	6.17	6.18	+.01

$\beta = -6.50$ $\alpha = 11.86$ p.e. $= \pm 0.022$

* Cramer Instantaneous Isochromatic with visual luminosity filter β 10.

avoid decimals it is found convenient to take for δ ten times the ratio $\frac{D-D'}{D}$. The quantity δ may now be defined as the difference in galvanometer-deflections between the stellar image and its background for a standard plate-reading of 10 cm. Column five therefore gives

$$\delta = \frac{D-D'}{D} \times 10 \, .$$

The fourth root of δ is entered for each star in column six and is easily obtained by use of a table of squares and square roots of numbers. If the number whose fourth root is desired is located in the column marked "Square," its fourth root may at once be taken from the column marked "Square Root"—such tables for numbers from 1 to 1600 are usually found in the numerous engineers' handbooks and require little interpolation.

The last three columns give the magnitudes found by Müller and Kempf, the magnitudes computed from the constants $\alpha = 11.86$, $\beta = -6.50$, obtained from the least-square solution, and the resulting residuals, indicating close agreement between the magnitudes photo-visually determined with the Zeiss camera and Müller and Kempf's scale, the probable error for a single star being $0^{M}.022$.

Ordinary plates taken without light-filter with the 2-foot reflector of the Yerkes Observatory were reduced in a similar manner and justified again the use of the fourth-root law. The results of reducing Pleiades-magnitudes from Plate R 531 is shown graphically in Fig. 8. The photographic magnitudes platted as abscissae are those of Münch determined at Potsdam. The solid dots indicate the reduced magnitudes after correction to the center of the plate. Although the aperture of the mirror was reduced to 12 inches for the particular exposure in question, the field is still subject to correction for distances greater than $\rho = 4'$ and amounts to $0^{M}.2$ or more at a distance of $\rho = 20'$. The method of calibration of this field correction was similar to that previously described in this *Journal*[1] and needs no detailed description here. In the present instance Plate R 2594 was selected, which contained four sets of images taken across the optical axis. The absorption of these images was

[1] *Astrophysical Journal*, **38**, 224, 1913.

measured with the thermopile in the usual manner and the $\delta^{\frac{1}{4}}$ corresponding to each image was platted against the distance of the images from the center of the plate, expressed in minutes of arc. Fig. 9 shows the resulting graph. The departures from the mean

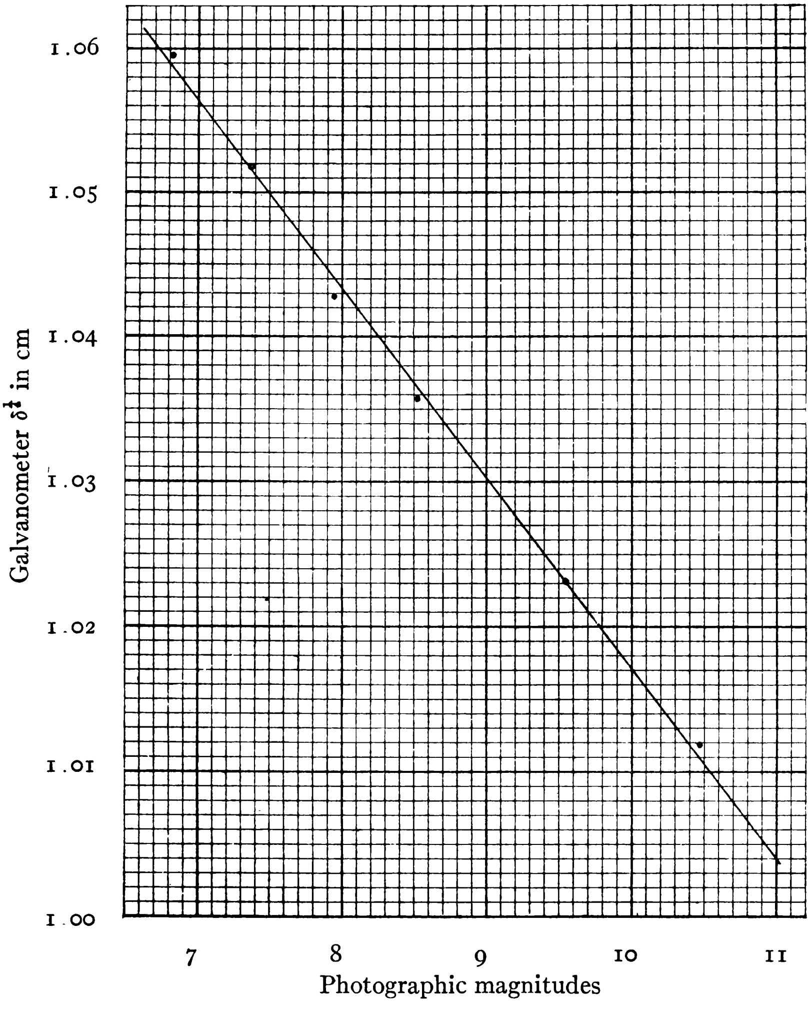

FIG. 8.—From Plate R 531, taken with 2-foot reflector at Yerkes Observatory

curve may be attributed to changes in "seeing," atmospheric transparency, and small errors in the equality of the times of exposure. These curves are all similar and express the variation of $\delta^{\frac{1}{4}}$ with distance from the optical axis for four different-sized images corresponding to four different values of $\delta^{\frac{1}{4}}$ for any given distance from the axis. Within the range of images measured, no particular dependence of the form or size of correction upon the size of the image seems apparent. It must be admitted that corrections made in this manner seem at best artificial, and at present it seems exceedingly doubtful if corrections taken from certain plates used experimentally can be applied generally to any other plate without large assumptions as to the order of accuracy maintained. In applying corrections from the curves of Fig. 9 to the magnitudes of R 531, Fig. 8, the constant β of equation (4) was so chosen as to reduce to a minimum the residuals of the corrected magnitudes of R 531.

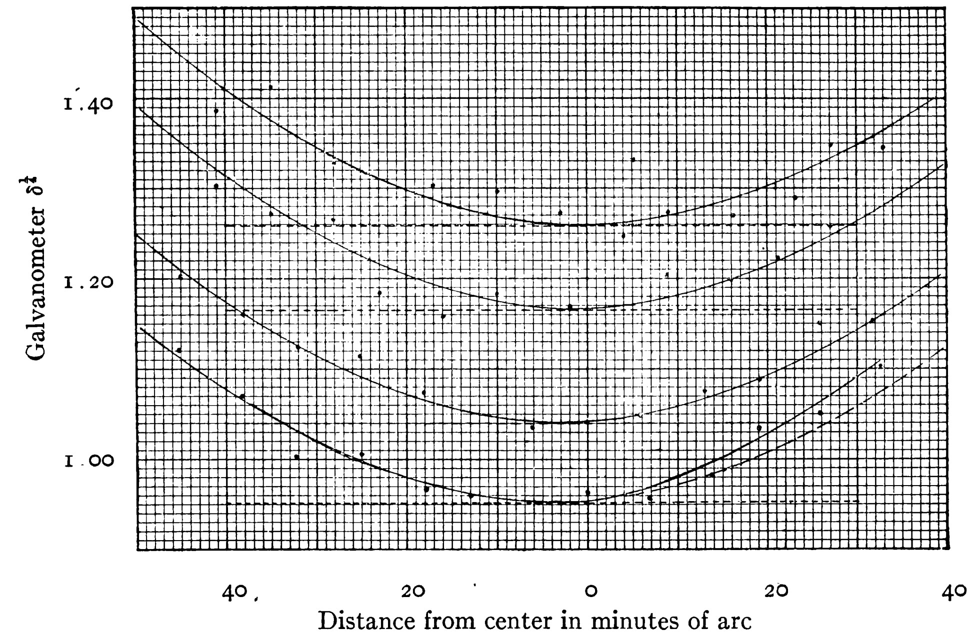

FIG. 9.—Calibration curves for 2-foot reflector from Plate R 2594

This gives a value of $0^{M}.026$ for a change in $\delta^{\frac{1}{4}}$ of 0.01 unit. Stars 24 and 27 (Bessel) receive the largest corrections, being 17′ and 26′ from the axis, respectively.

The whole matter of finding suitable methods of correction to the center of the plate is one of the most obstinate difficulties in the way of exact photometry on an absolute scale. A high degree of precision in the measurement of photographic images will therefore best be rewarded in the investigation of relative magnitude-changes or in the photometry of small areas where the field under investigation is restricted to the vicinity of the optical axis.

Other difficulties in the way of more accordant results from various observers is the lack of uniformity of equipment and spectral sensitiveness of the plates used. "Photographic magnitudes," as at present published are understood to be the results of measurement of the stellar images on "ordinary" (blue-sensitive) dry plates exposed at the focus of a reflector or a photographic refractor (corrected to blue rays). Unfortunately the difficulties of non-uniformity in color-sensitiveness of the various dry plates on the market is greatly accentuated by the wide divergence in the optical equipment employed. Unless a color-filter is used, the uncorrected, and out-of-focus rays will materially affect the size and blackness of the stellar image on the plate. Moreover, the effect will be quite different for differing spectral types, the difficulty being exaggerated for focal settings on either side of the normal position. Again, a photographic objective of special Jena glass will undoubtedly behave somewhat differently from a photographic lens of the usual flint and crown combination. In the case of the reflector, where radiations of all wave-lengths are combined, the effect must invariably differ from the case of any refracting telescope. These divergences, small or large, will become more apparent with greater refinement in the measurement of the photographic images and indicate further fields of investigation in stellar photometry.

It would seem that, for agreement on a precise scale for magnitudes, the reflector would be the ideal instrument for photography, as the question of a color-curve would thus be entirely eliminated so far as the instrument is concerned. The chief difficulty remaining would then be the adoption of a standard plate of given spectral sensitiveness, or, in want of such plates, employing a prescribed filter of known absorption which would produce as far as possible a certain effect for a given brand of plates.

For purposes of completeness in the application of the fourth-root law for the reduction of thermopile-measures, tests were made upon plates taken with the 40-inch refractor through color-filter and with isochromatic plates. Fig. 10 shows the magnitudes of the Pleiades on Müller and Kempf's scale platted against the corresponding $\delta^{\frac{1}{4}}$ of the galvanometer-readings. From other series of measurements covering a variety of times of exposure, the slope of the line appears quite independent of the time of exposure, at least within the range of exposures used.

Although considerable unpublished material exists relative to the magnitude-corrections to the center of the plate for the 40-inch refractor, and further investigations were made using the thermopile method for plate measurement, consistent results have not been obtained which may be trusted within the degree of precision

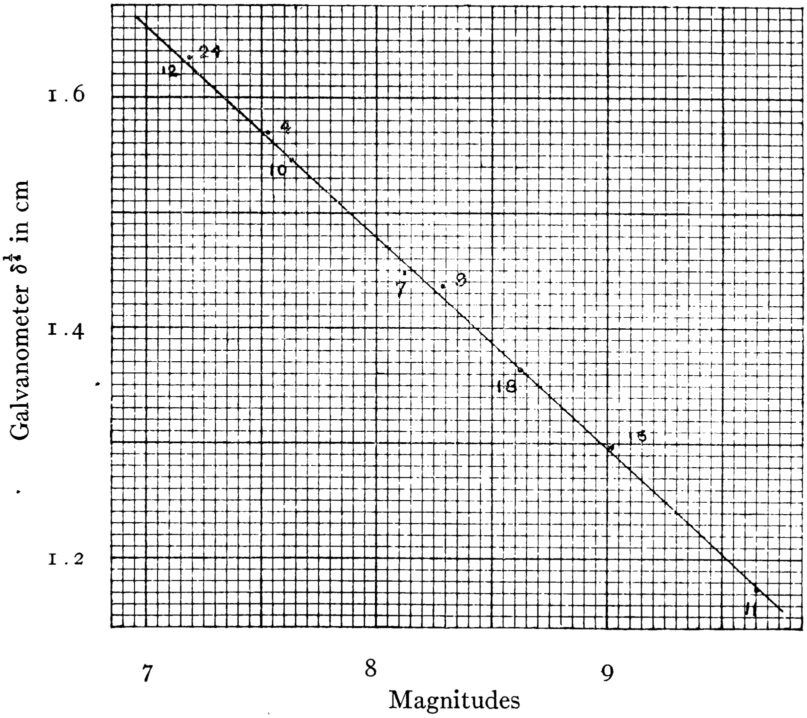

FIG. 10.—From Plate 015, taken with 40-inch refractor and color-filter at Yerkes Observatory.

desired. Owing to rapid and uncertain changes in seeing and the great focal length, it has been found practically impossible to use the simple method employed for the calibration of the reflector plates. Inasmuch as the stars of the Pleiades platted above cover a considerable area on the plate, the residuals indicated are doubtless attributable to this troublesome and uncertain correction to center which has yet to be applied. A still further difficulty in the way of applying this correction is the uncertainty of locating with exactness the position of this so-called "center." Although a rough approximation for the position of the optical axis with a given plate may be obtained from the intersection of the prolonged major axis of the elongated stellar images near the edge of the plate, the errors introduced by the uncertainty in the location will easily amount to several hundredths of a magnitude at large distances from the axis. Further, the stability of the position of this "center" from plate to plate should not be assumed without careful investigation.

The investigations of Schlesinger[1] in locating the optical axis in the case of the 40-inch refractor, and the subsequent precautions taken in obtaining plates for stellar parallax, may not be without application to work of precision in the line of photographic photometry.

SOURCES OF ERROR IN THE PLATE

Enough has already been said in regard to sources of error for which the telescope is responsible. To a large degree these errors are susceptible of evaluation and correction, if not of elimination. There remains, however, one serious source of error in all photographic photometry which has its effect in purely relative measurement as well as in determinations on a standard scale. Such is the plate itself. Residual errors due to a lack of uniformity in the sensitiveness and thickness of the emulsion on the plates must exist, and it remains to eliminate as far as possible dangers from this source. Fortunately the magnitude of such errors has not been large enough to interfere seriously with the tenth's place on the magnitude-scale, and for this reason such errors have in general aroused little attention. If, however, we hope to push forward another decimal place in astronomical photometry, here lies one of the fields for further investigation and research.

Neglecting the region of the plate within 1 cm from the edge, Parkhurst[2] found from several hundred measures on Seed and Cramer plates at the Yerkes Observatory that the effect of local variations in sensitiveness of film would in general be less than $0^{M}.04$, and under most unfavorable circumstances might rise to $0^{M}.1$.

[1] *Astrophysical Journal*, 32, 376–378, 1910.

[2] *Astrophysical Journal*, 31, 20, 1910.

This error could doubtless be much reduced by employing plates especially coated on plate glass. The concavity of ordinary commercial plates introduces another source of error, and has been investigated by Parkhurst, who estimates the probable effect for focal measures to be of the order of $0^{M}.01$ or $0^{M}.02$.

Investigations by Hartmann[1] reveal somewhat larger sources of error than those found by Parkhurst.

In a further study it is purposed by means of the thermopile to investigate extensively a large number of plates of both domestic and foreign manufacture on both commercial and plate glass to determine, if possible, the maximum degree of precision which may be relied upon in the use of the plates now upon the market. When one considers the extreme sensitiveness of the photographic plate as a detector of radiation, and the possibility of obtaining measurable images of faint stars hopelessly beyond the range of any other known "radiometer," the importance of perfecting such a medium for work of precision cannot be overemphasized. If the co-operation of observatories and other scientific institutions using large numbers of plates could be secured, it would doubtless be possible through special manufacture to secure plates much superior to any of the commercial brands now upon the market.

METHOD OF EXTRA-FOCAL MEASUREMENT

The adaptation of the thermo-electric photometer to the direct measurement of plate-opacities is at once apparent. For such work, since the area of the region to be measured is presumably more than sufficient to cover the diaphragm, a large aperture may be used and the galvanometer-resistance reduced for greater speed of manipulation. For such work galvanometer δ's are taken as directly proportional to plate-opacities. If extra-focal stellar images are being measured, correction for a lack of uniformity in the absorption of the glass can be made by taking readings on the free glass in close proximity to the image, as is done in the measurement of focal plates, already described. Here, as in the other case,

[1] "Ueber die Konstanz der Empfindlichkeit innerhalb einer photographischen Platte," *Eders Jahrbuch für Photographie*, 1906.

precision of results rests, first, on the assumption of uniform sensitiveness of the photographic film, and secondly, on the assumption that the absorption of the glass for the region immediately surrounding the stellar image is the same for the region beneath the image under measurement.

The advantage which the thermopile has over the "microphotometer" for extra-focal work is the elimination of the personal equation by the shifting of responsibility for correct judgment of opacities from the eye to a purely mechanical device. The order of accuracy of setting is about the same for the two methods, though, of course, the results of the Hartmann method are influenced by the skill of the observer.

To illustrate the degree of blackening due to various exposures, a plate containing a series of "standard squares," produced by simultaneous exposures to light-sources of known ratio of intensity, was measured with the thermo-electric photometer. The graph in Fig. 11 shows the relative blackening as measured by the galvanometer-deflections platted against the relative intensities expressed in magnitude-differences. Since the exposures were all made simultaneously, the validity of the reciprocity law $E = It^p$ does not enter, the values of E being to each other as the intensities of the source. It should be stated that the plates used for this test were selected from among those used by Parkhurst for another purpose and described by him in an earlier publication.[1]

Readings were taken upon the clear background of the plate adjacent to each image as usual, and subtracted from the galvanometer-readings for the image alone. Assuming that the galvanometer-deflections so corrected will be directly proportional to the transparency of the image, we should expect to find a nearly linear relation between the deflections so recorded and the logarithm of the "exposure" E, or a direct proportion between the quantity E as expressed in magnitudes. Data concerning the several apertures admitting the light to the several images have already been published (*loc. cit.*), together with the relative magnitudes. It will be observed that an approximate relation holds over a range of about one magnitude only.

[1] *Astrophysical Journal*, **26**, 244, 1907.

An extra-focal plate (blue-sensitive) of the Pleiades taken with the Zeiss camera was then measured with the thermo-electric photometer. The graph obtained from platting magnitudes against the logarithm of the galvanometer-deflections is shown in Fig. 12. The bending of the curve for the brighter stars due to overexposure

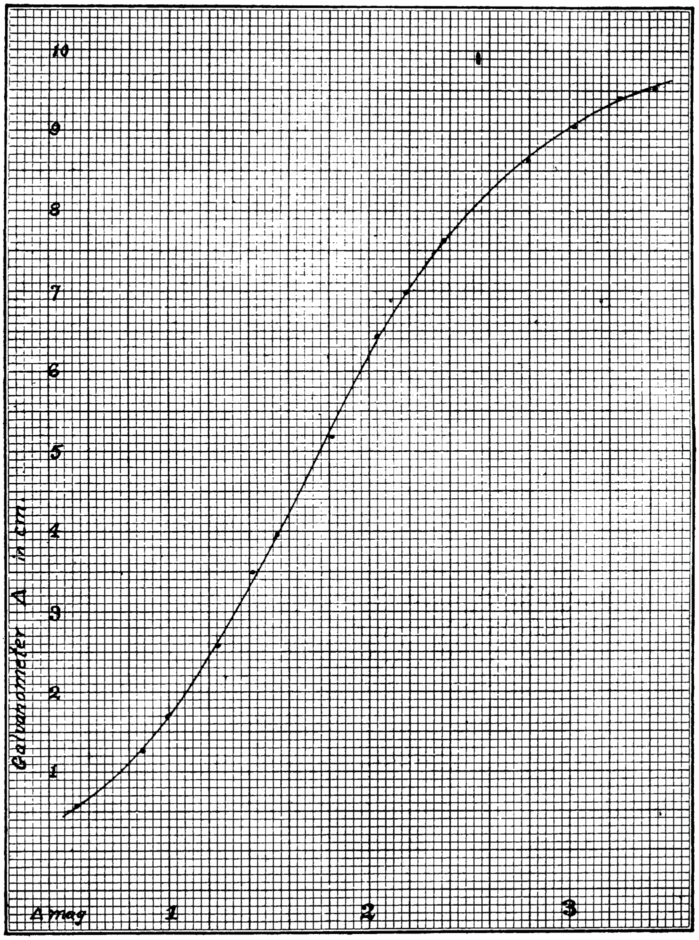

FIG. 11.—Curve showing plate opacities in terms of galvanometer deflections

indicates the comparatively narrow limits within which the law of blackening may be applied. In extra-focal work several exposures properly timed will ordinarily be needed to cover the desired range for a given field. The same limitation applies of course equally well to measurements of extra-focal images by any other method.

SPECTRO-PHOTOMETRY

The idea of measuring relative intensities in spectral plates by means similar to the foregoing is not new. The extended investigations of Koch[1] resulting in the so-called "self-registering" microphotometer have proved not only the feasibility of the method but

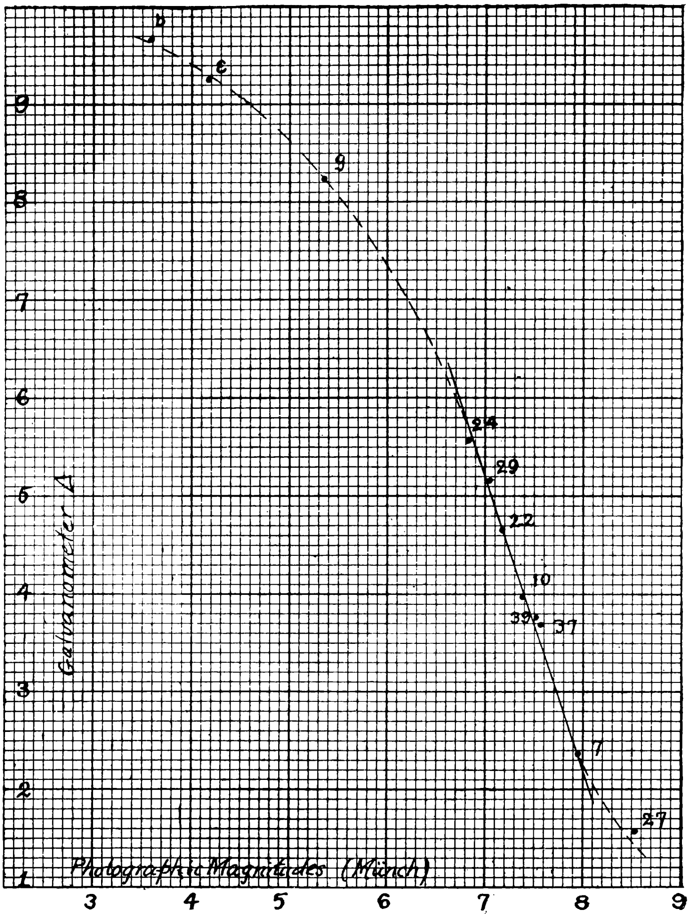

FIG. 12.—Extra-focal measures of magnitudes of stars of the Pleiades (the designation is that of Bessel).

the valuable results which may be obtained by such methods of measurement, especially in the study of laboratory spectra.[2] The apparatus under present consideration, when provided with a

[1] *Annalen der Physik*, **39**, 705, 1912.

[2] *Astrophysical Journal*, **39**, 213, 1914.

micrometer-screw stage, becomes at once adapted to such measurements. When so used, the small circular diaphragm is replaced by a small adjustable slit, which all but touches the under and film-side of the spectral plate to be measured. Arrangement is then made whereby a sliding platform moved by an accurately cut micrometer-screw may be attached to the main stage of the instrument. The spectral plate is held to the latter by clips in the usual manner, with the film down and in close contact with the slit-diaphragm. The spectrum is then exposed by setting the micrometer-screw for a given reading and noting the galvanometer-deflection corresponding to the opacity of the negative, then advancing the plate a known fraction of a millimeter, and recording the corresponding galvanometer-reading against the screw-setting, continuing the process till the region in question has been covered. This arrangement, though lacking the elegance of the "registering micro-photometer," has the advantage of simplicity, the thermopile replacing as detector the somewhat more troublesome photo-electric cell as used in the more elaborate apparatus of Koch.

In the case of objective-prism photographs, when the linear dimensions of the stellar spectra are not great, the task of mapping the intensity-curve is not difficult and may be accomplished in a comparatively short time. After the spectrogram has been properly lined up on the stage no further attention is necessary other than noting the reading of the galvanometer after each turn of the screw. Results are shown for measurements of an objective-prism plate of the Pleiades taken with the Zeiss ultra-violet doublet previously mentioned. The graphs in Fig. 13 show the intensity-curves for the photographic spectra of the three stars Alcyone, Taygeta, and Merope. Screw-readings are platted as abscissae against the direct readings of the galvanometer as ordinates, the plate being advanced 0.01 mm for each setting. The plate for which the curves are drawn is an enlarged positive, the hydrogen lines appearing dark and accordingly represented in the graph by depressions corresponding to the lower values of the galvanometer-reading. The slit is purposely adjusted somewhat wider than the absorption lines in order to minimize their effect and give a somewhat smoother form for the entire curve of distribution of energy. No shift in the point of maximum radiation is apparent, but the center of gravity of the area formed by the curve and its horizontal base-line is appreciably nearer the violet end for Alcyone than is the case for the other two stars. This would seem to indicate that the bright star is relatively somewhat "bluer" than the other two in question. Careful investigations of spectral types in this manner may afford a more refined

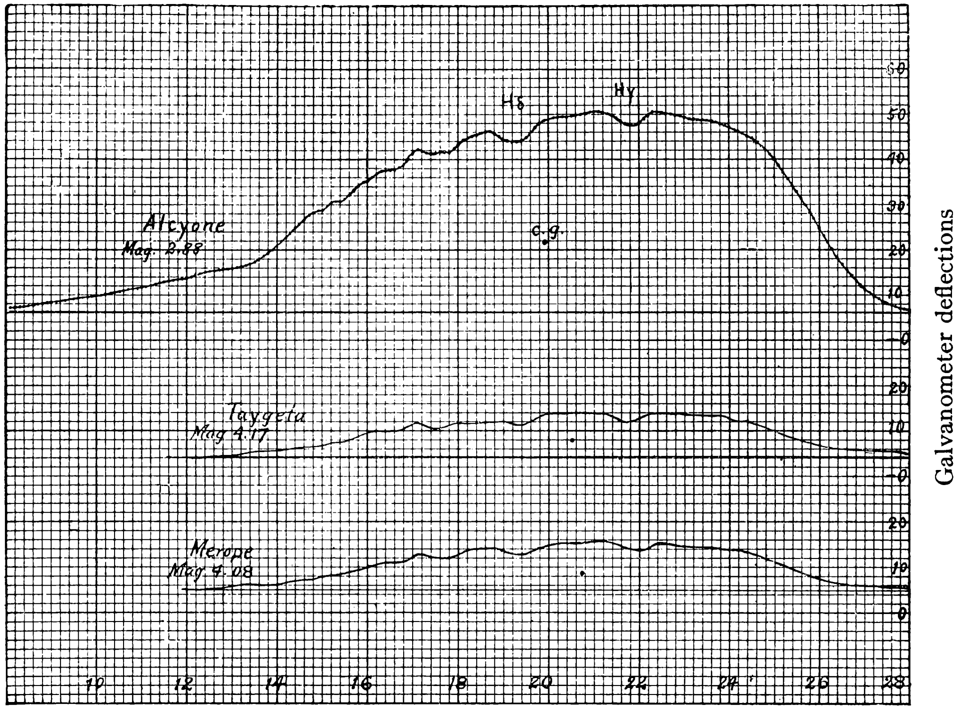

FIG. 13.—Spectral intensities of three stars of the Pleiades

method for establishing color-indices and make possible important quantitative measurements in spectro-photometry.

By narrowing the slit in the case of the arrangement described above, it becomes possible to determine with considerable degree of accuracy the mean position of broad and diffuse lines, which are so difficult of measurement by the usual methods. Further, by a special modification of the design of the thermopile it should become possible to measure with considerable rapidity the location of a given line, if its spectral intensity is not a question of importance. Such a modification would form an important accessory to the instrument and would prove of special value in the determination of the radial velocities of stars of early type.

A continuation of this subject of thermo-electric methods of measurements, with an application to variable stars, will appear in a subsequent number of this *Journal.*

YERKES OBSERVATORY
March 1916

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XLIII JUNE 1916 NUMBER 5

ON AN APPARATUS AND METHOD FOR THERMO-ELECTRIC MEASUREMENTS IN PHOTOGRAPHIC PHOTOMETRY. II

APPLICATION TO VARIABLE STARS

By HARLAN TRUE STETSON

In the application of the thermo-electric method of measurement to the study of the light-curves of variable stars we are concerned chiefly with relative measurements, and difficulties incident to the determination of the "absolute scale" are of less importance in this field. Since in a given series of measurements the comparison stars and variable will be compared image for image, the uncertain correction to plate center will be largely eliminated. Again, we should expect the larger errors due to progressive change in atmospheric transparency to be minimized, since presumably variable and comparison stars would be similarly affected image for image.

The eclipsing variable U Cephei was selected for measurement by the new method for several reasons: the general character of the light-curve is well known, elements have been determined, and the period is short enough to enable a large portion of the curve to be obtained from a single plate.

Variable-star observers will recall that the light-curve of U Cephei has been determined by numerous investigators with somewhat conflicting results, indicating the possible existence of peculiarities yet to be determined. As early as 1884 observations by Wilsing[1] and also by Knott[2] indicated that the minima observed in the spring of the year appeared lower than those deduced from autumnal observations by two or three tenths of a magnitude. The period of the star being nearly 2½ days (more exactly $2^{d}.493$), it is obvious that in lower latitudes two successive minima may never be seen from the same station. A given observer must content himself with observing *alternate* minima. Should we regard the period as double the foregoing value, the autumnal minima would then occupy parts of the light-curve opposite in phase to those corresponding to the minima observable in the spring of the year. In 1889 Chandler[3] published a light-curve showing the same peculiarity, indicating also a more rapid rate of increase than of decrease, corroborating the results of earlier observations by Schmidt. In Chandler's curve the lowest point is reached first, and a short inflection follows near the median of the curve. The reality of the seasonal variation, as well as that of the minor fluctuations, is regarded by Chandler as open to question. Bohlin,[4] working at Upsala from observations made in 1896, secured no foundation for any seasonal discrepancies, but obtained a light-curve with a slight elevation of about one-tenth of a magnitude at mid-minimum.

From an extended study of a large number of observations, Yendell[5] in 1903 came to the conclusion that all such irregularities were illusory and attributable to physiological causes, the seasonal variation depending upon the orientation of the observing field. He accordingly believed the light-curve to be symmetrical, with a constant minimum of two hours' duration due to the occurrence of an annular eclipse. That Yendell's statement, however, may not be regarded as final is evidenced by the fact that Bemporad,[6] of the Osservatorio di Capodimonte, in 1914 published light-curves for both the June and December minima indicating a difference of

[1] *Astronomische Nachrichten*, **109**, 48, 1884. [2] *Ibid.*, p. 59.

[3] *Astronomical Journal*, **9**, 49, 1889.

[4] *Astronomische Nachrichten*, **157**, 293, 1902.

[5] *Astronomical Journal*, **23**, 213, 1903.

[6] *Astronomische Nachrichten*, **199**, 217, 1914.

$0^{M}.25$ in the depth of the curve. The June curve indicates also a secondary fluctuation during the minimum phase, and the December curve suggests some similar peculiarity.

Still more recently Shapley,[1] in the discussion of a light-curve of U Cephei based upon Wendell's extensive observations, recognizes a non-symmetrical tendency of the primary minimum and attributes the downward slant of the bottom of the light-curve to the irregular luminosity of the dark companion. He adds also that the close proximity of the components suggests a prolateness which might well show its effect upon the light-curve, although Wendell's observations at maximum gave no definite indications of such a feature.

With such discussions in mind, any new method of approach to the problem becomes of interest, especially if it affords opportunity for measurements free from subjective errors. If any secondary fluctuation or other departures from the main curve do exist, it would seem that these could best be detected by a careful study of a single series of observations, since the very attempt to combine observations of different epochs and reducing to a mean curve would largely if not wholly mask any disturbance not commensurable with the principal period of the star. The advantage of photography in such a study is at once apparent, since it not only eliminates physiological difficulties, but makes possible a much larger number of independent observations during a given minimum than would be obtainable otherwise. With the thermoelectric photometer as a purely physical means for the measurement of the star-images, it would seem that any contributions through this means would be of special interest as a source of additional information of the behavior of the light-curve of this star.

Plates taken for measurement by the method of diameters were found in the collection of the Yerkes Observatory, and a light-curve was published by Parkhurst in 1906.[2] Plate R 71, taken on June 25, 1905, with the 2-foot reflector, was used as the basis of the results then published. The same plate was the one first selected for measurement by the present method. Comparison

[1] *Contributions from the Princeton University Observatory*, No. 3, p. 35, 1915.

[2] *Astrophysical Journal*, **23**, 79, 1906.

stars *f* (B.D. 81°30) and *g* (B.D. 81°27) were used. The exposures were made at approximately ten-minute intervals, the plate-carrier being moved by two revolutions of the screw after each exposure. Owing to the close proximity of the two series of images on the original plate, a second and somewhat enlarged negative was made from the original and used with the thermopile.

The use of enlarged second negatives in this connection has the advantage not only of separating images in close proximity, but of increasing the contrast between star and plate ground, which may be reduced to clear glass if desired. This in itself greatly increases the precision with which thermopile readings may be made, since it affords much larger values of the galvanometer deflections and, with a fine-grained film, makes possible a more uniform background. On the other hand, it is doubtful if the enlarging process does not amplify errors in the original plate as well as introduce new ones in each reproduction.

TABLE II

PLATE R 71. JUNE 25, 1905. U CEPHEI

Comparison Star B.D. 81°30, Magnitude 8.04
Comparison Star B.D. 81°27, Magnitude 8.73 $\beta=2.80$

Image	Mag.	Time	Image	Mag.	Time
		G.M.T.			G.M.T.
1..........	7.43	$15^h 12^m 0\pm$	11.........	9.15	$18^h 50^m 0\pm$
2..........	7.88	43.4	12.........	9.12	19 19.6
3..........	7.98	16 00.9	13.........	8.90	45.6
4..........	8.07	20.5	14.........	8.55	20 04.4
5..........	8.27	37.6	15.........	8.53	16.3
6..........	8.72	59.8	16.........	8.23	28.2
7..........	9.14	17 20.8	17.........	8.33	36.0
8..........	9.30	39.7	18.........	8.21	44.0
9..........	9.20	18 02.7	19.........	8.07	50.0
10..........	9.07	19.2			

However, it is interesting to note that the resulting light-curve (Fig. 14) bears a striking similarity to that of Chandler to which reference has already been made. The times and magnitudes are given in Table II. The magnitudes of the comparison stars were taken as previously published, all points on the variable curve being referred to B.D. 81°30, of adopted magnitude 8.04.

Measurements were made on a number of other plates taken under widely differing circumstances, and indicated large residuals from the theoretical curve at minimum which could hardly be accounted for on grounds of errors in measurement. Repeated measuring gave the same results for a given plate with a surprising degree of accuracy.

Through the courtesy of Professor E. C. Pickering of the Harvard College Observatory a number of plates, taken with the

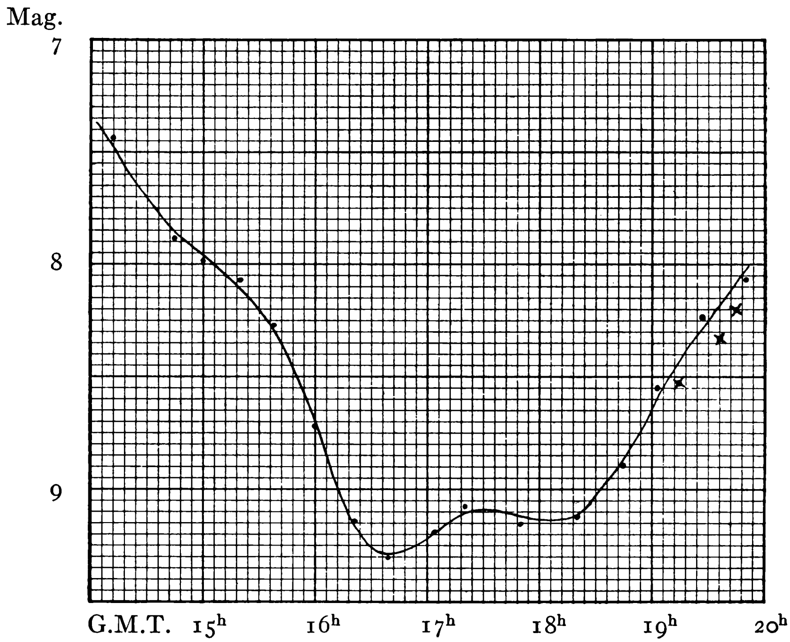

FIG. 14.—U Cephei, minimum of June 25, 1905. Plate R 71
x = points of half weight based on one comparison star

"variable-star apparatus" attached to the 11-inch Draper telescope, were secured. By means of this attachment the variable was automatically photographed with a minute's exposure at short intervals, the plate being displaced mechanically a fraction of a millimeter after each exposure. In this manner a long series of images may be impressed upon a 5×8-inch (12½×20 cm) plate, covering a considerable portion of the minimum phase of the eclipse. The short exposure gave very small gray images of the variable when at minimum ($9^{M}.2$) which were difficult to measure. The photometer, however, was modified to meet the requirements, a more efficient projection lens was substituted, a finding microscope was added above the stage, the galvanometer was removed to 2 m from the scale and read by a telescope magnifying 20 diameters. These improvements resulted in increasing the sensitiveness of the whole apparatus about five times and gave very good and consistent results.

Plates C 11686 and C 11687 were taken during the minimum of December 29, 1898. From among the images available, those best suited for measurement were selected. Care was taken to compare the measurement of each star-image with the film background in its own immediate vicinity, this precaution being observed to eliminate as far as possible any errors that might otherwise be introduced because of a changing transparency in the somewhat sky-fogged film. For the measurement of a single image not less than six settings were involved, the order of procedure being to set first on the image and then on the plate, alternating till the set of three pairs was secured. If the residuals appeared large for certain images, additional readings were sometimes taken in the order noted. The plate readings served also as a check on the voltage at the lamp terminals, and at the same time, when taken as indicated, eliminated any error introduced by the gradual decline, with consumption of current, of the potential of the storage battery.

As space would not permit the publication of the individual measures, a summary of the resulting data is given in Table III. The first column gives the number of the image on the plate, the second and third columns give the resulting $\Delta^{\frac{1}{2}}$ for the variable and comparison star, respectively, and the fifth column contains the differences of the second and third columns multiplied by the plate constant β, and therefore represents the differences of magnitude between the variable and comparison star. The value of β was determined from measures of stars B.D. 81°30 and B.D. 81°27, as was the case in the reduction of R 71. Columns 5 and 6 give the co-ordinates for plotting the light-curve, assuming the magnitude of B.D. 81°30 as 8.04 as before.

The graph resulting from these measures is shown in Fig. 15, and indicates a secondary fluctuation of about one-tenth of a magnitude, with an interval of 40 minutes between these apparent undulations. It is hard to account for this appearance on the grounds of systematic errors in measurement. The variable and comparison star were measured independently, and on different occasions, and it appears hardly likely that this curve is the delineation of accidental errors. Such a result might seem attributable to peculiarities in the plate or to atmospheric disturbances. Unfortunately, the overlapping of several rows of images on the plates restricted the use of suitable images other than those of comparison star B.D. 81°30.

TABLE III

PLATES C 11686 AND C 11687. DATE OF PLATES, DECEMBER 29, 1898

Comparison Star B.D. 81°30

Plate	Image	$\Delta^{\frac{1}{4}}$ U	$\Delta^{\frac{1}{4}}$ *f*	Diff.	Δ Mag.	Mag.	G.M.T.
C 11686	1	0.750	1.054	0.304	1.220	9.260	10h59m23s.9
	11	0.790	1.144	.354	1.436	9.476	11 9 23.9
	21	0.830	1.191	.361	1.465	9.505	19 23.9
	31	0.790	1.122	.332	1.345	9.385	29 23.9
	41	0.781	1.005	.324	1.315	9.355	39 23.9
	52	0.790	1.136	.346	1.404	9.444	50 23.9
	62	0.801	1.162	.361	1.465	9.505	12 0 23.9
	72	0.845	1.174	.329	1.338	9.378	10 23.9
	83	0.837	1.163	.326	1.322	9.362	21 23.9
	92	0.815	1.150	.335	1.360	9.400	30 23.9
	102	0.740	1.122	.382	1.551	9.591	40 23.9
	112	0.781	1.140	.359	1.457	9.497	50 23.9
	122	0.760	1.118	.358	1.453	9.493	13 00 23.9
C 11687	2	0.833	1.116	.283	1.145	9.188	9 23.8
	12	0.919	1.166	.247	1.003	9.043	19 23.8
	22	0.946	1.170	.224	0.909	8.949	29 23.8
	32	0.906	1.034	.128	0.520	8.560	39 23.8
	42	0.995	1.077	.082	0.333	8.373	49 23.8
	52	1.070	1.122	.052	0.211	8.251	59 23.8
	62	1.056	1.068	.012	0.049	8.089	14 9 23.8
	72	1.105	1.097	− .008	−0.032	8.008	19 23.8
	84	1.132	1.083	− .049	−0.199	7.841	21 23.8
	93	1.144	1.090	− .054	−0.219	7.721	30 23.8
	101	1.164	1.118	− .046	−0.187	7.853	48 23.8
	112	1.125	1.105	−0.020	−0.081	7.959	59 23.8

A set of three other plates was selected, Nos. C 9188, 9189, 9190, covering a portion of the decline, minimum, and rise of U Cephei under the date of June 13, 1896. The quality of the plates was better than that of the previous pair, and it was possible to measure images at more frequent intervals. Two good comparison stars, B.D. 81°30 and 81°29, were used throughout.

The results of the measures are given in Table IV. Columns 1 and 2 give the number of the image on the plate, and the corresponding Greenwich mean time of the exposure. Columns 3 and 4 give the resulting magnitude of the variable as determined from

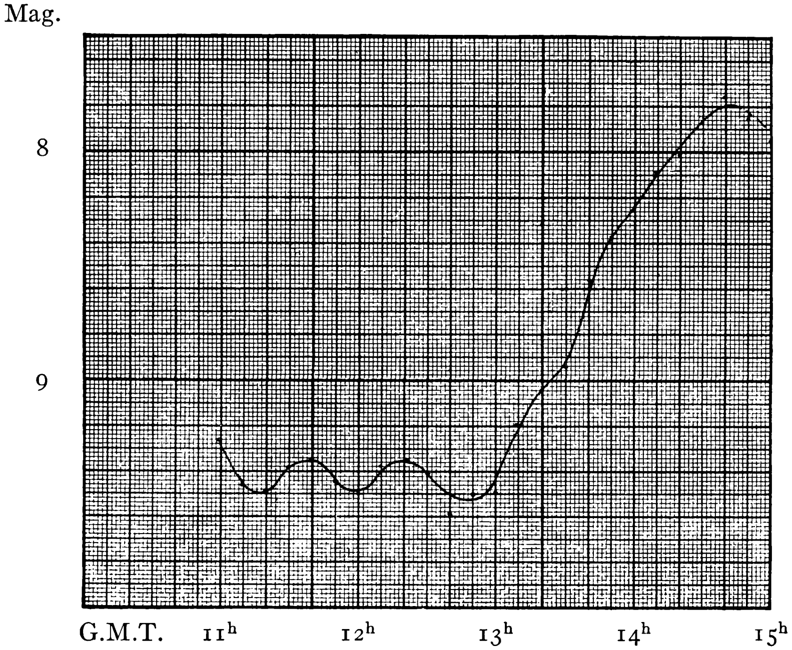

FIG. 15.—U Cephei, minimum of December 29, 1898. Plates C 11686 and C 11687.

the two comparison stars *f* and *h* respectively. The fifth column gives the direct difference of column 3 minus column 4 and forms the basis for suspected variability between *f* and *h*. The differences of magnitude have all been referred to B.D. 81°30 as before. The magnitudes of B.D. 81°30 and B.D. 81°27 were adopted as for Plate R 71, and the magnitude of B.D. 81°29 was determined from the measures of the plates.

The curve platted from these measures and represented in Fig. 16 was anything but what was expected, and indicated sources of difficulty not anticipated. Determinations of probable error pointed to variations of hundredths, whereas the curve advertised discrepancies in the tenths' place. Remeasures of offending points gave almost identical results with but slight variations in the last place, indicating that the source of trouble was not in the apparatus

TABLE IV

U CEPHEI, MINIMUM FOR JUNE 13, 1896. J.D. 2413693

Comparison Stars Used: *f*, B.D. 81°30; *g*, B.D. 81°29; *h*, B.D. 81°27
Adopted Magnitudes: *f*, 8.04; *g*, 8.73; *h*, 8.79

SUMMARY PLATE 9188

Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*–Mag. *h*	Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*.–Mag. *h*.
2.....	$14^h26^m29^s$	7.666	7.658	0.008	60.....	$15^h24^m29^s$	8.150	8.216	−0.066
4.....	28 29	7.704	7.678	.026	62.....	26 29	8.112	8.048	.064
8.....	32 29	7.670	7.764	.096	65.....	29 29	8.170	8.332	− .162
10.....	34 29	7.718	7.768	− .050	68.....	32 29	8.284	8.166	.118
12.....	36 29	7.766	7.708	.058	70.....	34 29	8.310	8.344	− .034
15.....	39 29	7.790	7.802	− .012	72.....	36 29	8.272	8.258	.014
18.....	42 29	7.826	7.752	.074	75.....	39 29	8.286	8.418	− .132
20.....	44 29	7.852	7.862	− .010	78.....	42 29	8.430	8.488	− .058
22.....	46 29	7.896	7.932	− .036	80.....	44 29	8.468	8.530	− .062
25.....	49 29	7.874	7.850	.024	82.....	46 29	8.348	8.518	− .170
28.....	52 29	7.862	7.894	− .032	85.....	49 29	8.474	8.762	− .288
30.....	54 29	7.800	7.824	− .024	88.....	52 29	8.392	8.356	.036
32.....	56 29	7.956	7.924	.032	90.....	54 29	8.474	8.374	.100
35.....	59 29	7.898	8.004	.094	92.....	56 29	8.678	8.680	− .002
38.....	15 02 29	7.970	7.902	.068	95.....	59 29	8.494	8.540	− .046
40.....	04 29	8.138	7.802	.336	98.....	16 02 29	8.688	8.800	− .112
42.....	06 29	8.060	8.032	.028	100.....	04 29	8.560	8.634	− .074
45.....	09 29	8.130	7.988	.142	102.....	06 29	8.788	8.884	− .096
48.....	12 29	8.064	8.036	.028	105.....	09 29	8.824	8.868	− .044
50.....	14 29	8.086	7.970	.116	107.....	11 29	8.968	9.014	− .046
52.....	16 29	8.064	7.918	.146	109.....	13 29	8.818	9.052	−0.234
55.....	19 29	8.064	8.040	0.024					

TABLE IV—*Continued*

SUMMARY PLATE 9189

Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*–Mag. *h*	Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*.–Mag. *h*.
5.....	$16^h32^m29^s$	9.374	9.482	−0.108	58.....	$17^h25^m29^s$	9.916	9.876	.040
7.....	34 29	9.408	9.464	− .056	61.....	28 29	9.658	9.662	− .004
10.....	37 29	9.296	9.416	− .120	65.....	32 29	9.580	9.506	.074
12.....	39 29	9.282	9.464	− .182	68.....	35 29	9.772	9.766	.006
15.....	42 29	....	....	....	71.....	38 29	9.764	9.768	−0.004
17.....	44 29	....	....	....	75.....	42 29	9.484	9.418	0.066
19.....	46 29	9.312	9.406	− .094	78.....	45 29	9.538	9.448	.090
23.....	50 29	9.388	9.400	− .012	81.....	48 29	9.618	9.508	.110
25.....	52 29	9.406	9.342	.064	85.....	52 29	9.488	9.348	.140
28.....	55 29	9.420	9.476	− .056	88.....	55 29	9.466	9.348	.128
31.....	58 29	....	....	....	91.....	58 29	9.710	9.530	.180
35.....	17 02 29	9.402	9.538	− .136	95.....	18 02 29	9.630	9.428	.202
41.....	08 29	9.532	9.476	.056	98.....	05 29	9.550	9.458	.092
45.....	12 29	9.344	9.346	− .002	100.....	07 29	....	....	....
48.....	15 29	9.264	9.262	.002	105.....	12 29	9.460	9.388	.072
51.....	18 29	9.320	9.418	− .098	107.....	14 29	9.554	9.438	0.126
55.....	22 29	9.588	9.748	− .160					

TABLE IV—*Concluded*

SUMMARY PLATE 9190

Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*.–Mag. *h*.	Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*.–Mag. *h*.
4.....	$18^h28^m29^s$	9.776	9.826	−0.050	58.....	$19^h22^m29^s$	8.582	8.704	−0.122
6.....	30 29	9.482	9.426	+ .056	61.....	25 29	8.416	8.446	−0.030
8.....	32 29	9.506	9.522	− .016	63.....	27 29	8.438	8.428	+ .010
11.....	35 29	9.338	9.440	− .102	66.....	29 29	8.432	8.400	+ .032
13.....	37 29	9.388	9.444	− .056	68.....	32 29	8.538	8.536	+ .002
15.....	39 29	9.404	9.356	+ .048	71.....	35 29	8.350	8.370	− .020
18.....	42 29	9.254	9.380	− .126	74.....	38 29	....	....	....
21.....	45 29	9.290	9.338	− .048	77.....	41 29	8.328	8.228	+ .100
23.....	47 29	9.528	9.556	− .028	81.....	45 29	8.302	8.142	+ .160
25.....	49 29	9.330	9.274	+ .056	83.....	47 29	8.264	8.216	+ .048
28.....	52 29	9.222	9.322	− .100	85.....	49 29	8.174	8.036	+ .138
31.....	55 29	9.122	9.160	− .038	88.....	52 29	8.140	8.058	+ .082
33.....	57 29	9.182	9.318	− .136	91.....	55 29	8.138	8.052	+ .086
35.....	59 29	8.778	8.868	− .090	93.....	57 29	8.196	8.084	+ .112
38.....	19 02 29	8.918	9.010	− .092	95.....	59 29	8.094	8.022	+ .072
41.....	05 29	8.742	8.770	− .028	98.....	20 02 29	8.052	7.988	+ .064
43.....	07 29	9.070	9.136	− .066	101.....	05 29	8.090	8.038	+ .052
45.....	09 29	....	8.848	....	103.....	07 29	7.984	8.040	− .056
48.....	12 29	8.712	8.856	− .144	105.....	09 29	8.038	7.828	+ .210
51.....	15 29	8.608	8.690	− .082	108.....	12 29	7.932	7.884	+ .048
53.....	17 29	8.764	8.724	+ .040	110.....	14 29	7.920	7.824	+0.096
55.....	19 29	8.876	8.748	+ .128					

nor in the method of measurement. The discrepancies were therefore photographic, to be interpreted as irregularities in the exposure due to rapid changes of atmospheric transparency, as an uneven condition of sensitiveness in the plate film, or, as seemed hardly likely, due to actual changes of light among the stars in question.

The singular feature is that for the most part the general character of the light-curve is the same for either comparison star, indicating that the chief cause of the discrepancy is in the images of U Cephei. If the large irregularities are due to atmospheric changes, it is difficult to see why the variable alone should be affected, while the two comparison stars fail to show a corresponding disturbance. The most outstanding difficulty is the behavior of the curve during minimum, around 17^h20^m, where, within the short space of half an hour, both comparison stars indicate the variable to experience a change of nearly half a magnitude in light. It has seemed wiser in this preliminary graph to leave the points connected with straight lines rather than to bias judgment by any attempt at drawing a mean curve until more complete data are at hand. The combination of several such curves at minimum would doubtless "smooth out" many irregularities, but the argument perhaps needs renewed emphasis that the combination of phenomena at separate minima only tends to mask any singularities not a definite function of the 2.49 days.

Another fact at once apparent from Fig. 16 is that the curves drawn from the data furnished by the two comparison stars cross at stated intervals. The curve belonging to B.D. 81°30, for example, clearly lies above that drawn for B.D. 81°29, from 16^h to 17^h, whereas from 17^h30^m to 18^h30^m the reverse is true. A

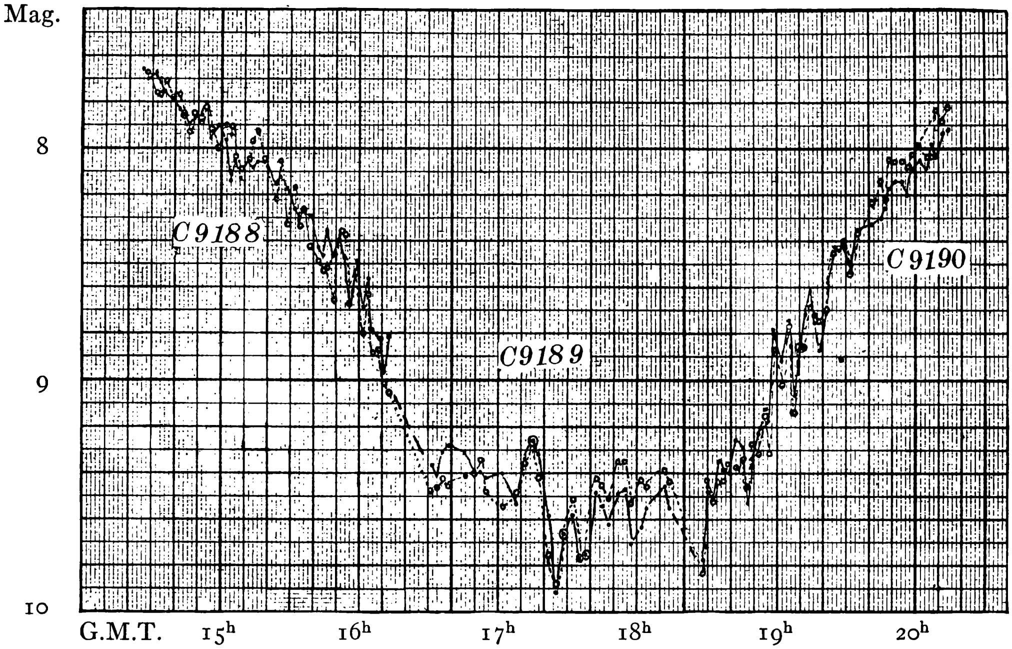

FIG. 16.—U Cephei, minimum of June 13, 1896
● = comparison star *f*; ○ = comparison star *h*

third crossing of the two curves occurs at about 19^h30^m. This behavior suggests a variability in one or the other of the comparison stars, and becomes more apparent when we examine the two comparison stars platted against each other as in Fig. 17. Here, in spite of the large residuals from a mean curve, the rise and fall of the curve at an approximate interval of an hour and three-quarters is at once evident. Sufficient measures of a third star, B.D. 81°26, were made to indicate that the eighth-magnitude star B.D. 81°30 was the cause of the discrepancy. We must therefore either try to account for the rise and fall in Fig. 17 on the ground of a progressive undulating change in atmospheric transparency continuing throughout the entire exposure, but not appreciably affecting the adjacent star, or else suspect the comparison star B.D. 81°30 of variability. If this should be the case, it might well be a contributing factor in the cause of the various forms of the light-curve of U Cephei previously published, when this star has been the principal comparison star used.[1]

Whatever may be gleaned from these preliminary results, it is hard to see how a straight flat-bottom curve, as demanded on the grounds of an eclipse theory alone, can be reconciled with the observations. If the binary system be held to account for the main period

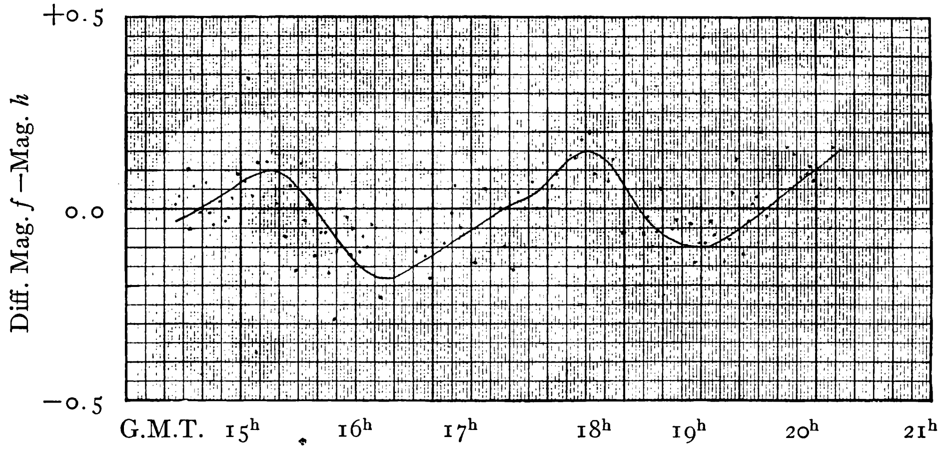

FIG. 17—Curve showing suspected variation in B.D. 81°30

of the star,[2] it may seem necessary to suppose other vital factors operative in the light-curve of the star. In any event, enough has been exhibited already to indicate that with every determination complexities arise which forbid an easy verification of anything more than a mean light-curve of general characteristics. With the continuance of the method of investigation here outlined it is hoped that the completion of further measures in all parts of the light-curve may at length afford sufficient data for the formation of more conclusive ideas regarding the behavior of the light-curve of this most interesting object.

[1] Compare with the results of Capodimonte, to which reference has already been made (*Astronomische Nachrichten*, **199**, 217, 1914).

[2] For a discussion of irregularities in the main period, see Yendell's article (*loc. cit.*). Further investigation of this point is being undertaken by Shapley.

In order to determine the order of precision which could be expected in the measurements of the Harvard plates, the probable error was computed from a number of measures of the same star-image, of U Cephei near minimum, and found to be $\pm 0^{M}.009$ for twenty settings or corresponding to about $\pm 0^{M}.025$ for a set of readings as ordinarily taken for each star. Results from measuring diameters with a micrometer microscope and reducing to magnitudes by means of the square-root formula yielded a probable error nearly five times as great. This large value resulted from the fact that, the plate being unguided, the images were somewhat enlarged, and the variable was so faint at the minimum that the images then taken were still gray and showed no well-defined periphery. The use of the thermopile promises well for the measurement of star-images not suitable for reduction by the diameter method. With longer exposures on the variable at minimum and special care in the development of the plate it should be possible to reduce the probable error to a still lower figure. Experiments with the ferrous-oxalate developer show a great advantage to be gained by its use for photometric purposes, especially for plates to be measured thermo-electrically, inasmuch as good contrast results without the slightest possibility of chemically fogging the unexposed portion of the film.

Sources of error in the film itself are without doubt a cause of many discrepancies, and further refinement must be looked for in adopting a *modus operandi* which will reduce the effect of such residual errors to a minimum.

CONCLUSION

In the development of the present method of investigation, the first consideration has been to adapt a piece of apparatus to the measurement of stellar magnitudes from photographic plates which should eliminate so far as possible some of the difficulties incident to other methods. In so doing care has been taken to consider the errors involved and to reduce them to a minimum. The degree of precision attainable by means of the thermo-electric measurements has been tested under widely different circumstances and enough numerical results tabulated to indicate what addition a contributions are to be expected from the use of such an instrument in stellar photometry.

The several lines of special investigation suggested by the results of these preliminary determinations indicate a fruitful field for extended study in many directions. It has seemed wise, therefore, in the present treatment to outline in considerable detail the essential steps in the development of the instrument and methods of reduction, with a preliminary application to several different problems, rather than to devote an early investigation in a new field to a more exhaustive treatment of a single topic.

The precise determination of magnitudes by means of the thermo-electric method of measurement has already been discussed. The difficulties mentioned are not new to the present method; rather, they are those which have long occupied the attention of investigators in stellar photometry. It is to be hoped that the measurement of plates in the manner herein described, by reducing another element of uncertainty, may facilitate more rapid progress in the field of photometry.

As has been shown, the thermopile may be used for the direct measurement of plate opacities, covering the same field as that of the Hartmann microphotometer with about the same degree of accuracy. The points of difference between the two instruments when so used may again be mentioned. The reductions of the Hartmann readings depend upon the calibration of the photographic wedge used in comparison, whereas with the thermopile arrangement the deflections of the galvanometer may be taken at once as proportional to opacities on the plate. The thermopile further has the advantages of restricting a much smaller area for examination than the microphotometer, and of making the comparison entirely independent of physiological effects or personal equation.

In measuring opacities of extended areas such as are found on solar, lunar, or nebular photographs, these advantages may be in part offset by the uncertainty in the assumption of constant absorption of glass and gelatine throughout the plate. Such a source of error will not prove troublesome whenever adjacent settings on the comparatively uniform unexposed portion of the film can be made.

The ease with which the apparatus may be adapted to the service of spectrophotometry in the measurement of objective-prism plates affords opportunity for an important study of the relation of color-index to spectral type, and may afford further means for a more precise definition of "magnitude."

In a similar manner the application of such spectral measures to the determination of the radial velocities of faint stars photographed with the objective prism is a promising field for investigation. In such a case it should be possible to locate with considerable accuracy the position of the absorption band of neodymium used as a comparison spectrum. The method becomes equally applicable to line-of-sight measures for stars having broad-line spectra.

A line of work most promising for the thermo-electric photometer is that of the photographic study of short-period variables. In the case of fields where close comparison stars are available and a long series of exposures are made on the same plate, the large and uncertain errors of atmospheric transparency, plate constants, and reduction to a standard scale are reduced to a minimum and the accuracy of the measures appears to be limited only by the quality and uniformity of the plate. Combining the advantages of photography with refinement of measurement, the thermopile so used may yet be able to accomplish for the fainter stars what the photo-electric cell and the thermopile in a more direct way are promising for the stars of brighter magnitudes.[1]

In conclusion, the writer wishes to express his appreciation to both astronomers and physicists who in any way have aided in the progress of this investigation. He especially wishes to acknowledge his indebtedness to Professor Fox, of the Dearborn Observatory, Northwestern University, for the provision and loan of important pieces of apparatus; to Professors Michelson and Millikan, of the University of Chicago, for so kindly placing at his disposal the resources of the Ryerson Physical Laboratory; to Professor E. C. Pickering, for the loan of valuable plates from the Harvard College Observatory, and for many important suggestions; to Professor Frost, for the use of the facilities of the Yerkes

[1] See W. W. Coblentz, "Radiometric Measurements of 110 Stars with the Crossley Reflector," *Lick Observatory Bulletin*, **8**, 104, 1914.

Observatory, and for his kindly interest in the progress of the work; and to Professor Parkhurst, for the use of certain plates taken by him, and for that constant appreciation and material encouragement which have done much to further the application of laboratory methods to the problems of astronomical photometry.

SUMMARY

The results of the foregoing paper may be briefly summarized as follows:

1. An instrument has been devised, using a delicate thermopile as a receiver, whereby the magnitudes of stars may be determined from either focal or extrafocal plates by purely physical means.

2. A reduction formula has been adopted which shows a fourth-root relation between galvanometer deflections and stellar magnitudes, and this relation has been checked by the measurements of magnitudes from plates taken under various conditions, and with four different telescopes ranging from six to forty inches in aperture.

3. Provision has been made for the adaptation of the apparatus to the measurement of spectral intensities, and the method illustrated in the measurement of objective-prism spectra of B-type stars for purposes of spectral photometry.

4. An extensive study of the eclipsing variable U Cephei has been begun, the preliminary measures here presented indicating the presence of light-changes not explained on the eclipse theory alone, and revealing a suspected variability in the much-used comparison star B.D. 81°30.

5. From the measurements of the variable-star plates a somewhat higher degree of accuracy is to be expected from thermoelectric measures than from measures of diameters, the thermopile promising distinct advantage in the case of images too ill-defined to be suitable for the micrometer microscope. The principal sources of error appear to be the changing atmospheric conditions during the exposure at the telescope and irregularities in the photographic plate.

YERKES OBSERVATORY
November 1915