The University of Chicago

TABLES FOR THE COMPUTATION OF THE JUPITER PERTURBATIONS OF THE GROUP OF SMALL PLANETS WHOSE MEAN DAILY MOTIONS ARE IN THE NEIGHBOURHOOD OF 750″

A DISSERTATION

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PREFACE.

The following tables, skillfully worked out by Professor D. T. WILSON of Cleveland, refer to the group of asteroids, whose mean motion is nearly $\frac{5}{2}$ of that of Jupiter, their mean motion being approximately:

$$n = 750''.$$

They are said to belong to the group $\frac{2}{5}$ of the asteroids or to the *Minerva*-type. The tables are fit to fill a space of asteroids for which the general perturbations hitherto had not been calculated.

Considering the commensurabilities of the mean motions of the small planets with regard to Jupiter, we will have the following table:

	corresponding to the commensurability
$n = 448.''7$	$2-3\,\mu$
478.6	$5-8\,\mu$
498.6	$3-5\,\mu$
523.5	$4-7\,\mu$
598.3	$1-2\,\mu$
698.0	$3-7\,\mu$
747.8	$2-5\,\mu$
797.7	$3-8\,\mu$
897.4	$1-3\,\mu$
1047.0	$2-7\,\mu$
1196.5	$1-4\,\mu$.

Of these ratios the following three are representative for the whole system of the asteroids:

		approximately
1:0	$\mu = \frac{1}{2}$	$n = 600''$
2:0	$\mu = \frac{2}{5}$	$n = 750''$
3:0	$\mu = \frac{1}{3}$	$n = 900''$,

the remaining commensurabilities either being very rare in the reality or corresponding to so high degrees of the eccentricity and of the inclination as to give remarkable types, only where the commensurabilities are very strong. Accordingly, the asteroids can in general be put in one of the three categories:

$\mu = \frac{1}{2}$ *Hecuba*-group;

$\mu = \frac{2}{5}$ *Minerva*-group;

$\mu = \frac{1}{3}$ *Hestia*-group.

The first[1] and third[2] of these groups were considered resp. by Dr. H. v. ZEIPEL and by myself and their general perturbations were carefully worked out and compared with other computations. Moreover the perturbations of Saturn[3] were calculated by Dr. H. G. BLOCK. Thus, after that the tables for the groups $\frac{2}{5}$ herewith are given by Professor D. T. WILSON[4] the computation of the general perturbations for any one of the asteroids may be worked out in a few hours in almost every case.

The tables have been used for computing the general *Jupiter*-perturbations of the following planets: (9) Metis; (32) Pomona; (29) Amphitrite[5]; (161) Athor[6]; (48) Doris[7]; (10) Hygiea[8]; (24) Themis[9]; (28) Bellona; (55) Pandora; (127) Johanna[10] and the gene-

[1] H. v. ZEIPEL. Angenäherte Jupiterstörungen für die Hecuba-Gruppe. Mémoires de l'académie impériale des sciences de S:t Petersbourg. 8 série, Vol. 12, n:o 11.

[2] K. BOHLIN. Sur le développement des perturbations planétaires. Application aux petites planètes. Astron. iakttagelser och undersökningar å Stockholms Observatorium. Band 7.

[3] H. G. BLOCK. Tafeln zur Berechnung der Störungen einer Gruppe kleiner Planeten durch Saturn. Astron. iakttagelser och undersökningar å Stockholms Observatorium. Band 8, n:o 5.

[4] D. T. WILSON. The present paper.

[5] Astron. iakttagelser och undersökningar å Stockholms Observatorium. Band 7 and Astron. Nachr., n:o 3396.

[6] Astron. Nachr., n:o 3605.

[7] Mémoires de l'académie impériale des sciences de S:t Petersbourg. 8 série, Vol. 12, n:o 11.

[8] Ibid. and in the Astron. Nachr., n:o 3793.

[9] Allgemeine Jupiter-störungen des Planeten (24) Themis. Astron. Nachr., n:o 4123 and 4551.

[10] The present memoir.

ral *Saturn*-perturbations of (32) Pomona and (29) Amphitrite.[1] In every case the agreement was satisfactory. In the several papers mentioned below all accounts for the application of the methods ought to be given.

There are nevertheless some remarks and corrections to the matter, which should be observed and which I take this opportunity to announce.

Corrections.

1. To the »Développement des perturbations planétaires». Astron. iakttagelser och undersökningar å Stockholms Observatorium, Band 7:

page	184 note:	to be read:	$n = 0$	instead of	$n = 1$
»	83	»	$\theta_1 = \frac{w}{1-w} nz$	»	$\theta_1 = -\frac{w}{1-w} nz$
»	103	»	$-\frac{w}{1-w} g$	»	$\frac{\varkappa - w}{1-w} g$
»	105 (212)	»	$n = \frac{n_0}{1-w}$	»	$n = n_0(1-w)$
»	106	»	»	»	»
»	106	»	$-\frac{w}{1-w}$	»	$\frac{\varkappa - w}{1-w}$
»	106	»	$\varkappa = w - a_0(1-w)$	»	$\varkappa = w - \frac{1}{2} a_0(1-w)$
»	184 by $P_{11}[n-1-n-1]_{n=0}^{i=2}$		0.90309	»	0.90309_n
»	258	»	$ndz_1 = -\frac{w}{1-w} g + \cdots$	»	$ndz_1 = \frac{\varkappa - w}{1-w} g + \cdots$

2. To the »Formeln und Tafeln zur gruppenweisen Berechnung der Störungen benachbarter Planeten». Nova Acta Regiae Societatis Scientiarum Uppsaliensis, Serie 3, Vol. 17, 1896.

Page 215 to be read: for $n = 1$:	$D_{01}(n-n)_{+\pi'} = 2.6026_n$	instead of	2.699511
» » :	$D_{01}(n-n)_{-\nu'} = 1.1256$	»	1.22259_n.

Determination of the constants of the orbite.

In order to determine the elements of the orbits one may put the constants of integration for all perturbations zero and use directly the perturbations as they are given in the tables. The terms in ndz:

[1] Astron. iakttagelser och undersökningar å Stockholms Observatorium. Band 8, n:o 5.

$$a_0 g = a_0 nt$$

may be omitted, as its effect is only to change the mean motion, which still is to be determined by observations or normal places. This is connected with the determination of $\varkappa$:

$$\varkappa = w - a_0(1 - w)$$

in the »Développement des perturbations planétaires» page 106. In order to employ the formulae (214), (242) and (284) pages 256—259 of the »Développement des perturbations planétaires» in the case of Professor WILSON, the constant

$$\varkappa$$

is to be changed into

$$\varkappa_2 = 2\varkappa,$$

because his developments, valid for $\mu_0 = \frac{2}{5}$, contain the argument 2θ instead of θ, resp. ϑ^2 instead of ϑ, as is the case of

$$\mu_0 = \frac{1}{3} \quad \text{or} \quad \mu_0 = \frac{1}{2}.$$

Calculation of heliocentric coordinates.

Having formed the perturbations of the

Mean anomaly:	$n\delta z$
Radius vector:	ν
Third coordinate:	$s = \frac{u}{\cos i}$

the corrected mean anomaly is to be found by applying the formula:

$$nz + c_0 = nt + c_0 + n\delta z.$$

Using this corrected mean anomaly, the coordinates in the orbit:

$$r_0, \; v$$

are computed according to the elliptic formulae, and thereupon the actual radius vector is found by:

$$r = r_0(1 + \nu).$$

Rectangular heliocentric coordinates.

a) Ecliptical:

$$x = r \sin a \sin (A + u) + s \, . \, a_0 \sin i_0 \sin \Omega_0$$
$$y = r \sin b \sin (B + u) - s \, . \, a_0 \sin i_0 \cos \Omega_0$$
$$z = r \sin c \sin (C + u) + s \, . \, a_0 \cos i_0$$

where

$$\sin a \sin A = \cos \Omega_0 \qquad \sin b \sin B = \sin \Omega_0$$

$$\sin a \cos A = - \cos i_0 \sin \Omega_0 \qquad \sin b \cos B = \cos i_0 \cos \Omega_0$$

and

$$a_0$$

design~~es~~ates the constant value of the half great axis of the orbite;

b) Equatorial:

$$x = r \sin a \sin (A + u) + s \, . \, a_0 \cos a$$

$$y = r \sin b \sin (B + u) + s \, . \, a_0 \cos b$$

$$z = r \sin c \sin (C + u) + s \, . \, a_0 \cos c$$

where

$$\cos a = \sin i_0 \sin \Omega_0$$

$$\cos b = -[\sin i_0 \cos \varepsilon \cos \Omega_0 + \cos i_0 \sin \varepsilon] = - m \sin (\varepsilon + M)$$

$$\cos c = [- \sin i_0 \sin \varepsilon \cos \Omega_0 + \cos i_0 \cos \varepsilon] = m \cos (\varepsilon + M)$$

$$m \sin M = \sin i_0 \cos \Omega_0$$

$$m \cos M = \cos i_0$$

and a, A; b, B; c, C are the Gaussian Equatoreal Constants computed with the constant elements:

$$i_0, \; \Omega_0 .$$

Stockholm 23 September 1911.

Karl Bohlin.

Tables for the Computation of the Jupiter Perturbations of the Group of Small Planets whose Mean Daily Motions are in the Neighbourhood of 750″.

By

D. T. WILSON.

Cleveland.

In a series of three memoirs entitled: *Auseinandersetzung einer zweckmässigen Methode zur Berechnung der absoluten Störungen der kleinen Planeten*, published in the *Memoirs of the Royal Scientific Society of Saxony*, V, VI and VII, Hansen set forth a method of computing the perturbations of a small planet, in which he developed the perturbative function according to sines and cosines of multiples of the eccentric anomaly. He employed the method of quadratures in such a way as to take rigorously into account all the powers of the eccentricities of the disturbed and disturbing planets. In 1896 Bohlin published in *Nova Acta Regiae Societatis Scientiarum Uppsaliensis* a memoir entitled: *Formeln und Tafeln zur gruppenweisen Berechnung der allgemeinen Störungen benachbarter Planeten,* in which he adapted Hansen's individual method to the group method of computing the perturbations of the small planets. He developed the mathematical theory and constructed tables for the Jupiter perturbations of the group $\mu = \frac{1}{3}$. He computed the perturbations of (9) Metis and of (32) Pomona, and compared his results with those of the same planets computed by Lesser by Hansen's method. The close agreement confirmed the mathematical theory. In 1902 he published a more comprehensive volume: *Sur le Dévelopment des Perturbations Planétaires*, Stockholm, in which he gives a more elegant treatment of the secular terms. He also extended his tables to the third and fourth powers of the eccentricities and to the second power of w for certain terms. The results show a closer agreement between the perturbations computed by the two methods.

The following memoir is an application of the Hansen-Bohlin method to the Jupiter perturbations of the group $\mu = \frac{2}{5}$. Tables have been constructed for the perturbative function beginning with $\gamma_i^{s \cdot n}$, the point where the different groups diverge. In this case the integration divisors for certain values of the integers n, r and s become as small as 0.2. These terms increase rapidly as the series advance. They were computed to the third power of the eccentricities and to the fourth power of w. It was found that all the terms of the third and fourth powers of w and some of those of the third power of the eccentricities were negligible when the eccentricity of the disturbed planet does not exceed 0.34 ($\varphi = 20°$) and when the mean daily motion lies within the limits 720″ to 780″. Therefore only those terms of the third power of the eccentricities which are appreciable within the above limits have been retained. All the secular terms and those of arguments (2—6) have been computed to the fourth power of the eccentricities.

By means of these tables the Jupiter perturbations of (55) Pandora, of (28) Bellona, and of (127) Johanna were computed and compared with the results previously obtained by HANSEN's method by MÖLLER (Astr. Nach., n:o 1791), by BOHLIN (manuscript in the office of the Royal Astronomical Recheninstitute, Berlin), and by OLSSON (Ueber die allgemeinen Jupiterstörungen des Planeten (127) Johanna. K. Svenska Vetenskaps Akademiens Handl., Band 28, n:o 4, Stockholm). The comparisons, given herewith, show a satisfactory agreement.

I wish to express my gratitude to Prof. LAVES of Chicago and to Professor BOHLIN of Stockholm for valuable suggestions and material aid in the execution of this work.

Perturbations of (55) Pandora.

$n = 773''.9479$	$n' = 299''.1286$	$\log \eta = 8.85136$	$\Pi = 10°\ 53'\ 29''$
$\varphi = 8°\ 9'\ 55''.7$	$\varphi' = 2°\ 45'\ 56''.7$	$\log \eta' = 8.38249$	$\Pi' = 11°\ 22'\ 19''$
$i = 7°\ 13'\ 29''.6$	$i' = 1°\ 18'\ 38''.3$	$\log j^2 = 7.60503$	
		$\log \iota = 9.10364$	
		$\log w = 8.52837$	

		$n\delta z$ WILSON		$n\delta z$ MÖLLER	
		cos	sin	cos	sin
1	0	$-9''.18\,nt$	$+0''.31\,nt$	$-9''.61\,nt$	$+0''.36\,nt$
2	0	$+0\,.33\,nt$	$-0\,.02\,nt$	$+0\,.34\,nt$	$-0\,.01\,nt$
−1	−1	+ 0.4	− 1.4	+ 0.8	− 1.9
0	−1	+ 0.8	− 23.7	+ ~~1.8~~ 1.2	− 23.8
1	−1	+ 2.0	− 233.9	+ ~~1.2~~ 1.8	− 226.1
2	−1	0.0	− 1.7	− 0.2	− 1.6
3	−1	+ 0.1	+ 0.8	+ 0.1	+ 0.7

	$n\delta z$ Wilson cos	$n\delta z$ Wilson sin	$n\delta z$ Möller cos	$n\delta z$ Möller sin
0 − 2	− 2.9	+ 7.0	− 2.9	+ 7.8
1 − 2	− 18.3	+ 770.7	− 16.7	+ 769.1
2 − 2	− 7.9	+ 468.2	− 7.7	+ 467.6
3 − 2	+ 0.4	− 20.5	+ 0.3	− 20.8
4 − 2	0.0	+ 0.2	0.0	+ 0.2
0 − 3	− 0.2	− 1.1	− 0.2	+ 0.8
1 − 3	+ 6.1	+ 11.2	+ 6.5	+ 12.7
2 − 3	− 7.6	+ 175.4	− 7.6	+ 178.2
3 − 3	− 0.9	+ 36.6	− 0.8	+ 36.4
4 − 3	0.0	− 2.3	0.0	− 2.5
1 − 4	0.0	+ 0.4	+ 0.1	+ 0.2
2 − 4	+ 4.4	− 18.3	+ 4.4	− 22.9
3 − 4	+ 1.0	− 20.5	+ 0.9	− 19 3
4 − 4	− 0.3	+ 10.0	− 0.4	+ 10.4
5 − 4	0.0	− 0.7	0.0	− 0.6
1 − 5	− 0.1	+ 0.2	− 0.1	+ 0 2
2 − 5	− 16.6	+ 47.4	− 16.2	+ 57.3
3 − 5	− 6.7	+ 36.7	− 6.8	+ 37.3
4 − 5	+ 0.5	− 5.2	+ 0.5	− 4.8
5 − 5	− 0.1	+ 3.0	− 0.1	+ 3.0
2 − 6	0.0	− 0.1	0.0	− 0.1
3 − 6	− 0.6	+ 1.1	− 0.7	+ 2.3
4 − 6	− 0.4	+ 1.6	− 0.2	+ 1.3
5 − 6	+ 0.1	− 1.1	+ 0.1	− 1.0
6 − 6	0.0	+ 1.0	0.0	+ 1.0

	ν Wilson cos	ν Wilson sin	ν Möller cos	ν Möller sin
0 0	$-0''.02\,nt$		$-0''.03\,nt$	
1 0	$-0\ .16\,nt$	$-4''.59\,nt$	$-0\ .18\,nt$	$-4''.81\,nt$
−1 − 1	− 1.3	− 0.3	− 1.4	− 0.6
0 − 1	+ 3.4	− 0.2	+ 3.9	− 0.3
1 − 1	+ 78.1	+ 0.7	+ 76.0	+ 0.6
2 − 1	+ 5.5	+ 0.1	+ 5.4	− 0.1
3 − 1	− 0.3	+ 0.1	− 0.3	+ 0.1
0 − 2	− 7.2	+ 0.9	− 7.0	+ 0.9
1 − 2	− 151.5	− 3.0	− 153.6	− 3.0
2 − 2	− 275.2	− 4.7	− 273.8	− 4.6
3 − 2	+ 1.0	0.0	+ 1.5	0.0
4 − 2	− 0.1	0.0	− 0.1	0.0

	ν Wilson cos	ν Wilson sin	ν Möller cos	ν Möller sin
0 — 3	— 1.4	+ 0.1	+ 0.2	+ 0.1
1 — 3	— 10.2	— 1.2	— 9.7	— 1.3
2 — 3	— 82 0	— 3.3	— 84.5	— 3.5
3 — 3	— 30.1	— 0.8	— 30.0	— 0.8
4 — 3	+ 0.3	0.0	+ 0.4	0.0
1 — 4	+ 1.0	+ 0.1	+ 0.8	+ 0.1
2 — 4	+ 8.8	+ 1.3	+ 8.1	+ 1.3
3 — 4	+ 11.8	+ 0.6	+ 11.0	+ 0 6
4 — 4	— 7.3	— 0.2	— 7 6	— 0.2
5 — 4	+ 0.1	0.0	+ 0.1	0.0
1 — 5	— 0.2	0.0	— 0.2	0.0
2 — 5	— 3.9	— 2.2	— 5.4	— 1.2
3 — 5	— 19.1	— 3.6	— 19.3	— 3.6
4 — 5	+ 2.2	+ 0.1	+ 2.1	+ 0.1
5 — 5	— 2.3	— 0.1	— 2.4	— 0.1
2 — 6	— 0.1	0.0	— 0.1	0.0
3 — 6	0.0	— 0.3	— 1.0	— 0.3
4 — 6	— 1.4	— 0.2	— 0.8	— 0.2
5 — 6	+ 0.7	0.0	+ 0.6	0.0
6 — 6	— 0.7	0.0	— 0.9	0.0

	$\frac{u}{\cos i}$ Wilson cos	$\frac{u}{\cos i}$ Wilson sin	$\frac{u}{\cos i}$ Möller cos	$\frac{u}{\cos i}$ Möller sin
0 0	$-0''.73\,nt$		$-0''.72\,nt$	
1 0	$+4.99\,nt$	$+1''.09\,nt$	$+5.07\,nt$	$+1''.13\,nt$
— 1 — 1	— 0.4	+ 1.8	— 0.5	+ 2.3
0 — 1	— 1.7	+ 8.6	— 1.0	+ 8.0
1 — 1	+ 1.0	+ 0.2	+ 1.5	— 2.4
2 — 1	+ 0 7	+ 4.0	+ 0.8	+ 3.9
3 — 1	0.0	— 0.2	0.0	— 0.2
0 — 2	+ 2.3	— 10.6	+ 2.2	— 10.0
1 — 2	— 2.6	+ 12.7	— 2.4	+ 11.3
2 — 2	— 1.6	+ 2.4	— 1.5	+ 2.3
3 — 2	+ 0.2	+ 1.0	+ 0.2	+ 0.9
4 — 2	0.0	— 0.1	0.0	— 0.1

	$\frac{u}{\cos i}$ Wilson		$\frac{u}{\cos i}$ Möller	
	cos	sin	cos	sin
0 — 3	+ 0.3	— 1.0	+ 0.4	— 1.2
1 — 3	+ 1.0	— 4.7	+ 1.1	— 4.5
2 — 3	— 5.1	+ 22.3	— 5.0	+ 21.6
3 — 3	— 0.2	+ 0.2	— 0.2	+ 0.2
4 — 3	+ 0.1	+ 0.3	+ 0.1	+ 0.3
1 — 4	— 0.3	+ 0.8	— 0.2	+ 0.6
2 — 4	+ 0.2	— 0.8	+ 0.2	— 0.8
3 — 4	+ 0.8	— 3.5	+ 0.8	— 3.4
4 — 4	0.0	0.0	0.0	0.0
5 — 4	0.0	+ 0.1	0.0	+ 0.1
2 — 5	+ 0.2	— 0.7	+ 0.1	— 0.4
3 — 5	— 1.2	+ 4.7	— 1.3	+ 4.5
4 — 5	+ 0.2	— 0.7	+ 0.2	— 0.7
5 — 5	0.0	0.0	0.0	0.0
3 — 6	0.0	+ 0.2	— 0.1	+ 0.2
4 — 6	— 0.1	+ 0.2	— 0.1	+ 0.2
5 — 6	0.0	— 0.2	0.0	— 0.2
6 — 6	0.0	0.0	0.0	0.0

Perturbations of (28) Bellona.

$n = 766''.3957$ $\quad n' = 299''.1142$ $\quad \log \eta = 8.87776$ $\quad \Pi = 332° \; 57' \; 1''$

$\varphi = 8° \; 40' \; 51''.9$ $\quad \varphi' = 2° \; 48' \; 48''.0$ $\quad \log \eta' = 8.38472$ $\quad \Pi' = 222° \; 1' \; 2''$

$i = 9° \; 21' \; 41''.1$ $\quad i' = 1° \; 18' \; 31''.9$ $\quad \log j^2 = 7.73630$

$\log \iota = 9.16911$

$\log w = 8.38530$

	$n\delta z$ Wilson		$n\delta z$ Bohlin	
	cos	sin	cos	sin
0 0	— 43''.~~8~~ 9 nt		— 43''. 9 nt	
1 0	— 13 .67 nt	— 3''.37 nt	— 13 .76 nt	— 3''.38 nt
2 0	+ 0 .52 nt	+ 0 .13 nt	+ 0 .52 nt	+ 0 .13 nt
— 1 — 1	— 1.7	+ 5.4	— 1.4	+ 5.3
0 — 1	— 47.8	+ 43.8	— 46.3	+ 45.8
1 — 1	— 218.3	+ 94.2	— 216.6	+ 91.7
2 — 1	+ 0.9	+ 1.2	+ 0.1	+ 0.2
3 — 1	+ ~~0.9~~ 0.8	+ 0.1	+ 0.7	+ 0.1

	$n\delta z$ Wilson cos	$n\delta z$ Wilson sin	$n\delta z$ Bohlin cos	$n\delta z$ Bohlin sin
0 — 2	— 25.3	+ 1.0	— 25.8	+ 1.6
1 — 2	— 771.9	— 676.1	— 782.0	— 681.2
2 — 2	— 333.5	— 359.4	— 333.1	— 361.7
3 — 2	+ 15.3	+ 15.8	+ 15.5	+ 16.0
4 — 2	0.0	— 0.2	0.0	— 0.2
0 — 3	+ 2.5	+ 5.1	+ 4.3	+ 3.1
1 — 3	— 62.7	— 303.1	— 67.3	— 290.6
2 — 3	— 26.5	+ 539.9	— 31.1	+ 548.9
3 — 3	— 16.9	+ 18.2	— 17.7	+ 19.8
4 — 3	+ 0.6	— 2.0	+ 0.7	— 2.1
1 — 4	+ 1.8	— 2.8	+ 1.1	— 1.8
2 — 4	— 149.4	+ 128.0	— 136.0	+ 120.9
3 — 4	— 64.8	+ 22.3	— 64.0	+ 19.7
4 — 4	+ 10.7	— 0.8	+ 12.2	— 0.3
5 — 4	— 0.5	+ 0.2	— 0.4	+ 0.1
1 — 5	— 3.8	+ 2.3	— 3.2	+ 2.5
2 — 5	— 2474.1	— 155.1	— 2339.7	— 119.9
3 — 5	— 386.8	— 196.2	— 388.8	— 220.0
4 — 5	+ 25.4	+ 19.3	+ 23.5	+ 18.0
5 — 5	— 1.3	— 2.9	— 1.2	— 3.0
2 — 6	— 4.0	— 4.7	— 5.6	— 5.6
3 — 6	+ 15.7	+ 55.1	+ 15.9	+ 49.7
4 — 6	— 1.4	+ 15.5	— 1.6	+ 11.8
5 — 6	+ 2.2	— 3.7	+ 1.7	— 3.9
6 — 6	— 0.9	+ 0.9	— 0.8	+ 0.7

	ν Wilson cos	ν Wilson sin	ν Bohlin cos	ν Bohlin sin
0 0	+ 0″.25 *nt*		+ 0″.26 *nt*	
1 0	+ 1 .70 *nt*	— 6″.83 *nt*	+ 1 .71 *nt*	— 6″.88 *nt*
—1 — 1	+ 4.3	+ 1.7	+ 4.2	+ 1.5
0 — 1	+ 4.8	+ 0.6	+ 6.0	+ 0.5
1 — 1	— 30.7	— 72.1	— 30.1	— 71.6
2 — 1	— 2.5	— 4.0	— 2.0	— 4.4
3 — 1	— 0.2	+ 0.5	— 0.2	+ 0.4
0 — 2	+ 9.3	+ 0.3	+ 9.8	+ 0.1
1 — 2	+ 132.9	— 139.9	+ 130.6	— 139.6
2 — 2	+ 21~~4.4~~ 2.0	— ~~207.6~~ 197.5	+ 212.2	— 196.4
3 — 2	+ 0.1	+ 0.7	— 0.4	+ 0.9
4 — 2	+ 0.2	0.0	+ 0.2	0.0

	ν Wilson cos	ν Wilson sin	ν Bohlin cos	ν Bohlin sin
0 − 3	+ 0.9	− 2.2	− 0.6	− 2.3
1 − 3	− 70.7	+ 4.3	− 68.9	+ 6.0
2 − 3	− 242.7	− 16.2	− 248.9	− 15.9
3 − 3	− 27.8	− 13.1	− 27.6	− 13.3
4 − 3	0.0	− 0.3	− 0.1	− 0.2
1 − 4	− 4.8	− 5.4	− 4.4	− 3.2
2 − 4	− 35.4	− 47.2	− 34.0	− 43.0
3 − 4	− 15.9	− 41.1	− 14.3	− 39.9
4 − 4	− 0.4	+ 7.0	− 0.6	+ 7.4
5 − 4	− 0.2	0.0	− 0.1	0.0
1 − 5	+ 2.5	− 4.4	+ 2.1	− 4.4
2 − 5	+ 22 9	− 116.8	+ 21.8	− 107.2
3 − 5	+ 100.8	− 207.8	+ 113.3	− 205.0
4 − 5	− 7.5	+ 6.0	− 6.6	+ 5.8
5 − 5	+ 2.0	− 0.6	+ 2 2	− 0.6
2 − 6	− 3.0	+ 2.2	− 3.8	+ 2.1
3 − 6	− 22.3	+ 5.7	− 20.0	+ 5.8
4 − 6	− 9.9	− 0.6	− 8.4	− 0.5
5 − 6	+ 2.9	+ 1.6	+ 2.6	+ 1.2
6 − 6	− 0.3	− 0.7	− 0.4	− 0.6

	$\frac{u}{\cos i}$ Wilson cos	$\frac{u}{\cos i}$ Wilson sin	$\frac{u}{\cos i}$ Bohlin cos	$\frac{u}{\cos i}$ Bohlin sin
0 0	$-0''.84\,nt$		$-0''.82\,nt$	
1 0	$+5\ .55\,nt$	$+3''.36\,nt$	$+5\ .40\,nt$	$+3''.27\,nt$
−1 − 1	+ 2.5	− 4.8	+ 2.1	− 4.9
0 − 1	+ 4.1	− 7.0	+ 4.9	− 8.1
1 − 1	+ 1.9	+ 3.9	+ 2.2	+ 4.0
2 − 1	+ 5.0	+ 0.5	+ 5.3	+ 0.7
3 − 1	− 0.4	− 0.1	− 0.4	− 0.1
0 − 2	+ 20.0	+ 3.5	+ 18.6	+ 2.1
1 − 2	− 14.2	− 5.0	− 14.0	− 4.4
2 − 2	− 2.9	+ 5.1	− 1.9	+ 5.1
3 − 2	− 0.1	− 1.2	+ 0.2	− 1.3
4 2	0.0	+ 0.1	0.0	+ 0.1

	$\frac{u}{\cos i}$ Wilson		$\frac{u}{\cos i}$ Bohlin	
	cos	sin	cos	sin
0 — 3	— 4.3	— 6.3	— 4.8	— 5.8
1 — 3	— 2.7	— 10.2	— 2.5	— 7.9
2 — 3	+ 4.4	+ 25.1	+ 3.7	+ 23.8
3 — 3	+ 0.9	— 0.4	+ 0.8	— 0.5
4 — 3	— 0.3	+ 0.2	— 0.4	+ 0.3
1 — 4	+ 2.8	— 4.9	+ 1.8	— 3.8
2 — 4	— 3.7	+ 4.1	— 3.2	+ 4.5
3 — 4	— 4.3	+ 1.6	— 3.1	+ 2.0
4 — 4	— 0.3	— 0.2	— 0.3	— 0.2
5 — 4	+ 0.1	+ 0.1	+ 0.2	+ 0.1
2 — 5	+ 2.8	+ 2.0	+ 1.0	+ 1.8
3 — 5	— 34.1	— 14.1	— 30.5	— 8.3
4 — 5	+ 0.7	+ 0.7	+ 0.5	+ 0.3
5 — 5	0.0	+ 0.2	0.0	+ 0.2
3 — 6	+ 1.2	+ 2.7	+ 1.4	+ 2.2
4 — 6	0.0	+ 1.5	+ 0.1	+ 1.1
5 — 6	+ 0.1	— 0.2	0.0	— 0.1
6 — 6	+ 0.1	0.0	+ 0.1	0.0

Perturbations of (127) Johanna.

$n = 775''.8987$ $n' = 299''.1142$ $\log \eta = 8.51936$ $\Pi = 99^\circ\ 19'\ 30''$

$\varphi = 3^\circ\ 47'\ 30''$ $\varphi' = 2^\circ\ 46'\ 52''$ $\log \eta' = 8.38490$ $\Pi' = 350^\circ\ 7'\ 46''$

$i = 8^\circ\ 15'\ 35''$ $i' = 1^\circ\ 18'\ 33''$ $\log j^2 = 7.67075$

$\log \iota = 9.13549$

$\log w = 8.55910$

	$n\delta z$ Wilson		$n\delta z$ Olsson	
	cos	sin	cos	sin
0 0	— ~~39''.04~~ 40''.33 nt		— 40''.50 nt	
1 0	— 5 .24 nt	— 2''.47 nt	— 5 .26 nt	— 2''.46 nt
2 0	+ 0 .09 nt	+ 0 .04 nt	+ 0 .09 nt	+ 0 .04 nt
— 1 — 1	+ 2.9	+ 0.9	+ 2.7	+ 0.7
0 — 1	— 18.6	+ 31.5	— 17.6	+ 29.3
1 — 1	— 201.5	+ 75.0	— 203.3	+ 74.7
2 — 1	+ 0.9	+ 1.2	+ 1.4	+ 0.6
3 — 1	— 0.2	+ 0.2	— 0.2	+ 0.2

	$n\delta z$ Wilson cos	$n\delta z$ Wilson sin	$n\delta z$ Olsson cos	$n\delta z$ Olsson sin
0 — 2	— 1.8	+ 10.6	— 1.1	+ 10.1
1 — 2	— 333.1	— 265.1	— 335.8	— 270.2
2 — 2	— 288.3	— 358.3	— 290.0	— 361.5
3 — 2	+ 6.0	+ 6.4	+ 5.8	+ 6.3
4 — 2	+ 0.1	+ 0.1	+ 0.1	+ 0.1
0 — 3	0.0	— 1.6	+ 0.6	— 2.1
1 — 3	— 61.2	— 94.6	— 59.7	— 89.7
2 — 3	+ 47.9	+ 346.6	+ 48.6	+ 347.5
3 — 3	— 22.5	+ 30.7	— 22.3	+ 29.5
4 — 3	+ 0.1	— 0.7	+ 0.1	— 0.8
1 — 4	+ 0.1	— 0.6	+ 0.8	— 1.2
2 — 4	— 17.6	+ 65.6	— 15.4	+ 60.4
3 — 4	— 36.8	+ 20.4	— 35.7	+ 18.4
4 — 4	+ 9.7	+ 1.3	+ 9.5	+ 1.6
5 — 4	— 0.2	+ 0.1	— 0.1	+ 0.1
1 — 5	+ 1.1	+ 0.1	+ 1.1	+ 0.1
2 — 5	— 133.4	+ 207.2	— 130.9	+ 209.9
3 — 5	— 105.0	+ 5.9	— 107.4	+ 6.1
4 — 5	+ 8.0	+ 5.1	+ 7.3	+ 4.9
5 — 5	— 0.5	— 2 6	— 0.4	— 2.7
2 — 6	— 0.7	+ 0.3	— 1.2	+ 0.7
3 — 6	+ 11.2	+ 3.7	+ 11.0	+ 3.1
4 — 6	+ 2.6	+ 4.4	+ 1.6	+ 4.3
5 — 6	+ 1.0	— 2.3	+ 0.8	— 2.2
6 — 6	— 0.9	+ 0.4	— 0.8	+ 0.4

	ν Wilson cos	ν Wilson sin	ν Olsson cos	ν Olsson sin
0 0	$+ 0''.08\,nt$		$+ 0''.08\,nt$	
1 0	$+ 1\,.24\,nt$	$- 2''.62\,nt$	$+ 1\,.23\,nt$	$- 2''.63\,nt$
— 1 — 1	+ 0.8	— 1.8	+ 0.6	— 1.7
0 — 1	+ 4.7	+ 0.2	+ 4.4	— 0.4
1 — 1	— 24.8	— 67.5	— 24.7	— 67.6
2 — 1	— 1.4	— 1.4	— 1.0	— 0.9
3 — 1	— 0.2	— 0.2	— 0.2	— 0.2
0 — 2	+ 5.6	— 1.2	+ 5.5	— 1.5
1 — 2	+ 53.5	— 58.6	+ 54.5	— 59.9
2 — 2	+ 213.3	— 170.7	+ 207 8	— 167.0
3 — 2	+ 0.4	+ 0.4	+ 0.2	+ 0.4
4 — 2	— 0.1	+ 0.1	— 0.1	+ 0.1

	ν Wilson cos	ν Wilson sin	ν Olsson cos	ν Olsson sin
0 − 3	− 1.2	− 0.1	− 1.5	− 0.2
1 − 3	− 19.2	+ 8.1	− 19.1	+ 8.1
2 − 3	− 155.3	+ 20.7	− 156.4	+ 20.4
3 − 3	− 24.8	− 15.1	− 24.7	− 15.4
4 − 3	+ 0.1	− 0.3	0.0	− 0.3
1 − 4	− 1.2	− 0.4	− 1.2	− 0.4
2 − 4	− 18.3	− 6.8	− 17.2	− 6.0
3 − 4	− 10.7	− 21.0	− 12.1	− 21.9
4 − 4	− 1.4	+ 7.0	− 1.6	+ 7.0
5 − 4	− 0.2	+ 0.1	− 0.1	+ 0.1
1 − 5	− 0.1	− 0.8	− 0.3	− 0.7
2 − 5	− 9.3	− 10.8	− 9.8	− 9.8
3 − 5	− 3.9	− 56.3	− 3.8	− 56.1
4 − 5	− 3.9	+ 4.4	− 3.5	+ 4.1
5 − 5	+ 2.1	− 0.3	+ 2.2	− 0.2
2 − 6	+ 0.1	+ 0.4	+ 0.1	+ 0.5
3 − 6	− 1.6	+ 4.7	− 1.4	+ 4.3
4 − 6	− 3.3	+ 1.9	− 2.8	+ 1.3
5 − 6	+ 1.9	+ 0.7	+ 1.7	+ 0.6
6 − 6	− 0.2	− 0.7	− 0.3	− 0.7

	$\frac{u}{\cos i}$ Wilson cos	$\frac{u}{\cos i}$ Wilson sin	$\frac{u}{\cos i}$ Olsson cos	$\frac{u}{\cos i}$ Olsson sin
0 0	$+ 0''.06\, nt$		$+ 0''.05\, nt$	
1 0	$- 0\ .83\, nt$	$- 5''.74\, nt$	$- 0\ .81\, nt$	$- 5''.61\, nt$
− 1 − 1	+ 1.6	+ 2.0	+ 1.6	+ 1.9
0 − 1	+ 2.3	+ 9.6	+ 2.3	+ 9.4
1 − 1	− 3.3	− 4.5	− 3.3	− 4.4
2 − 1	− 2.1	− 3.8	− 2.1	− 3.9
3 − 1	0.0	+ 0.1	0.0	+ 0.1
0 − 2	− 5.5	+ 7.0	− 5.0	+ 6.7
1 − 2	+ 10.2	− 6.4	+ 10.0	− 6.3
2 − 2	+ 2.8	− 5.6	+ 2.4	− 5.3
3 − 2	− 0.7	+ 0.8	− 0.7	+ 0.8
4 − 2	0.0	0.0	0.0	0.0

	$\frac{u}{\cos i}$ Wilson		$\frac{u}{\cos i}$ Olsson	
	cos	sin	cos	sin
0 — 3	+ 2.1	— 0.8	+ 1.8	— 1.2
1 — 3	+ 5.2	+ 1.1	+ 4.6	+ 1.0
2 — 3	— 20.8	— 14.6	— 19.7	— 14.2
3 — 3	— 0.9	0.0	— 0.9	+ 0.1
4 — 3	+ 0.3	+ 0.1	+ 0.3	+ 0.1
1 — 4	+ 1.3	+ 1.1	+ 1.1	+ 0.7
2 — 4	— 1.2	— 3.0	— 1.4	— 2.9
3 — 4	+ 1.5	— 3.5	+ 0.9	— 3.6
4 — 4	+ 0.1	+ 0.2	+ 0.1	+ 0.2
5 — 4	0.0	— 0.1	0.0	— 0.1
2 — 5	— 0.6	— 0.2	— 0.6	— 0.2
3 — 5	+ 10.3	— 8.6	+ 9.4	— 9.2
4 — 5	— 0.7	0.0	— 0.7	+ 0.1
5 — 5	+ 0.1	— 0.2	+ 0.1	— 0.1
3 — 6	— 1.1	+ 0.2	— 1.0	+ 0.4
4 — 6	— 0.6	— 0.4	— 0.7	— 0.3
5 — 6	0.0	+ 0.2	+ 0.1	+ 0.2
6 — 6	— 0.1	0.0	0.0	0.0

I give here a resume of the theory on which these tables have been constructed.

In order to determine the perturbations in mean longitude and radius vector Hansen derived the differential equations

$$\frac{dz}{dt} = 1 + \overline{W} + \frac{h_0}{h}\left(\frac{\nu}{1+\nu}\right)^2 \tag{1}$$

$$\frac{d\nu}{dt} = -\frac{1}{2}\frac{\partial \overline{W}}{\partial \tau}. \tag{2}$$

$\overline{W}$ is defined by the equation

$$\overline{W} = \frac{2h}{h_0} - \frac{h_0}{h} - 1 + 2\frac{h}{h_0}\xi_1\frac{\bar{r}}{a_0}\cos\bar{f} + 2\frac{h}{h_0}\eta_1\frac{\bar{r}}{a_0}\sin\bar{f}. \tag{3}$$

The quantities ξ_1 and η_1 are of the order of the disturbing mass and h_0, $\bar{r}$ and $\bar{f}$ are determined from the equations

$$\begin{aligned} \bar{\varepsilon} - e_0\sin\bar{\varepsilon} &= n_0 z + c_0 \\ n_0^2 a_0^3 &= h_0^2 p_0 \end{aligned} \tag{4}$$

$$\begin{aligned} \tan \frac{1}{2}\bar{f} &= \sqrt{\frac{1+e_0}{1-e_0}} \tan \frac{1}{2}\bar{\varepsilon} \\ \bar{r} = a_0(1 - e_0 \cos \bar{\varepsilon}) &= \frac{p_0}{1 + e_0 \cos \bar{f}}\,. \end{aligned} \tag{4}$$

These equations contain the constant elements. Neglecting the perturbative force, equation (1) gives

$$z = t.$$

We may assume then

$$z = t + \delta z. \tag{5}$$

From equations (1) and (5) we get

$$\frac{d\delta z}{dt} = \frac{h_0}{h}\left(\frac{\nu}{1+\nu}\right)^2 + \overline{W}. \tag{6}$$

If we limit ourselves to perturbations of the first order with respect to the mass of the disturbing body, (6) may be written

$$\frac{d\delta z}{dt} = \overline{W}. \tag{7}$$

Introducing ε in place of t in (7) by means of the equation

$$\varepsilon - e \sin \varepsilon = nt + c \tag{8}$$

gives

$$\frac{dn\delta z}{d\varepsilon} = \overline{W}(1 - e \cos \varepsilon). \tag{9}$$

To get the inequalities of the third coordinate HANSEN found a function U similar to W which is defined by the equation

$$\frac{dU}{dt} = hr\frac{\bar{\varrho}}{a_0} \sin(\bar{\omega} - \bar{f}) \frac{\partial \Omega}{\partial Z} \cos i. \tag{10}$$

When U is determined u, the quantity sought, is obtained from the differential equation

$$\frac{du}{dt} = \overline{\frac{\partial U}{\partial \tau}} \tag{11}$$

$\overline{W}$ and $\overline{U}$ refer to the ellipse of equations (4), while W and U are similar functions of an auxiliary ellipse

$$\begin{aligned} \psi - e \sin \psi &= n\tau + c \\ \frac{\varrho}{a} \cos \omega &= \cos \psi - e \\ \frac{\varrho}{a} \sin \omega &= \sqrt{1 - e^2} \sin \psi \end{aligned} \tag{12}$$

such that changing τ to t, and ψ, ω and ϱ to ε, f and r changes W and U to $\overline{W}$ and $\overline{U}$. The expressions $\overline{\frac{\partial W}{\partial \tau}}$ and $\overline{\frac{\partial U}{\partial \tau}}$ mean that τ is changed to t after W and U are differentiated with respect to τ. Making use of (8) and (12) equation (2) may be written

$$\frac{d\nu}{d\varepsilon} = -\frac{1}{2}\frac{\partial \overline{W}}{\partial \psi} \tag{13}$$

and equation (11) becomes

$$\frac{du}{d\varepsilon} = \frac{\partial \overline{U}}{\partial \psi}. \tag{14}$$

The three inequalities $n\delta z$, ν and u are obtained from the solution of equations (9), (13) and (14). W is determined from the equation

$$\frac{dW}{d\varepsilon} = Ma\frac{\partial \Omega}{\partial \varepsilon} + Nar\frac{\partial \Omega}{\partial r} = T. \tag{15}$$

$$M = \frac{1}{\cos^2 \varphi}\Big[-\left(3 - \frac{3}{2}e^2\right) + 2e\cos\varepsilon - \frac{1}{2}e^2\cos 2\varepsilon + e^2\cos(\psi+\varepsilon)$$
$$-3e\cos\psi + (4-e^2)\cos(\psi-\varepsilon) - e\cos(\psi - 2\varepsilon)\Big]$$

$$N = \frac{1}{\cos^2\varphi}\Big[e\sin\varepsilon - \frac{1}{2}e^2\sin 2\varepsilon + e^2\sin(\psi+\varepsilon) - e\sin\psi$$
$$-(2-e^2)\sin(\psi-\varepsilon) + e\sin(\psi-2\varepsilon)\Big]$$

U is determined from the equation

$$\sec i\frac{dU}{d\varepsilon} = Qa^2\frac{\partial\Omega}{\partial Z}. \tag{16}$$

$$Q = e\sin\varepsilon - \frac{1}{2}e^2\sin 2\varepsilon + \frac{1}{2}e^2\sin(\psi+\varepsilon)$$
$$-\frac{3}{2}e\sin\psi + \left(1+\frac{1}{2}e^2\right)\sin(\psi-\varepsilon) - \frac{1}{2}e\sin(\psi-2\varepsilon)$$

The symbols Ω, r and a are the perturbative function, radius vector, and semi-major axis respectively. The coordinate perpendicular to the orbit is given by the equation

$$Z = au.$$

In order to adapt the above equations to the group method and thus make it possible to compute the perturbations of a group of small planets of nearly the same mean daily motion with one set of tables, Bohlin introduced a function θ defined by the equation

$$\mu_0\theta = \mu_0(\varepsilon - e\sin\varepsilon) - g', \tag{17}$$

μ_0 being a rational number and differing by a small amount from μ, the ratio of n' to n, and g' being the mean anomaly of the disturbing body. Here μ_0 is equal to $\frac{2}{5}$. From the relations

$$g' = n't + n'\delta z'$$

$$nz = nt + n\delta z$$

we have

$$\mu_0 \theta = (\mu_0 - \mu) nz + \mu n \delta z - n'\delta z'.$$

We now assume a small quantity w defined by the equation

$$1 - \frac{\mu}{\mu_0} = w. \tag{18}$$

Equation (17) may now be written in the form

$$\theta = wnz + (1 - w) n\delta z - \frac{n'\delta z'}{\mu_0} \tag{19}$$

from which

$$n\delta z = \frac{1}{1-w}\left[\theta - wnz + \frac{n'\delta z'}{\mu_0}\right]. \tag{20}$$

Differentiating (19) with respect to ε gives

$$\frac{d\theta}{d\varepsilon} = \left[w + (1-w)\overline{W}\right](1 - e\cos\varepsilon) - \frac{dn'\delta z'}{\mu_0 d\varepsilon}. \tag{21}$$

Determining θ from (21) and substituting it in (20) gives the inequality $n\delta z$. When Jupiter is the disturbing body $\frac{n'\delta z'}{\mu_0}$ is a small quantity of the second order and can be neglected.

Developing the perturbative function in powers of the eccentricities and inclinations we have

$$\left.\begin{array}{r} -\frac{1}{\mathfrak{J}} a \frac{\partial \Omega}{\partial \varepsilon} \\ \frac{1}{\mathfrak{R}} ar \frac{\partial \Omega}{\partial r} \\ \frac{1}{\mathfrak{J}} a^2 \frac{\partial \Omega}{\partial Z} \end{array}\right\} = \Sigma \left\{\begin{array}{l} P_i[n+r.-n+s]\, 2\gamma_i^{1.n} \\ Q_i[n+r.-n+s]\, 2\gamma_i^{1.n} \\ R_i[n+r.-n+s]\, \frac{1}{2\alpha} 2\gamma_i^{3.n} \end{array}\right\} y^{n+r} x'^{-n+s} \tag{22}$$

where $y = e^{\sqrt{-1}\,\varepsilon}$, $x' = e^{\sqrt{-1}\,g'}$, $\alpha = \frac{a}{a'}$, $\mathfrak{R}$=real part, and $\mathfrak{J}$=imaginary part divided by $\sqrt{-1}$. The coefficients $\gamma_i^{s.n}$ depend on the ratio $\mu_0 = \frac{2}{5}$ and the mass of the disturbing planet. The mass of Jupiter is here taken as $\frac{1}{1048}$. Instead of $\gamma_i^{s.n}$ a more convenient symbol, $\omega_i^{s.n}$ is used, defined by the relations

$$\begin{aligned}
\omega_i^{1\cdot n} &= 2\,\gamma_i^{1\cdot n}\\
\omega_i^{3\cdot n} &= \frac{1}{2\,\alpha}\,2\,\gamma_i^{3\cdot n}\\
\omega_i^{5\cdot n} &= \frac{3}{4\,\alpha^2}\,2\,\gamma_i^{5\cdot n}.
\end{aligned} \tag{23}$$

In the same way $\bar{\omega}$ and $\bar{\bar{\omega}}$ are obtained from $\bar{\gamma}$ and $\bar{\bar{\gamma}}$.

Eliminating the mean anomaly of the disturbing planet by means of the equation

$$g' = \mu(\varepsilon - e\sin\varepsilon) - \mu\theta, \tag{24}$$

in which μ is the rational fraction, $\frac{2}{5}$, we have

$$\left.\begin{matrix} -\frac{1}{\mathfrak{I}}\,a\,\frac{\partial\Omega}{\partial\varepsilon}\\[2ex] \frac{1}{\mathfrak{R}}\,ar\,\frac{\partial\Omega}{\partial r}\\[2ex] \frac{1}{\mathfrak{I}}\,a^2\frac{\partial\Omega}{\partial Z}\end{matrix}\right\} = \Sigma\left\{\begin{matrix} P(n+r\,.-n+s)\\[2ex] Q(n+r\,.-n+s)\\[2ex] R(n+r\,.-n+s)\end{matrix}\right\} y^{n+r-(n-s)\mu}\vartheta^{(n-s)\mu}. \tag{25}$$

Substituting the values of M, N, and Q from (15) and (16) in (25) we have

$$\begin{aligned}
T = Ma\frac{\partial\Omega}{\partial\varepsilon} + Nar\frac{\partial\Omega}{\partial r} &= \mathfrak{I}\left[F + \frac{y}{v}G + \frac{v}{y}H\right]\\
Qa^2\frac{\partial\Omega}{\partial Z} &= -\,\mathfrak{R}\left[F + \frac{y}{v}G + \frac{v}{y}H\right]
\end{aligned} \tag{26}$$

F, G and H, which represent different quantities in the two equations, are defined by the relations

$$\left.\begin{matrix} F\\ G\\ H\end{matrix}\right\} = \Sigma\left\{\begin{matrix} F(n+r\,.-n+s)\\ G(n+r\,.-n+s)\\ H(n+r\,.-n+s)\end{matrix}\right\} y^{n+r-(n-s)\mu}\vartheta^{(n-s)\mu}. \tag{27}$$

In that which precedes the following notations have been used

$$x' = e^{\sqrt{-1}\,g'},\ y = e^{\sqrt{-1}\,\varepsilon},\ \vartheta = e^{\sqrt{-1}\,\theta},\ v = e^{\sqrt{-1}\,\varphi}. \tag{28}$$

The development so far is based on μ, a rational fraction. In order to obtain the Jupiter perturbations of a planet which is in the neigbourhood of this ideal planet BOHLIN developed T in a power series in w.

Let

$$T = T_0 + T_1 w + T_2 w^2 + \cdots \tag{29}$$

be this series, where T_0 represents the coefficients given in equation (27). Those represented by T_1 and T_2 are derived from (22) in the same way as T_0, except that $-2\bar{\gamma}_i^{s \cdot n}$ and $2\bar{\bar{\gamma}}_i^{s \cdot n}$ are used as factors in place of $2\gamma_i^{s \cdot n}$. These factors are given by the equations

$$
\begin{aligned}
-\bar{\gamma} &= -\frac{2}{3}\frac{d\gamma}{d\alpha} \\
\bar{\bar{\gamma}} &= \frac{2}{9}\alpha^2\frac{d^2\gamma}{d\alpha^2} - \frac{1}{9}\alpha\frac{d\gamma}{d\alpha}\,.
\end{aligned}
\tag{30}
$$

The differential equation

$$\frac{dW}{d\varepsilon} = T$$

must now be intregrated. The coefficients T are multiplied by the factors

$$y^{n+r-(n-s)\mu}\,\vartheta^{(n-s)\mu} = e^{\sqrt{-1}\,[n+r-(n-s)\mu]\varepsilon}\,e^{\sqrt{-1}\,(n-s)\mu\theta}. \tag{31}$$

Since μ is a rational fraction and n, r and s are integers, there will be certain terms of F, G and H independent of ε — those for which $n + r - (n - s)\mu = 0$. These terms must be treated differently from the others, since their integration directly would give a zero divisor.

Differentiating equation (19) with respect to g, where

$$g = \varepsilon - e \sin \varepsilon \tag{32}$$

we have

$$\frac{d\theta}{dg} = w + (1 - w)\,\overline{W}\,. \tag{33}$$

It is convenient to separate the function

$$w + (1 - w)\,W$$

into two parts:

$$(1 - w)\,W_1,$$

containing terms independent of ε, and

$$(1 - w)\,V$$

which contains ε explicitly. W_1 contains the quantity v which may be considered constant. Set

$$\theta = \theta_1 + (1 - w)\zeta \tag{34}$$

where θ_1 refers to the terms independent of ε and ζ takes the place of θ in those terms which contain both ε and θ. We can now write (33) in the form

$$
\begin{aligned}
\frac{d\theta_1}{dg} &= (1 - w)\,\overline{W}_1 \\
\frac{d\zeta}{dg} &= \overline{V}
\end{aligned}
\tag{35}
$$

BOHLIN, in a very ingenious way, finds the function

$$W_1 = \frac{\varkappa}{1 - w} - \Re \left\{ \frac{f_2}{2\varkappa} e^{\sqrt{-1} F_2} \vartheta_1^2 + \left(\frac{v_2}{2\varkappa} e^{\sqrt{-1} v_2} \vartheta_1^2 - \frac{v'_2}{2\varkappa} e^{\sqrt{-1} v'_2} \vartheta_1^{-2} \right) (v + \eta) \right\}. \qquad (36)$$

The numbers f_2, F_2, v_2, V_2, v'_2, V'_2 are defined by the relations

$$\begin{aligned} f_2 e^{\sqrt{-1} F_2} \vartheta_1^2 &= (F) - \eta[(H) - (G')] \\ v_2 e^{\sqrt{-1} V_2} \vartheta_1^2 + v'_2 e^{\sqrt{-1} V'_2} \vartheta_1^{-2} &= -[(H) - (G')] \end{aligned} \qquad (37)$$

(F), (G') and (H) include all the corresponding terms of the perturbative function which are independent of ε, and which would consequently have a zero integration divisor. The numbers marked prime (′) have had the sign of the exponent of ϑ changed.

Changing ψ to ε in (36) we have

$$\overline{W} = \frac{\varkappa}{1 - w} - \Re \left\{ \frac{f_2}{2\varkappa} e^{\sqrt{-1} F_2} \vartheta_1^2 + \right.$$
$$\left. \left(\frac{v_2}{2\varkappa} e^{\sqrt{-1} V_2} \vartheta_1^2 - \frac{v'_2}{2\varkappa} e^{\sqrt{-1} V'_2} \vartheta_1^{-2} \right) \left(x + \eta x^2 + \frac{3}{2} \eta^2 x^3 - \eta^3 x - \frac{1}{2} \eta^2 x^{-1} \right\}. \qquad (38)$$

The first power of ϑ_1 does not appear in this problem. In the above equations

$$\vartheta_1 = e^{\sqrt{-1}\theta_1}, \quad x = e^{\sqrt{-1} g}, \quad \eta = \frac{e}{2}, \quad \eta' = \frac{e'}{2}, \quad \varkappa = w. \qquad (39)$$

Substituting the value of $\overline{W}_1$ from (38) in the first of (35) we have

$$\begin{aligned} \frac{1}{1 - w} \frac{d\theta_1}{dg} = \frac{\varkappa}{1 - w} &- \frac{f_2}{2\varkappa} \cos(2\theta_1 + F_2) + \frac{v_2}{2\varkappa} \cos(g + 2\theta_1 + V_2) - \frac{v'_2}{2\varkappa} \cos(g - 2\theta_1 + V'_2) \\ &+ \eta \frac{v_2}{2\varkappa} \cos(2g + 2\theta_1 + V_2) - \eta \frac{v'_2}{2\varkappa} \cos(2g - 2\theta_1 + V'_2) \\ &+ \frac{3}{2} \eta^2 \frac{v_2}{2\varkappa} \cos(3g + 2\theta_1 + V_2) - \eta^2 \frac{v_2}{2\varkappa} \cos(g + 2\theta_1 + V_2) - \\ &- \frac{1}{2} \eta^2 \frac{v_2}{2\varkappa} \cos(-g + 2\theta_1 + V_2). \end{aligned} \qquad (40)$$

Equation (40) can be integrated only by successive approximations. Neglecting all but the first term in the right member we have

$$\theta_1 = \varkappa g + G$$

where G is the constant of integration. Substituting this value of θ_1 in (40) and integrating again, we obtain

$$\frac{\theta_1}{1-w} = \frac{\Theta}{1-w} - \frac{f_2}{4\varkappa^2}\sin(2\Theta + F_2)$$

$$+ \frac{V_2}{2\varkappa(1+2\varkappa)}\sin(g+2\Theta+V_2) - \frac{V'_2}{2\varkappa(1-2\varkappa)}\sin(g-2\Theta+V'_2)$$

$$+ \eta\frac{V_2}{2\varkappa(2+2\varkappa)}\sin(2g+2\Theta+V_2) - \eta\frac{V'_2}{2\varkappa(2-2\varkappa)}\sin(2g-2\Theta+V'_2)$$

$$+ \frac{3}{2}\eta^2\frac{V_2}{2\varkappa(3+2\varkappa)}\sin(3g+2\Theta+V_2) \tag{41}$$

$$- \eta^2\frac{V_2}{2\varkappa(1+2\varkappa)}\sin(g+2\Theta+V_2)$$

$$- \frac{1}{2}\eta^2\frac{V_2}{2\varkappa(1-2\varkappa)}\sin(g-2\Theta-V_2)$$

where

$$\Theta = \varkappa g + G.$$

Writing equation (41) in the exponential form and eliminating x by means of the relations

$$x^\lambda = y^\lambda\left[1 - \lambda\eta(y - y^{-1}) + \frac{\lambda^2\eta^2}{2}(y^2 - 2 + y^{-2}) - \cdots\right] \tag{42}$$

and rejecting all terms higher than the second degree in the eccentricity, we have

$$\begin{aligned}
\frac{\theta_1}{1-w} = \frac{\Theta}{1-w} + \mathfrak{J}\Bigg[&\left(-\eta\frac{f_2}{2\varkappa}e^{\sqrt{-1}(2G+F_2)} + \frac{\eta^2}{2(1+2\varkappa)}\frac{V_2}{2\varkappa}e^{\sqrt{-1}(2G+V_2)}\right)y^{-1+2\varkappa} \\
&+ \left(-\frac{f_2}{4\varkappa^2}e^{\sqrt{-1}(2G+F_2)} + \eta\frac{V_2}{2\varkappa}e^{\sqrt{-1}(2G+V_2)}\right)y^{2\varkappa} \\
&+ \left(\frac{1-\eta^2}{1+2\varkappa}\cdot\frac{V_2}{2\varkappa}e^{\sqrt{-1}(2G+V_2)} + \eta\frac{f_2}{2\varkappa}e^{\sqrt{-1}(2G+F_2)}\right)y^{1+2\varkappa} \\
&- \eta\frac{1+2\varkappa}{2+2\varkappa}\cdot\frac{V_2}{2\varkappa}e^{\sqrt{-1}(2G+V_2)}\cdot y^{2+2\varkappa} + \frac{3}{2}\frac{\eta^2}{3+2\varkappa}\frac{V_2}{2\varkappa}e^{\sqrt{-1}(2G+V_2)}\cdot y^{3+2\varkappa} \\
&- \eta\frac{V'_2}{2\varkappa}e^{\sqrt{-1}(-2G+V'_2)}\cdot y^{-2\varkappa} \\
&- \left(\frac{1}{1-2\varkappa}\frac{V'_2}{2\varkappa}e^{\sqrt{-1}(-2G+V'_2)} + \frac{\eta^2}{2(1-2\varkappa)}\cdot\frac{V_2}{2\varkappa}e^{\sqrt{-1}(-2G-V_2)}\right)y^{1-2\varkappa} \\
&+ \eta\frac{1-2\varkappa}{2-2\varkappa}\frac{V'_2}{2\varkappa}e^{\sqrt{-1}(-2G+V'_2)}\cdot y^{2-2\varkappa}\Bigg].
\end{aligned} \tag{43}$$

$V = l\varepsilon + m\varepsilon\cos\psi + n\varepsilon\sin\psi$

$$+ \Re \Sigma \left\{ \begin{array}{l} \left[\tilde{F}_{0pq} + \frac{y}{v} \tilde{G}_{0pq} + \frac{v}{y} \tilde{H}_{0pq} \right] \\ \left[\tilde{F}_{1pq} + \frac{y}{v} \tilde{G}_{1pq} + \frac{v}{y} \tilde{H}_{1pq} \right] w \\ \left[\tilde{F}_{2pq} + \frac{y}{v} \tilde{G}_{2pq} + \frac{v}{y} \tilde{H}_{2pq} \right] w^2 \end{array} \right\} y^{n+r-(n-s)\mu} \vartheta_1^{(n-s)\mu} \tag{44}$$

$$- \Re \left[(F) + \frac{1}{v}(G) + v(H) \right] \eta (y - y^{-1})$$

where $\tilde{F}$, $\tilde{G}$ and $\tilde{H}$ come from integrating F, G and H, and $l + m \cos \psi + n \sin \psi$ represent the terms independent of ε and θ.

Changing ψ to ε and multiplying both sides of (44) by

$$1 - e \cos \varepsilon$$

we obtain

$$\begin{aligned} \overline{V}(1 - e \cos \varepsilon) = & (l - m\eta) \varepsilon + (m - 2 l\eta) \varepsilon \cos \varepsilon + n\varepsilon \sin \varepsilon \\ & + m\eta \varepsilon \cos 2\varepsilon - n\eta\varepsilon \sin 2\varepsilon \\ & + \Re \Sigma \left\{ \begin{array}{l} A_{0pq}(n + r \,.\, - n + s) \\ A_{1pq}(n + r \,.\, - n + s) \,.\, w \\ A_{2pq}(n + r \,.\, - n + s) \,.\, w^2 \end{array} \right\} y^{n+r-(n-s)\mu} \vartheta_1^{(n-s)\mu} \\ & - \Re \{ + \eta (y - y^{-1})(F) + \eta(1 - y^{-2})(G) + \eta(y^2 - 1)(H) \} \end{aligned} \tag{45}$$

where

$$\overline{A}_{pq}(n + r \,.\, - n + s) = \tilde{F}_{pq}(n + r \,.\, - n + s) + \tilde{G}_{pq}(n + r \,.\, - n + s) + \tilde{H}_{pq}(n + r \,.\, - n + s)$$

and (46)

$$A_{pq}(n + r \,.\, - n + s) = \overline{A}_{pq}(n + r \,.\, - n + s) - \overline{A}_{p-1\,q}(n + r + 1 \,.\, - n + s) - \\ - \overline{A}_{p-1\,q}(n + r - 1 \,.\, - n + s).$$

Equation (45) contains terms (A_1) which are independent of ε. Neglecting those of the third order these terms will be given by the equation

$$\Re (A_1) = \Re \{ (A) + \eta [(H) - (G')] + \eta [(H) + (G')] w \}. \tag{47}$$

The differential equation for ζ (35) now becomes

$$\frac{d\zeta}{d\varepsilon} = \overline{V}(1 - e \cos \varepsilon) - (1 - w) \frac{\partial \zeta}{\partial \theta_1} \overline{W}_1 (1 - e \cos \varepsilon) - (1 - w) \frac{\partial \zeta_\theta}{\partial \theta_1} \overline{W}_1 (1 - e \cos \varepsilon). \tag{48}$$

The first and second parts of the right member are functions of ε and can be integrated immediately. The coefficient of $(1 - e \cos \varepsilon)$ of the third part is independent of ε, hence we may write

$$(1 - w) \overline{W}_1 \frac{d\zeta_\theta}{d\theta_1} = \Re (A_1) = a_0 + \Re [a_2 e^{\sqrt{-1} A_2} \vartheta_1^2], \tag{49}$$

from which is derived by a process similar to that of (43)

$$\zeta_\theta = a_0 g + \mathfrak{I}\frac{a_2}{2\varkappa} e^{\sqrt{-1}(A_2+G)} \cdot y^{2\varkappa}. \tag{50}$$

Let

$$n\delta z = (n\delta z)_1 + (n\delta z)_2 + (n\delta z)_3 + (n\delta z)_4 \tag{51}$$

where $(n\delta z)_1$ includes the secular terms, $(n\delta z)_2$ those which are functions of θ_1 alone, $(n\delta z)_3$ those which are functions of both ε and θ_1, and $(n\delta z)_4$ the supplementary terms. Of the last named I have retained only those of the first power of the eccentricity.

Collecting the results we have

$$\begin{aligned}(n\delta z)_1 = {} & \frac{l' - m\eta}{2}\varepsilon^2 + (m - 2l\eta)\varepsilon\sin\varepsilon - n\varepsilon\cos\varepsilon \\ & - \frac{m\eta}{2}\varepsilon\sin 2\varepsilon + \frac{n\eta}{2}\varepsilon\cos 2\varepsilon \\ & + (m - 2l\eta)\cos\varepsilon + n\sin\varepsilon \\ & - \frac{m\eta}{4}\cos 2\varepsilon - \frac{n\eta}{4}\sin 2\varepsilon\end{aligned} \tag{52}$$

$$\begin{aligned} l &= \text{constant part of } \mathfrak{I}(F) \\ m &= \quad » \qquad » \qquad » \quad \mathfrak{I}[(H) - (G')] \\ n &= \quad » \qquad » \qquad » \quad \mathfrak{R}[(H) - (G')]\end{aligned} \tag{53}$$

$$\begin{aligned}(n\delta z)_2 = {} & \frac{\Theta}{1 - w} + a_0 g \\ & + r_0^{(2)}\sin(2\Theta + R_0^{(2)}) \\ & + r_1^{(2)}\sin(2\Theta + \varepsilon + R_1^{(2)}) \\ & + r_2^{(2)}\sin(2\Theta + 2\varepsilon + R_2^{(2)}) \\ & + r_3^{(2)}\sin(2\Theta + 3\varepsilon + R_3^{(2)}) \\ & + r_{-0}^{(2)}\sin(2\Theta + R_{-0}^{(2)}) \\ & + r_{-1}^{(2)}\sin(2\Theta - \varepsilon + R_{-1}^{(2)}) \\ & + r_{-2}^{(2)}\sin(2\Theta - 2\varepsilon + R_{-2}^{(2)}).\end{aligned} \tag{54}$$

The above coefficients are determined from the following formulae:

$$\begin{aligned} r_0^{(2)}\cos R_0^{(2)} &= -\frac{f_2}{4\varkappa^2}\cos F_2 + \eta\frac{\mathrm{V}_2}{2\varkappa}\cos \mathrm{V}_2 + \frac{a_2}{2\varkappa}\cos A_2 \\ r_0^{(2)}\sin R_0^{(2)} &= -\frac{f_2}{4\varkappa^2}\sin F_2 + \eta\frac{\mathrm{V}_2}{2\varkappa}\sin \mathrm{V}_2 + \frac{a_2}{2\varkappa}\sin A_2\end{aligned} \tag{55}$$

$$
\begin{aligned}
r_1^{(2)} \cos R_1^{(2)} &= \frac{1-\eta^2}{1+2\varkappa} \cdot \frac{v_2}{2\varkappa} \cos V_2 + \eta \frac{f_2}{2\varkappa} \cos F_2 \\
r_1^{(2)} \sin R_1^{(2)} &= \frac{1-\eta^2}{1+2\varkappa} \cdot \frac{v_2}{2\varkappa} \sin V_2 + \eta \frac{f_2}{2\varkappa} \sin F_2 \\
r_2^{(2)} \cos R_2^{(2)} &= -\eta \frac{1+2\varkappa}{2+2\varkappa} \cdot \frac{v_2}{2\varkappa} \cos V_2 \\
r_2^{(2)} \sin R_2^{(2)} &= -\eta \frac{1+2\varkappa}{2+2\varkappa} \cdot \frac{v_2}{2\varkappa} \sin V_2 \\
r_3^{(2)} \cos R_3^{(2)} &= \frac{3}{2} \frac{\eta^2}{3+2\varkappa} \cdot \frac{v_2}{2\varkappa} \cos V_2 \\
r_3^{(2)} \sin R_3^{(2)} &= \frac{3}{2} \frac{\eta^2}{3+2\varkappa} \cdot \frac{v_2}{2\varkappa} \sin V_2 \\
r_{-0}^{(2)} \cos R_{-0}^{(2)} &= +\eta \frac{v'_2}{2\varkappa} \cos V'_2 \\
r_{-0}^{(2)} \sin R_{-0}^{(2)} &= -\eta \frac{v'_2}{2\varkappa} \sin V'_2 \\
r_{-1}^{(2)} \cos R_{-1}^{(2)} &= + \frac{1}{1-2\varkappa} \cdot \frac{v'_2}{2\varkappa} \cos V'_2 - \eta \frac{f_2}{2\varkappa} \cos F_2 + \frac{\eta^2}{1-4\varkappa^2} \cdot \frac{v_2}{2\varkappa} \cos V_2 \\
r_{-1}^{(2)} \sin R_{-1}^{(2)} &= - \frac{1}{1-2\varkappa} \cdot \frac{v'_2}{2\varkappa} \sin V'_2 - \eta \frac{f_2}{2\varkappa} \sin F_2 + \frac{\eta^2}{1-4\varkappa^2} \cdot \frac{v^2}{2\varkappa} \sin V_2 \\
r_{-2}^{(2)} \cos R_{-2}^{(2)} &= -\eta \cdot \frac{1-2\varkappa}{2-2\varkappa} \cdot \frac{v'_2}{2\varkappa} \cos V'_2 \\
r_{-2}^{(2)} \sin R_{-2}^{(2)} &= +\eta \cdot \frac{1-2\varkappa}{2-2\varkappa} \cdot \frac{v'_2}{2\varkappa} \sin V'_2
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
(n\delta z)_3 = &+ \Im \Sigma R_{0\,pq}(n+r\,.\,-n+s)\, y^{n+r-(n-s)\mu}\, \vartheta_1^{(n-s)\mu} \\
&+ \Im \Sigma R_{1\,pq}(n+r\,.\,-n+s)\, y^{n+r-(n-s)\mu}\, \vartheta_1^{(n-s)\mu}\,.\, w \\
&+ \Im \Sigma R_{2\,pq}(n+r\,.\,-n+s)\, y^{n+r-(n-s)\mu}\, \vartheta_1^{(n-s)\mu}\,.\, w^2.
\end{aligned}
\tag{56}
$$

The subscripts p and q indicate the powers of η and η' by which each term is multiplied.

$$
(n\delta z)_4 = -\Im \left\{ +\eta(y+y^{-1})\,(F) + \frac{\eta}{2} y^{-2}(G) + \frac{\eta}{2} y^2 (H) \right\} \cdot \tag{57}
$$

Inequalities of the Radius Vector.

Breaking up ~~V~~ ν into two parts as in the case of $n\delta z$, we have from (44)

$$\frac{\partial \overline{V}}{\partial \psi} = - m\varepsilon \sin \varepsilon + n\varepsilon \cos \varepsilon$$

$$+ \Im \Sigma \left\{ \begin{array}{l} B_{0\,pq}(n + r\,.\, - n + s) \\ B_{1\,pq}(n + r\,.\, - n + s)\,.\,w \\ B_{2\,pq}(n + r\,.\, - n + s)\,.\,w^2 \end{array} \right\} y^{n+r-(n-s)\mu}\, \vartheta_1^{(n-s)\mu} \tag{58}$$

$$+ \Im \{ - \eta(1 - y^{-2})(G) + \eta(y^2 - 1)(H)\},$$

where the coefficients have the following values:

$$B_{pq}(n + r\,.\, - n + s) = \tilde{G}_{pq}(n + r\,.\, - n + s) - \tilde{H}_{pq}(n + r\,.\, - n + s)\,. \tag{59}$$

Differentiating (36) with respect to ψ and then writing ε for ψ we have

$$- \frac{\partial \overline{W}_1}{\partial \psi} = \Im \left[\frac{\mathrm{v}_2}{2\varkappa} e^{\sqrt{-1}\,\mathrm{V}_2}\,.\, y\vartheta_1^2 - \frac{\mathrm{v}'_2}{2\varkappa} e^{\sqrt{-1}\,\mathrm{V}'_2}\,.\, y\vartheta_1^{-2} \right]. \tag{60}$$

Integrating (~~57~~ 58) and (~~59~~ 60) in a manner similar to that of $n\delta z$, and placing

$$\nu = (\nu)_1 + (\nu)_2 + (\nu)_3 + (\nu)_4 \tag{61}$$

we have from (13)

$$\begin{aligned} 2\,(\nu)_1 = &- m\varepsilon \cos \varepsilon - n\varepsilon \sin \varepsilon \\ &- n \cos \varepsilon + m \sin \varepsilon \end{aligned} \tag{62}$$

$$\begin{aligned} 2\,(\nu)_2 = &- b_0 g \\ &+ s^{(2)} \cos (2\,\Theta + S^{(2)}) \\ &+ s_1^{(2)} \cos (2\,\Theta + \varepsilon + S_1^{(2)}) \\ &+ s_2^{(2)} \cos (2\,\Theta + 2\,\varepsilon + S_2^{(2)}) \end{aligned} \tag{63}$$

where

$$\begin{aligned} s^{(2)} \cos S^{(2)} &= - \eta \frac{\mathrm{v}_2}{2\varkappa} \cos \mathrm{V}_2 + \frac{b_2}{2\varkappa} \cos B_2 \\ s^{(2)} \sin S^{(2)} &= - \eta \frac{\mathrm{v}_2}{2\varkappa} \sin \mathrm{V}_2 + \frac{b_2}{2\varkappa} \sin B_2 \\ s_1^{(2)} \cos S_1^{(2)} &= - \frac{1}{1 + 2\varkappa} \cdot \frac{\mathrm{v}_2}{2\varkappa} \cos \mathrm{V}_2 + \eta \frac{f_2}{2\varkappa} \cos F_2 \\ s_1^{(2)} \sin S_1^{(2)} &= - \frac{1}{1 + 2\varkappa} \cdot \frac{\mathrm{v}_2}{2\varkappa} \sin \mathrm{V}_2 + \eta \frac{f_2}{2\varkappa} \sin F_2 \end{aligned} \tag{64}$$

$$
\begin{aligned}
s_2^{(2)} \cos S_2^{(2)} &= \eta \frac{\mathrm{v}_2}{2+2\varkappa} \cos \mathrm{V}_2 \\
s_2^{(2)} \sin S_2^{(2)} &= \eta \frac{\mathrm{v}_2}{2+2\varkappa} \sin \mathrm{V}_2
\end{aligned}
\tag{64}
$$

The numbers b_0, b_2 and B_2 are determined from the relation

$$
b_0 + \mathfrak{I}[b_2 e^{\sqrt{-1}B_2} \vartheta_1^2] = \mathfrak{I}(B_1) = \mathfrak{I}\{(B) - \eta[(H) - (G')] - \eta[(H) + (G')]\xi w\}
\tag{65}
$$

$$
\begin{aligned}
2(\nu)_3 = &+ \mathfrak{R}\Sigma S_{0\,pq}(n+r\,.-n+s)\, y^{n+r-(n-s)\mu} \vartheta_1^{(n-s)\mu} \\
&+ \mathfrak{R}\Sigma S_{1\,pq}(n+r\,.-n+s)\, y^{n+r-(n-s)\mu} \vartheta_1^{(n-s)\mu}\,.\,w \\
&+ \mathfrak{R}\Sigma S_{2\,pq}(n+r\,.-n+s)\, y^{n+r-(n-s)\mu} \vartheta_1^{(n-s)\mu}\,.\,w^2
\end{aligned}
\tag{66}
$$

$$
2(\nu)_4 = + \mathfrak{R}\left\{-\frac{\eta}{2} y^{-2}(G) + \frac{\eta}{2} y^2 (H)\right\}
\tag{67}
$$

Inequalities of the third coordinate.

From (16) and (26) we have

$$
\sec i\ \frac{dU}{d\varepsilon} = -\mathfrak{R}\Sigma \left\{
\begin{array}{l}
[F_{0\,pq} + \frac{y}{v} G_{0\,pq} + \frac{v}{y} H_{0\,pq}] \\
[F_{1\,pq} + \frac{y}{v} G_{1\,pq} + \frac{v}{y} H_{1\,pq}]w \\
[F_{2\,pq} + \frac{y}{v} G_{2\,pq} + \frac{v}{y} H_{2\,pq}]w^2
\end{array}
\right\} y^{n+r-(n-s)\mu} \vartheta_1^{(n-s)\mu}
\tag{68}
$$

Let

$$
U = V + U_1
\tag{69}
$$

where V contains ε explicitly and U_1 is a function of θ_1 alone. From (16), (26) and (35) we get

$$
\sec i\ \frac{dU_1}{d\theta_1}(1-w)\overline{W}_1 = -\mathfrak{R}[v^{-1}(G) + v(H)]
\tag{70}
$$

and for the determination of V

$$
\sec i\ \frac{\partial V}{\partial \varepsilon} = Qa^2 \frac{\partial \Omega}{\partial Z} - (1-w)\sec i\ \frac{\partial V}{\partial \theta_1} \overline{W}_1 (1 - e\cos\varepsilon).
\tag{71}
$$

From (~~69~~ 70) is obtained by a process similar to that of (43)

$$
\sec i\ U_1 = -\mathfrak{I}\left\{v\left[\frac{\mathrm{v}_2}{2\varkappa} e^{\sqrt{-1}(\mathrm{V}_2+2G)} y^{2\varkappa} - \eta \mathrm{v}_2 e^{\sqrt{-1}(\mathrm{V}_2+2G)}\,.\,y^{1+2\varkappa} + \eta \mathrm{v}_2 e^{\sqrt{-1}(\mathrm{V}_2+2G)} y^{-1+2\varkappa}\right]\right\}
\tag{72}
$$

where v_2 and V_2 are determined from the relation

$$
\mathrm{v}_2 e^{\sqrt{-1}\mathrm{V}_2} \vartheta_1^2 = (H) + (G').
\tag{73}
$$

The integration of (71) gives

$$\sec i \;\; V = -m\varepsilon\cos\psi - n\varepsilon\sin\psi$$

$$+\Im\Sigma\left\{\begin{array}{l}\left[\tilde{F}_{0\,pq}+\frac{y}{v}\tilde{G}_{0\,pq}+\frac{v}{y}\tilde{H}_{0\,pq}\right]\\ \left[\tilde{F}_{1\,pq}+\frac{y}{v}\tilde{G}_{1\,pq}+\frac{v}{y}\tilde{H}_{1\,pq}\right]w\\ \left[\tilde{F}_{2\,pq}+\frac{y}{v}\tilde{G}_{2\,pq}+\frac{v}{y}\tilde{H}_{2\,pq}\right]w^2\end{array}\right\}y^{n+r-(n-s)\mu}\,\vartheta_1^{(n-s)\mu} \tag{74}$$

where m and n are defined by the relations

$$\begin{aligned} m &= \Re[(H)+(G')] \\ n &= -\Im[(H)+(G')]. \end{aligned} \tag{75}$$

Collecting the values of V and U_1 and differentiating with respect to ψ, we have, after changing ψ to ε,

$$\sec i\;\frac{\partial\overline{U}}{\partial\psi} = m\varepsilon\sin\varepsilon - n\varepsilon\cos\varepsilon$$

$$-\Re\left[\frac{\mathrm{v}_2}{2\varkappa}e^{\sqrt{-1}(\mathrm{v}_2+2G)}\,.\,y^{1+2\varkappa} - \eta\mathrm{v}_2e^{\sqrt{-1}(\mathrm{v}_2+2G)}\,.\,y^{2+2\varkappa} + \eta\mathrm{v}_2e^{\sqrt{-1}(\mathrm{v}_2+2G)}\,.\,y^{2\varkappa}\right]$$

$$+\Re\Sigma\left\{\begin{array}{l}C_{0\,pq}(n+r\,.-n+s)\\ C_{1\,pq}(n+r\,.-n+s)w\\ C_{2\,pq}(n+r\,.-n+s)w^2\end{array}\right\}y^{n+r-(n-s)\mu}\,\vartheta_1^{(n-s)\mu} \tag{76}$$

where

$$C_{pq}(n+r\,.-n+s) = -\tilde{G}_{pq}(n+r\,.-n+s)+\tilde{H}_{pq}(n+r\,.-n+s). \tag{77}$$

Equation (14) can now be integrated. The part independent of ε arising from the integration will be given by

$$s_0 = c_0 g + \frac{c_2}{2\varkappa}\sin(2\Theta + C_2). \tag{78}$$

For convenience let

$$\frac{\mu}{\cos i} = s = (s)_1 + (s)_2 + (s)_3. \tag{79}$$

Collecting results we have

$$\begin{aligned} (s)_1 &= -m\varepsilon\cos\varepsilon - n\varepsilon\sin\varepsilon \\ &\quad - n\cos\varepsilon + m\sin\varepsilon \end{aligned} \tag{80}$$

$$\begin{aligned}(s)_2 = c_0 g & \\ & + t^{(2)} \sin (2\,\Theta + T^{(2)}) \\ & + t_1^{(2)} \sin (2\,\Theta + \varepsilon + T_1^{(2)}) \\ & + t_2^{(2)} \sin (2\,\Theta + 2\,\varepsilon + T_2^{(2)})\end{aligned} \tag{81}$$

where

$$t^{(2)} \cos T^{(2)} = -\eta \frac{\mathrm{v}_2}{2\,\varkappa} \cos \mathrm{V}_2 + \frac{c_2}{2\,\varkappa} \cos C_2$$

$$t^{(2)} \sin T^{(2)} = -\eta \frac{\mathrm{v}_2}{2\,\varkappa} \sin \mathrm{V}_2 + \frac{c_2}{2\,\varkappa} \sin C_2$$

$$\begin{aligned}t_1^{(2)} \cos T_1^{(2)} &= -\frac{1}{1 + 2\,\varkappa} \frac{\mathrm{v}_2}{2\,\varkappa} \cos \mathrm{V}_2 \\ t_1^{(2)} \sin T_1^{(2)} &= -\frac{1}{1 + 2\,\varkappa} \frac{\mathrm{v}_2}{2\,\varkappa} \sin \mathrm{V}_2\end{aligned} \tag{82}$$

$$t_2^{(2)} \cos T_2^{(2)} = \eta \frac{\mathrm{v}_2}{2 + 2\,\varkappa} \cos \mathrm{V}_2$$

$$t_2^{(2)} \sin T_2^{(2)} = \eta \frac{\mathrm{v}_2}{2 + 2\,\varkappa} \sin \mathrm{V}_2$$

c_0, c_2 and C_2 are determined from the relation

$$\Re(C_1) = c_0 + \Re[c_2 e^{\sqrt{-1}\,C_2} \vartheta_1^2] = \Re\{(C) + \eta[(H) + (G')]\} \tag{83}$$

$$\begin{aligned}(s)_3 = & + \Im \Sigma T_{0\,pq}(n + r\,.\, - n + s) y^{n+r-(n-s)\mu} \vartheta_1^{(n-s)\mu} \\ & + \Im \Sigma T_{1\,pq}(n + r\,.\, - n + s) y^{n+r-(n-s)\mu} \vartheta_1^{(n-s)\mu}\,.\,w \\ & + \Im \Sigma T_{2\,pq}(n + r\,.\, - n + s) y^{n+r-(n-s)\mu} \vartheta_1^{(n-s)\mu}\,.\,w^2.\end{aligned} \tag{84}$$

I give here a brief summary of the formulae necessary for the computation of the Jupiter perturbations of those small planets whose mean daily motions are in the neighborhood of 750″. From the given elements of the planet and from those of Jupiter we first determine Π, Π' and J by means of the formulae:

$$\begin{aligned}\sin \frac{1}{2} J \sin \frac{1}{2} (\Psi + \Phi) &= \sin \frac{1}{2} (\Omega - \Omega') \sin \frac{1}{2} (i + i') \\ \sin \frac{1}{2} J \cos \frac{1}{2} (\Psi + \Phi) &= \cos \frac{1}{2} (\Omega - \Omega') \sin \frac{1}{2} (i - i') \\ \cos \frac{1}{2} J \sin \frac{1}{2} (\Psi - \Phi) &= \sin \frac{1}{2} (\Omega - \Omega') \cos \frac{1}{2} (i + i') \\ \cos \frac{1}{2} J \cos \frac{1}{2} (\Psi - \Phi) &= \cos \frac{1}{2} (\Omega - \Omega') \cos \frac{1}{2} (i - i').\end{aligned} \tag{85}$$

Check formula

$$\frac{\sin \Psi}{\sin i} = \frac{\sin \Phi}{\sin i'} = \frac{\sin (\Omega - \Omega')}{\sin J} \tag{86}$$

$$\begin{aligned}
&\Pi = \pi - \Omega - \Phi && \Pi' = \pi' - \Omega' - \Psi \\
&\Delta = \Pi - \Pi' && \Sigma = \Pi + \Pi' \\
&e = \sin \varphi && e' = \sin \varphi' \\
&\eta = \frac{e}{2} && \eta' = \frac{e'}{2} \\
&j^2 = \sin^2 \tfrac{1}{2} J \cos^2 \frac{1}{2}\varphi \cos^2 \frac{1}{2}\varphi' && \iota = \sin J \cos^2 \frac{1}{2}\varphi' \\
&w = \frac{2 - 5\mu}{2} = \frac{2n - 5n'}{2n} && \varkappa = w.
\end{aligned} \tag{87}$$

The symbols i, Ω, Π and i', Ω', Π' represent the inclination, longitude of ascending node, and longitude of the perihelion of the disturbed planet and of Jupiter respectively. J is the mutual inclination of the orbits of the two planets. Φ and Ψ are the distances in their orbits from the ecliptic to the ascending node of one planet on the other.

The numbers l, m, n, f_2, F_2, v_2, V_2, v'_2, V'_2, a_0, a_2, A_2, b_0, b_2, B_2 are determined from the equations

$$\begin{aligned}
l &= \text{constant part of } \Im(F) \\
m &= \quad » \quad » \quad » \ \Im[(H) - (G')] \\
n &= \quad » \quad » \quad » \ \Re[(H) - (G')]
\end{aligned} \tag{53}$$

$$\begin{aligned}
f_2 e^{\sqrt{-1} F_2} \vartheta_1^2 &= (F) - \eta[(H) - (G')] \\
v_2 e^{\sqrt{-1} V_2} \vartheta_1^2 + v'_2 e^{\sqrt{-1} V'_2} \vartheta_1^{-2} &= -[(H) - (G')]
\end{aligned} \tag{37}$$

$$a_0 + \Re[a_2 e^{\sqrt{-1} A_2} \vartheta_1^2] = \Re\{(A) + \eta[(H) - (G')] + \eta[(H) + (G')]w\} \tag{47), (49}$$

$$b_0 + \Im[b_2 e^{\sqrt{-1} B_2} \vartheta_1^2] = \Im\{(B) - \eta[(H) - (G')] - \eta[(H) + (G')]w\} \tag{65}$$

$$r_0^{(2)} \cos R_0^{(2)} = -\frac{f_2}{4\varkappa^2} \cos F_2 + \eta \frac{v_2}{2\varkappa} \cos V_2 + \frac{a_2}{2\varkappa} \cos A_2$$

$$r_0^{(2)} \sin R_0^{(2)} = -\frac{f_2}{4\varkappa^2} \sin F_2 + \eta \frac{v_2}{2\varkappa} \sin V_2 + \frac{a_2}{2\varkappa} \sin A_2$$

$$r_1^{(2)} \cos R_1^{(2)} = \frac{1 - \eta^2}{1 + 2\varkappa} \cdot \frac{v_2}{2\varkappa} \cos V_2 + \eta \frac{f_2}{2\varkappa} \cos F_2$$

$$r_1^{(2)} \sin R_1^{(2)} = \frac{1 - \eta^2}{1 + 2\varkappa} \cdot \frac{v_2}{2\varkappa} \sin V_2 + \eta \frac{f_2}{2\varkappa} \sin F_2$$

$$r_2^{(2)} \cos R_2^{(2)} = -\eta \frac{1+2\varkappa}{2+2\varkappa} \cdot \frac{v_2}{2\varkappa} \cos V_2$$

$$r_2^{(2)} \sin R_2^{(2)} = -\eta \frac{1+2\varkappa}{2+2\varkappa} \cdot \frac{v_2}{2\varkappa} \sin V_2 \qquad (55)$$

$$r_3^{(2)} \cos R_3^{(2)} = \frac{3}{2} \frac{\eta^2}{3+2\varkappa} \cdot \frac{v_2}{2\varkappa} \cos V_2$$

$$r_3^{(2)} \sin R_3^{(2)} = \frac{3}{2} \frac{\eta^2}{3+2\varkappa} \cdot \frac{v_2}{2\varkappa} \sin V_2$$

$$r_{-0}^{(2)} \cos R_{-0}^{(2)} = +\eta \frac{v'_2}{2\varkappa} \cos V'_2$$

$$r_{-0}^{(2)} \sin R_{-0}^{(2)} = -\eta \frac{v'_2}{2\varkappa} \sin V'_2$$

$$r_{-1}^{(2)} \cos R_{-1}^{(2)} = +\frac{1}{1-2\varkappa} \cdot \frac{v'_2}{2\varkappa} \cos V'_2 - \eta \frac{f_2}{2\varkappa} \cos F_2 + \frac{\eta^2}{1-4\varkappa^2} \cdot \frac{v_2}{2\varkappa} \cos V_2$$

$$r_{-1}^{(2)} \sin R_{-1}^{(2)} = -\frac{1}{1-2\varkappa} \cdot \frac{v'_2}{2\varkappa} \sin V'_2 - \eta \frac{f_2}{2\varkappa} \sin F_2 + \frac{\eta^2}{1-4\varkappa^2} \cdot \frac{v_2}{2\varkappa} \sin V_2$$

$$r_{-2}^{(2)} \cos R_{-2}^{(2)} = -\eta \cdot \frac{1-2\varkappa}{2-2\varkappa} \cdot \frac{v'_2}{2\varkappa} \cos V'_2$$

$$r_{-2}^{(2)} \sin R_{-2}^{(2)} = +\eta \cdot \frac{1-2\varkappa}{2-2\varkappa} \cdot \frac{v'_2}{2\varkappa} \sin V'_2$$

. . . .

$$s^{(2)} \cos S^{(2)} = -\eta \frac{v_2}{2\varkappa} \cos V_2 + \frac{b_2}{2\varkappa} \cos B_2$$

$$s^{(2)} \sin S^{(2)} = -\eta \frac{v_2}{2\varkappa} \sin V_2 + \frac{b_2}{2\varkappa} \sin B_2$$

$$s_1^{(2)} \cos S_1^{(2)} = -\frac{1}{1+2\varkappa} \cdot \frac{v_2}{2\varkappa} \cos V_2 + \eta \frac{f_2}{2\varkappa} \cos F_2$$

$$s_1^{(2)} \sin S_1^{(2)} = -\frac{1}{1+2\varkappa} \cdot \frac{v_2}{2\varkappa} \sin V_2 + \eta \frac{f_2}{2\varkappa} \sin F_2 \qquad (64)$$

$$s_2^{(2)} \cos S_2^{(2)} = \eta \frac{v_2}{2+2\varkappa} \cos V_2$$

$$s_2^{(2)} \sin S_2^{(2)} = \eta \frac{v_2}{2+2\varkappa} \sin V_2$$

$$(n\delta z)_1 = \frac{l - m\eta}{2}\varepsilon^2 + (m - 2\,l\eta)\varepsilon \sin\,\varepsilon - n\varepsilon\,\cos\,\varepsilon$$
$$- \frac{m\eta}{2}\varepsilon\,\sin\,2\,\varepsilon + \frac{n\eta}{2}\varepsilon\,\cos\,\varepsilon \tag{52}$$
$$+ (m - 2\,l\eta)\,\cos\,\varepsilon + n\,\sin\,\varepsilon$$
$$- \frac{m\eta}{4}\cos\,2\,\varepsilon - \frac{n\eta}{4}\sin\,2\,\varepsilon$$

$$(n\delta z)_2 = \frac{\Theta}{1 - w} + a_0 g$$
$$+ r_0^{(2)}\,\sin\,(2\,\Theta + R_0^{(2)})$$
$$+ r_1^{(2)}\,\sin\,(2\,\Theta + \varepsilon + R_1^{(2)})$$
$$+ r_2^{(2)}\,\sin\,(2\,\Theta + 2\,\varepsilon + R_2^{(2)}) \tag{54}$$
$$+ r_3^{(2)}\,\sin\,(2\,\Theta + 3\,\varepsilon + R_3^{(2)})$$
$$+ r_{-0}^{(2)}\,\sin\,(2\,\Theta + R_{-0}^{(2)})$$
$$+ r_{-1}^{(2)}\,\sin\,(2\,\Theta - \varepsilon + R_{-1}^{(2)})$$
$$+ r_{-2}^{(2)}\,\sin\,(2\,\Theta - 2\,\varepsilon + R_{-2}^{(2)})$$

$$(n\delta z) = \Sigma\,\eta^p\eta'^q \left\{\begin{matrix} R_{0\,pq}(n + r\,.\,- n + s) \\ R_{1\,pq}(n + r\,.\,- n + s)w \\ R_{2\,pq}(n + r\,.\,- n + s)w^2 \end{matrix}\right\} \sin\,\{[n + r - (n - s)\mu]\varepsilon + (n - s)\mu\theta_1 + n\Delta\} \tag{56}$$

$$(n\delta z)_4 = -\,\Im\left\{+\,\eta\,(y + y^{-1})(F) + \frac{\eta}{2}y^{-2}(G) + \frac{\eta}{2}y^2(H)\right\} \tag{57}$$

$$2\,(\nu)_1 = -\,m\varepsilon\,\cos\,\varepsilon - n\varepsilon\,\sin\,\varepsilon \tag{62}$$
$$-\,n\,\cos\,\varepsilon + m\,\sin\,\varepsilon$$

$$2\,(\nu)_2 = -\,b_0 g$$
$$+ s^{(2)}\,\cos\,(2\,\Theta + S^{(2)})$$
$$+ s_1^{(2)}\,\cos\,(2\,\Theta + \varepsilon + S_1^{(2)}) \tag{63}$$
$$+ s_2^{(2)}\,\cos\,(2\,\Theta + 2\,\varepsilon + S_2^{(2)})$$

$$2\,(\nu)_3 = \left\{\begin{matrix} S_{0\,pq}(n + r\,.\,- n + s) \\ S_{1\,pq}(n + r\,.\,- n + s)w \\ S_{2\,pq}(n + r\,.\,- n + s)w^2 \end{matrix}\right\} \cos\,\{[n + r - (n - s)\mu]\varepsilon + (n - s)\mu\theta_1 + n\Delta\} \tag{66}$$

$$2\,(\nu)_4 = +\,\Re\left\{-\frac{\eta}{2}y^{-2}(G) + \frac{\eta}{2}y^2(H)\right\}. \tag{67}$$

Inclination.

The numbers m, n, v_2, V_2, c_0, c_2, C_2 are determined from the equations

$$\begin{aligned} m &= \Re[(H) + (G')] \\ n &= -\Im[(H) + (G')] \end{aligned} \tag{75}$$

$$\mathrm{v}_2 e^{\sqrt{-1}\,\mathrm{V}_2}\, \vartheta_1^2 = (H) + (G) \tag{73}$$

$$c_0 + \Re[c_2 e^{\sqrt{-1}\,C_2}\, \vartheta_1^2] = \Re\{(C) + \eta[(H) + (G')]\} \tag{83}$$

$$\begin{aligned} t^{(2)} \cos T^{(2)} &= -\eta \frac{\mathrm{v}_2}{2\varkappa} \cos \mathrm{V}_2 + \frac{c_2}{2\varkappa} \cos C_2 \\ t^{(2)} \sin T^{(2)} &= -\eta \frac{\mathrm{v}_2}{2\varkappa} \sin \mathrm{V}_2 + \frac{c_2}{2\varkappa} \sin C_2 \\ t_1^{(2)} \cos T_1^{(2)} &= -\frac{1}{1+2\varkappa} \cdot \frac{\mathrm{v}_2}{2\varkappa} \cos \mathrm{V}_2 \\ t_1^{(2)} \sin T_1^{(2)} &= -\frac{1}{1+2\varkappa} \cdot \frac{\mathrm{v}_2}{2\varkappa} \sin \mathrm{V}_2 \\ t_2^{(2)} \cos T_2^{(2)} &= \eta \frac{v_2}{2+2\varkappa} \cos \mathrm{V}_2 \\ t_2^{(2)} \sin T_2^{(2)} &= \eta \frac{v_2}{2+2\varkappa} \sin \mathrm{V}_2 \end{aligned} \tag{82}$$

$$\begin{aligned} (s)_1 = &-m\varepsilon \cos \varepsilon - n\varepsilon \sin \varepsilon \\ &-n \cos \varepsilon + m \sin \varepsilon \end{aligned} \tag{80}$$

$$\begin{aligned} (s)_2 = \;& c_0 g \\ &+ t^{(2)} \sin (2\Theta + T^{(2)}) \\ &+ t_1^{(2)} \sin (2\Theta + \varepsilon + T_1^{(2)}) \\ &+ t_2^{(2)} \sin (2\Theta + 2\varepsilon + T_2^{(2)}) \end{aligned} \tag{81}$$

$$(s)_3 = \iota \Sigma \eta^p \eta'^q \begin{Bmatrix} T_{0pq}(n+r\,.-n+s)_{\pm\pi'} \\ T_{1pq}(n+r\,.-n+s)_{\pm\pi'} w \\ T_{2pq}(n+r\,.-n+s)_{\pm\pi'} w^2 \end{Bmatrix} \sin\{[n+r-(n-s)\mu]\varepsilon + (n-s)\mu\theta_1 + n\Delta \pm \Pi'\}. \tag{84}$$

Those coefficients which have the suffix $\pm\delta$ or $\pm\sigma$ must be multiplied by j^2 also, and the angles $\pm\Delta$ or $\pm\Sigma$ must be added to the arguments of those terms. The suffix $\pm\pi'$ in the inequalities of the third coordinate means that the angle $\pm\Pi'$ must be added to the argument of the corresponding term.

I.

	$n=0$	1	2	3	4	5	6	7
$\gamma_0^{1\,n}$	2.06574	[0.59732]	1.13380	0.79247	0.47106	0.1612	9.8590	9.5621
$\gamma_1^{1\,n}$	1.06500	0.69273	0.35746	0.03978	9.73249	9.4321	9.1366	8.8449
$\gamma_2^{1\,n}$	0.42958	0.09973	9.78544	9.48048	9.18181	8.8877	8.5969	8.3089
$\gamma_3^{1\,n}$	9.88887	9.57764	9.27483	8.97773	8.68480	8.3950	8.1078	7.8226
$\gamma_4^{1\,n}$	9.39177	9.09091	8.79529	8.50349	8.21464	7.9281	7.6436	7.3606
$\gamma_5^{1\,n}$	8.91975	8.62547	8.33474	8.04675	7.76094	7.4769	7.1945	6.9132
$\gamma_6^{1\,n}$	8.46402	8.17428	7.88708	7.60194	7.31849	7.0365	—	—
$\gamma_7^{1\,n}$	8.01971	7.73328	7.44875	7.16583	6.88425	6.6038	—	—
$\gamma_0^{3\,n}$	[1.56236]	1.69456	1.50801	1.30001	1.07985	0.8518	0.6182	0.3805
$\gamma_1^{3\,n}$	1.78961	1.59407	1.38101	1.15756	0.92722	0.6919	0.4529	0.2111
$\gamma_2^{3\,n}$	1.59882	1.38145	1.15514	0.92274	0.68592	0.4457	0.2029	9.9579
$\gamma_0^{5\,n}$	1.79707	1.75052	1.65093	1.51842	1.36373	1.1931	1.0106	0.8187
$\gamma_1^{5\,n}$	2.11521	1.99777	1.85376	1.69106	1.51459	1.3275	1.1322	0.9300
$\bar{\gamma}_0^{1\,n}$	1.96871	[1.07361]	1.48077	1.25522	1.02455	0.7895	0.5509	0.3095
$\bar{\gamma}_1^{1\,n}$	1.48287	1.21744	0.96467	0.71516	0.46626	0.2170	9.9672	9.7166
$\bar{\gamma}_2^{1\,n}$	1.08135	0.81592	0.55637	0.29938	0.04356	9.7882	9.5329	9.2776
$\bar{\gamma}_3^{1\,n}$	0.69264	0.42719	0.16513	9.90491	9.64574	9.3871	9.1289	8.8708
$\bar{\gamma}_4^{1\,n}$	0.30814	0.04271	9.77946	9.51755	9.25651	8.9960	8.7360	8.4761
$\bar{\gamma}_5^{1\,n}$	9.92556	9.66013	9.39621	9.13332	8.87113	8.6095	—	—
$\bar{\gamma}_6^{1\,n}$	9.54400	9.27857	9.01450	8.75072	8.48779	8.2253	—	—
$\bar{\gamma}_0^{3\,n}$	[2.19075]	2.26564	2.14435	1.99555	1.82842	1.6481	1.4578	1.2596
$\bar{\gamma}_1^{3\,n}$	2.49307	2.34491	2.17702	1.99555	1.80401	1.6047	1.3990	1.1883
$\bar{\gamma}_2^{3\,n}$	2.43930	2.25846	2.06686	1.86716	1.66106	1.4498	1.2341	1.0148
$\bar{\gamma}_0^{5\,n}$	2.58291	2.55371	2.48304	2.38279	2.26065	2.1217	1.9695	1.8065
$\bar{\bar{\gamma}}_0^{1\,n}$	0.67748	[1.11306]	1.32782	1.26620	1.15521	1.0144	0.8535	0.6781
$\bar{\bar{\gamma}}_1^{1\,n}$	1.49527	1.35171	1.19189	1.01876	0.83494	0.6425	0.4430	0.2376
$\bar{\bar{\gamma}}_2^{1\,n}$	1.37886	1.18087	0.97892	0.77259	0.56210	0.3478	0.1301	9.9094
$\bar{\bar{\gamma}}_3^{1\,n}$	1.16003	0.94133	0.72110	0.49889	0.27463	0.0484	9.8202	9.5902
$\bar{\bar{\gamma}}_4^{1\,n}$	0.89712	0.66748	0.43708	0.20562	9.97301	9.7392	—	—
$\bar{\bar{\gamma}}_5^{1\,n}$	0.60935	0.37291	0.13612	9.89854	9.66034	9.4214	—	—
$\bar{\bar{\gamma}}_0^{3\,n}$	[2.45493]	2.47876	2.41976	2.33135	2.21998	2.0906	1.9467	1.7909
$\bar{\bar{\gamma}}_1^{3\,n}$	2.86317	2.75866	2.63459	2.49506	2.34300	2.1806	2.0096	1.8312
$\bar{\bar{\gamma}}_2^{3\,n}$	2.95384	2.80792	2.65031	2.48312	2.30792	2.1259	—	—

II.

	$n=0$	1	2	3	4	5	6	7
$\omega_0^{1.n}$	116 342	7.913	27.217	12.402	5.917	2.899	1.445	0.730
$\omega_1^{1.n}$	11.614	9.857	4.555	2.192	1.080	0.541	0.274	0.140
$\omega_2^{1.n}$	2.689	2.516	1.220	0.605	0.304	0.154	0.079	0.041
$\omega_3^{1.n}$	0.774	0.756	0.377	0.190	0.097	0.050	0.026	0.013
$\omega_4^{1.n}$	0.246	0.247	0.125	0.064	0.033	0.017	0.009	—
$\omega_0^{3.n}$	33 633	91.199	59.352	36.766	22.145	13.099	7.650	4.426
$\omega_1^{3.n}$	56.757	72.360	44.304	26.485	15.583	9.065	5.228	2.996
$\omega_2^{3.n}$	36 578	44.348	26.337	15.423	8.940	5.142	2.940	1.672
$\omega_0^{5.n}$	79.793	143.365	113.985	84.012	58.837	39.772	52.181	—
$\omega_1^{5.n}$	166.000	253.330	181.84	125.023	83.274	54.134	69.043	—
$\bar{\omega}_0^{1.n}$	93.048	23.694	60.507	35.995	21.163	12.318	7.111	4.078
$\bar{\omega}_1^{1.n}$	30.400	32.997	18.438	10.380	5.852	3.297	1.854	1.042
$\bar{\omega}_2^{1.n}$	12.060	13.090	7.201	3.985	2.211	1.228	0.682	0.379
$\bar{\omega}_3^{1.n}$	4.928	5 348	2.925	1.607	0.885	0.488	0.269	—
$\bar{\omega}_4^{1.n}$	2.033	2.207	1.204	0.644	0.361	0.198	0.109	—
$\bar{\omega}_0^{3.n}$	120.519	278.880	217.344	157.870	109.358	73.214	47.770	30.548
$\bar{\omega}_1^{3.n}$	248.894	359.465	247.447	164.724	106.950	68.102	42.696	26.428
$\bar{\omega}_2^{3.n}$	228.955	304.545	197.372	125.420	78.470	48.474	29.631	17.952
$\bar{\bar{\omega}}_0^{1.n}$	4.758	25.947	42.545	36.917	28.592	20.674	14.273	9.531
$\bar{\bar{\omega}}_1^{1.n}$	31.280	44.951	31.111	20.883	13.676	8.780	5.546	3.456
$\bar{\bar{\omega}}_2^{1.n}$	23.925	30.332	19.053	11.847	7.297	4.455	2.699	1.624
$\bar{\bar{\omega}}_3^{1.n}$	14.455	17.473	10.523	6.308	3.764	2.236	1.322	—
$\bar{\bar{\omega}}_4^{1.n}$	7.891	9.300	5.471	3.211	1.880	1.097	0.484	—
$\bar{\bar{\omega}}_0^{3.n}$	186.02	379 08	346.08	294 01	235.34	179.63	131.97	93.99
$\bar{\bar{\omega}}_1^{3.n}$	512.69	825.44	634.33	469.22	336.35	234.89	160.48	107.62
$\bar{\bar{\omega}}_2^{3.n}$	679.84	985.99	695.000	478.56	323.10	214.48	—	—

Mean anomaly.

	$n = 0$	1	2	3	4	5	6	7
$R_{0.0}(n . - n)$	$-\infty$	2.41380_n	2.79389	1.72115	1.08221	0.56984	0.12103	9.71042
$R_{1.0}(n + 1 . - n)$	2.06178	0.57267_n	2.55454_n	1.54685_n	0.86854_n	0.22858_n	9.49479_n	8.11793
$R_{1.0}(n - 1 . - n)$	2.06178_n	2.88020_n	4.22571	3.77155	2.99287_n	2.23461_n	1.71611_n	1.28578_n
$R_{0.1}(n . - n + 1)$	2.73468_n	1.89800_n	1.80829_n	1.59330_n	1.28079_n	0.96617_n	0.65729_n	0.37354_n
$R_{0.1}(n . - n - 1)$	2.73468	3.78129_n	3.98755_n	3.22380	2.47939	1.97290	1.55278	1.17689
$R_{2.0}(n + 2 . - n)$	0.69069	1.91888	1.16984	9.97231	9.98318_n	9.88008_n	9.60785_n	9.25578_n
$R_{2.0}(n . - n)$	$-\infty$	3.51905_n	3.34411_n	3.43325_n	2.69695	1.81951	0.94275	0.29983_n
$R_{2.0}(n - 2 . - n)$	0.69069_n	2.37794_n	3.29185	4.35617_n	4.37325_n	3.25873_n	3.30186	2.70117
$R_{1.1}(n + 1 . - n + 1)$	2.68386_n	2.07344_n	1.30612	1.36960	1.05636	0.64832	0.12704	9.14176
$R_{1.1}(n - 1 . - n + 1)$	3.42607_n	$-\infty$	2.96358_n	3.58120	2.88777	2.49807	2.17353	1.87622
$R_{1.1}(n + 1 . - n - 1)$	3.42607	3.35207_n	3.61955	2.97363_n	2.18418_n	1.50182_n	0.55537_n	0.43257_n
$R_{1.1}(n - 1 . - n - 1)$	2.68386	3.80395_n	4.88586	4.89171	3.83784	3.83691_n	3.24691_n	2.81397_n
$R_{0.2}(n . - n + 2)$	3.38065_n	2.76661	1.75977	1.19212	1.02070	0.89775	0.74636	0.56875
$R_{0.2}(n . - n)$	$-\infty$	3.60100_n	3.56531_n	3.00341_n	2.66683_n	2.37395_n	2.09853_n	1.83164_n
$R_{0.2}(n . - n - 2)$	3.38065	3.97466_n	4.80287_n	3.81737_n	3.76834	3.19010	2.76779	2.40952
$R_{3.0}(n - 1 . - n)$	—	3.66755_n	4.03528_n	4.37401_n	3.79706	—	—	—
$R_{3.0}(n - 3 . - n)$	—	—	—	3.22686	3.54814	$-\infty$	4.79326	—
$R_{2.1}(n - 2 . - n + 1)$	—	—	3.69391	4.66483	4.63008	4.11900_n	—	—
$R_{2.1}(n . - n - 1)$	3.88080	4.96524_n	4.45922	4.43494_n	—	—	—	—
$R_{2.1}(n - 2 . - n - 1)$	—	—	3.98002_n	4.23177_n	$-\infty$	5.49278_n	—	—
$R_{1.2}(n - 1 . - n + 2)$	4.60023_n	—	—	—	—	—	—	—
$R_{1.2}(n - 1 . - n)$	—	4.26766_n	4.88081_n	5.00679_n	4.55339	—	—	—
$R_{1.2}(n + 1 . - n - 2)$	4.60023	4.47384	—	—	—	—	—	—
$R_{1.2}(n - 1 . - n - 2)$	—	4.24396_n	4.45837	$-\infty$	5.71163	5.51593_n	—	—
$R_{0.3}(n . - n - 3)$	—	—	$-\infty$	5.45084_n	5.26269	—	—	—

	$n=0$	1	2	3	4	5	6	7
$R_{0.0}(n+1.-n+1)_{+\sigma}$	2.48802	2.33332	1.90441	1.53287	1.19052	0.86561	0.55214	0.24666
$R_{0.0}(n-1.-n-1)_{-\sigma}$	2.48802_n	3.35635	3.28352_n	3.46532_n	2.30924_n	2.17738	1.49537	0.98634
$R_{0.0}(n+1.-n-1)_{+\delta}$	3.09929	3.40093_n	2.45540_n	1.91385_n	1.48114_n	1.09986_n	0.74796_n	0.41466_n
$R_{0.0}(n-1.-n+1)_{-\delta}$	3.09929_n	$-\infty$	3.35304	3.10481_n	2.10552_n	1.52729_n	1.06850_n	0.66810_n
$R_{1.0}(n.-n-1)_{-\sigma}$	3.73461_n	4.60141	4.01132	2.81889_n	—	—	—	—
$R_{1.0}(n-2.-n-1)_{-\sigma}$	—	—	3.76656	3.60293_n	$-\infty$	4.21365	—	—
$R_{1.0}(n-2.-n+1)_{-\delta}$	—	—	4.00083	4.66347_n	4.30376_n	—	—	—
$R_{1.0}(n.-n-1)_{+\delta}$	3.73152	4.90148_n	4.57880_n	3.87961	—	—	—	—
$R_{0.1}(n-1.-n)_{-\sigma}$	—	3.55756	4.05259	3.46471	—	—	—	—
$R_{0.1}(n-1.-n-2)_{-\sigma}$	—	4.08676_n	3.86599	$-\infty$	4.47996_n	—	—	—
$R_{0.1}(n-1.-n+2)_{-\delta}$	4.51005_n	—	—	—	—	—	—	—
$R_{0.1}(n-1.-n)_{-\delta}$	—	4.05687_n	4.76852	4.44240	—	—	—	—
$R_{0.1}(n+1.-n-2)_{+\delta}$	4 51005	4.67329	—	—	—	—	—	—

Mean Anomaly w.

	$n=0$	1	2	3	4	5	6
$R_{0.0}(n\,.-n)$	$-\infty$	2.9302	3.6688n	2.4660n	1.8443n	1.365n	0.953n
$R_{1.0}(n+1\,.-n)$	2.5076n	1.9037	3.4786	2.3880	1.7621	1.208	0.640
$R_{1.0}(n-1\,.-n)$	2.5076	3.3747	5.1925n	2.8799	3.9790	3.136	2.614
$R_{0.1}(n\,.-n+1)$	3.1219	2.4523	1.9904	2.1512	1.9415	1.696	1.443
$R_{0.1}(n\,.-n-1)$	3.1219n	4.7605	3.8732n	4.1789n	3.3415n	2.831n	2.428n
$R_{2.0}(n+2\,.-n)$	1.8207n	—	—	—	—	—	—
$R_{2.0}(n\,.-n)$	$-\infty$	4.1517	4.3858	3.7071n	3.8017n	2.934n	—
$R_{2.0}(n-2\,.-n)$	1.8207	2.9248	4.4213n	5.3918n	5.3222	4.195	4.367n
$R_{1.1}(n+1\,.-n+1)$	—	2.7382	—	—	—	—	—
$R_{1.1}(n-1\,.-n+1)$	3.9958	$-\infty$	3.5843n	4.4434n	3.6527n	—	—
$R_{1.1}(n+1\,.-n-1)$	3.9958n	4.3656	3.5918n	—	—	—	—
$R_{1.1}(n-1\,.-n-1)$	3.0756n	4.6639	5.933 5	5.7931n	4.7390n	4.8760	4.225
$R_{0.2}(n\,.-n+2)$	4.1597	—	2.5631n	—	—	—	—
$R_{0.2}(n\,.-n)$	$-\infty$	4.2334	4.3654	3.6297	3.4048	3.154	—
$R_{0.2}(n\,.-n-2)$	4.1597n	5.1429n	5.6891	4.6784	4.6809n	4.130n	—
$R_{3.0}(n-1\,.-n)$	—	—	4.9019	4.8309n	—	—	—
$R_{3.0}(n-3\,.-n)$	—	—	—	4.8598	—	$-\infty$	5.624n
$R_{2.1}(n-2\,.-n+1)$	—	—	—	5.6281n	3.6879	—	—
$R_{2.1}(n\,.-n-1)$	—	5.9864	4.2412n	—	—	—	—
$R_{2.1}(n-2\,.-n-1)$	—	—	5.0813n	—	$-\infty$	6.209	—
$R_{1.2}(n-1\,.-n+2)$	5.6062	—	—	—	—	—	—
$R_{1.2}(n-1\,.-n)$	—	—	5.7375	5.0012n	—	—	—
$R_{1.2}(n+1\,.-n-2)$	5.6062n	4.7313n	—	—	—	—	—
$R_{1.2}(n-1\,.-n-2)$	—	5.2418n	—	$-\infty$	6.4590n	6.870	—
$R_{0.3}(n\,.-n-3)$	—	—	$-\infty$	6.1418	6.6042n	—	—

	$n=0$	1	2	3	4	5	6
$R_{0.0}(n+1.-n+1)_{+\sigma}$	—	2.8756_n	—	—	—	—	—
$R_{0.0}(n-1.-n-1)_{-\sigma}$	2.8949	4.1664_n	4.5135_n	4.4247	3.2665	3.266_n	2.526_n
$R_{0.0}(n+1.-n-1)_{+\delta}$	3.7101_n	4.3363	—	—	—	—	—
$R_{0.0}(n-1.-n+1)_{-\delta}$	3.7101	$-\infty$	3.9736_n	4.0286_n	—	—	—
$R_{1.0}(n.-n-1)_{-\sigma}$	—	5.4131_n	—	—	—	—	—
$R_{1.0}(n-2.-n-1)_{-\sigma}$	—	—	4.1505	—	$-\infty$	5.057_n	—
$R_{1.0}(n-2.-n+1)_{-\delta}$	—	—	—	5.6588	—	—	—
$R_{1.0}(n.-n-1)_{+\delta}$	—	5.6844	—	—	—	—	—
$R_{0.1}(n-1.-n-2)_{-\sigma}$	—	4.6002_n	—	$-\infty$	5.320	—	—
$R_{0.1}(n-1.-n)_{-\delta}$	—	—	5.5722_n	—	—	—	—

Mean Anomaly w^2.

	$n=0$	1	2	3	4	5
$R_{0.0}(n.-n)$	$-\infty$	3.137_n	4.405	2.986	2.348	1.88
$R_{1.0}(n+1.-n)$	2.577	2.651_n	4.239_n	2.985_n	2.381_n	1.88_n
$R_{1.0}(n-1.-n)$	2.577_n	3.479_n	6.040	4.975	4.776_n	3.80_n
$R_{0.1}(n.-n+1)$	3.150_n	2.639_n	2.002	2.392_n	2.294_n	2.11_n
$R_{0.1}(n.-n-1)$	3.150	5.603_n	5.208_n	4.963	3.979	3.43
$R_{2.0}(n+2.-n)$	2.327	—	—	—	—	—
$R_{2.0}(n.-n)$	$-\infty$	4.493_n	5.261_n	4.601_n	4.668	3.74
$R_{2.0}(n-2.-n)$	2.327_n	3.012	5.304	6.236_n	5.763_n	4.90_n
$R_{1.1}(n+1.-n+1)$	—	2.934_n	—	—	—	—
$R_{1.1}(n-1.-n+1)$	4.307_n	$-\infty$	4.267	5.158	4.169	—
$R_{1.1}(n+1.-n-1)$	4.307	4.937_n	4.820	—	—	—
$R_{1.1}(n-1.-n-1)$	2.981	$5\ 399_n$	6.888	6.661	5.424	5.71_n
$R_{0.2}(n.-n+2)$	4.842_n	—	2.803	—	—	—
$R_{0.2}(n.-n)$	$-\infty$	4.544_n	5.037_n	4.111_n	3.879_n	3.652_n
$R_{0.2}(n.-n-2)$	4.842	5.992_n	6.533_n	5.342_n	4.681	4.84
$R_{3.0}(n-1.-n)$	—	—	5.680_n	5.841_n	—	—
$R_{3.0}(n-3.-n)$	—	—	—	5.480	—	$-\infty$
$R_{2.1}(n-2.-n+1)$	—	—	—	6.458	5.803	—
$R_{2.1}(n.-n-1)$	—	6.684_n	5.840	—	—	—
$R_{2.1}(n-2.-n-1)$	—	—	6.126_n	—	$-\infty$	—
$R_{1.2}(n-1.-n+2)$	6.462_n	—	—	—	—	—
$R_{1.2}(n-1.-n)$	—	—	6.557_n	6.210_n	—	—
$R_{1.2}(n+1.-n-2)$	6.462	5.560	—	—	—	—
$R_{1.2}(n-1.-n-2)$	—	5.920_n	—	$-\infty$	—	8.09_n
$R_{0.3}(n.-n-3)$	—	—	$-\infty$	—	7.821	—
$R_{0.0}(n+1.-n+1)_{+\sigma}$	—	3.101	—	—	—	—
$R_{0.0}(n-1.-n-1)_{-\sigma}$	2.725_n	4.858	5.342_n	5.268_n	3.922_n	4.13
$R_{0.0}(n+1.-n-1)_{+\delta}$	4.034	4.387_n	—	—	—	—
$R_{0.0}(n-1.-n+1)_{-\delta}$	4.034_n	$-\infty$	4.306	4.783_n	—	—

Radius Vector.

	$n=0$	1	2	3	4	5	6	7
$S_{0.0}(n.-n)$	$-\infty$	2.22729	2.84585_n	1.86950_n	1.28192_n	0.8000_n	0.3706_n	9.973_n
$S_{1.0}(n+1.-n)$	1.34967	2.16516	1.46224	0.27279	0.35137_n	0.3222_n	0.1445_n	9.921_n
$S_{1.0}(n-1.-n)$	1.34967	1.49752_n	3.76188_n	3.71768_n	3 06551	2.3714	1.8912	1.486
$S_{0.1}(n.-n+1)$	2.33674	1.29934_n	1.90453	1.76638	1.49386	1.2038	0.9112	0.619
$S_{0.1}(n.-n-1)$	2.33674	3.16600	3.91766	3.31570_n	2.64834_n	2.1847_n	1.7906_n	1.432_n
$S_{2.0}(n+2.-n)$	1.89564_n	1.63251_n	0.79397_n	0.05729_n	9.66300_n	9.6998_n	9.6932_n	9.614_n
$S_{2.0}(n.-n)$	$-\infty$	3.34650	3.10969_n	1.78761	2.00537	1.8641	1.6662	1.447
$S_{2.0}(n-2.-n)$	1.89564_n	2.75476_n	3.56504_n	4.19244_n	4.14939	2.6707	3.3864_n	2.832_n
$S_{1.1}(n+1.-n+1)$	2.95096	2.44134	1.64469	1.02490	0.82070	0.7604	0.6692	0.537
$S_{1.1}(n-1.-n+1)$	3.14090_n	$-\infty$	2.96267	3.61305_n	2.99293_n	2.6495_n	2.3556_n	2.080_n
$S_{1.1}(n+1.-n-1)$	3.14090_n	3.57796	2.34323	1.78351_n	1.93295_n	1.8223_n	1.6483_n	1.447_n
$S_{1.1}(n-1.-n-1)$	2.95096	3.54008_n	4.56164	4.62562_n	3.14670_n	3.9392	3.4031	3.003
$S_{0.2}(n.-n+2)$	3.28374	2.88804_n	1.98444_n	1.44702_n	1.26951_n	1.1518_n	1.0088_n	0.839_n
$S_{0.2}(n.-n)$	$-\infty$	3.40919	3.61060	3.14361	2.85977	2.5988	2.3438	2.091
$S_{0.2}(n.-n-2)$	3.28374	3.49302_n	4.48415	3.01528	3.89154_n	3.3759_n	2.9896_n	2.654_n
$S_{3.0}(n-1.-n)$	—	2.75159	3.66473_n	3 98438	3.54033_n	—	—	—
$S_{3.0}(n-3.-n)$	—	—	—	3.68790_n	4.29914	$-\infty$	4.6854_n	—
$S_{2.1}(n-2.-n+1)$	—	—	3.48430	4.32706_n	4.59263_n	4.1726	—	—
$S_{2.1}(n.-n-1)$	3.40649	4.36412	3.07878	3.62767	—	—	—	—
$S_{2.1}(n-2.-n-1)$	—	—	3.98002_n	4.83283_n	$-\infty$	5.3677	—	—
$S_{1.2}(n-1.-n+2)$	3.78138_n	—	—	—	—	—	—	—
$S_{1.2}(n-1.-n)$	—	3.50541_n	4.47000	4.95758	4.62224_n	—	—	—
$S_{1.2}(n+1.-n-2)$	3.78138_n	4.42380_n	—	—	—	—	—	—
$S_{1.2}(n-1.-n-2)$	—	4.15975_n	4.82625	$-\infty$	5.56701_n	5.5601	—	—
$S_{0.3}(n.-n-3)$	—	—	$-\infty$	5.28312	5.31450_n	—	—	—

	$n=0$	1	2	3	4	5	6	7
$S_{0.0}(n+1.-n+1)_{+\sigma}$	2.60922_n	2.53107_n	2.14659_n	1.80066_n	1.47403_n	1.1592_n	0.8524_n	0.551_n
$S_{0.0}(n-1.-n-1)_{-\sigma}$	2.60922_n	3.25943	2.83312_n	3.13086	0.65420_n	2.3180_n	1.7114_n	1.247_n
$S_{0.0}(n+1.-n-1)_{+\delta}$	2.90858_n	3.45767	2.61777	2.13207	1.73181	1.3706	1.0317	0.707
$S_{0.0}(n-1.-n+1)_{-\delta}$	2.90858_n	$-\infty$	$3\ 16184_n$	3.16622	2.28465	1.7707	1.3498	0.973
$S_{1.0}(n.-n-1)_{-\sigma}$	3.49196_n	3.79031_n	$3\ 94645_n$	3.28780	—	—	—	—
$S_{1.0}(n-2.-n-1)_{-\sigma}$	—	—	3.84541	1.23045_n	$-\infty$	4.0576_n	—	—
$S_{1.0}(n-2.-n+1)_{-\delta}$	—	—	3.14058	4.15045	4.23790	—	—	—
$S_{1.0}(n.-n-1)_{+\delta}$	2.92007	4.40992	4.51855	3.96142_n	—	—	—	—
$S_{0.1}(n-1.-n)_{-\sigma}$	—	3.15963	2.57548_n	3.39375_n	—	—	—	—
$S_{0.1}(n-1.-n-2)_{-\sigma}$	—	4.16590_n	3.57054	$-\infty$	4.30205	—	—	—
$S_{0.1}(n-1.-n+2)_{-\delta}$	3.88722_n	—	—	—	—	—	—	—
$S_{0.1}(n-1.-n)_{-\delta}$	—	3.65893_n	4.14489_n	4.36389_n	—	—	—	—
$S_{0.1}(n+1.-n-2)_{+\delta}$	3.88722_n	4.59836_n	—	—	—	—	—	—

Radius Vector w.

	$n=0$	1	2	3	4	5	6
$S_{0.0}(n.-n)$	$-\infty$	2.6585_n	3.6963	2.5905	2.0275	1.584	1.194
$S_{1.0}(n+1.-n)$	1.8956_n	2.7637_n	2.8274_n	1.9604_n	1.1579_n	0.394	0.704
$S_{1.0}(n-1.-n)$	1.8956_n	2.6790	4.5320	3.7291_n	4.0294_n	3.256_n	2.778_n
$S_{0.1}(n.-n+1)$	2.8732_n	1.9312	1.9746_n	2.3017_n	2.1408_n	1.925_n	1.690_n
$S_{0.1}(n.-n-1)$	2.8732_n	3.9304_n	4.1969	4.2388	3.4852	3.025	2.654
$S_{2.0}(n+2.-n)$	2.5219	—	—	—	—	—	—
$S_{2.0}(n.-n)$	$-\infty$	3.9178_n	3.8210	3.8173_n	3.0792	1.732_n	—
$S_{2.0}(n-2.-n)$	2.5219	3.2885	4.2415	4.5776_n	4.9330_n	3.463_n	4.436
$S_{1.1}(n+1.-n+1)$	—	3.0887_n	—	—	—	—	—
$S_{1.1}(n-1.-n+1)$	3.6380	$-\infty$	3.5222_n	4.4587	3.7410	—	—
$S_{1.1}(n+1.-n-1)$	3.6380	4.5536_n	3.9400	—	—	—	—
$S_{1.1}(n-1.-n-1)$	3.3780_n	4.4660	5.1128	5.3376	3.9087	4.951_n	4.367_n
$S_{0.2}(n.-n+2)$	4.1296_n	—	2.6807	—	—	—	—
$S_{0.2}(n.-n)$	$-\infty$	3.9747_n	4.3836_n	3.7082_n	3.5778_n	3.366_n	—
$S_{0.2}(n.-n-2)$	4.1296_n	4.2970_n	5.1017_n	3.7619_n	4.0724	4.291	—
$S_{3.0}(n-1.-n)$	—	—	4.5209	4.1965	—	—	—
$S_{3.0}(n-3.-n)$	—	—	—	4.4382	—	$-\infty$	5.335
$S_{2.1}(n-2.-n+1)$	—	—	—	5.1078	4.4680_n	—	—
$S_{2.1}(n.-n-1)$	—	5.2304_n	5.3651	—	—	—	—
$S_{2.1}(n-2.-n-1)$	—	—	4.4415	—	$-\infty$	5.724_n	—
$S_{1.2}(n-1.-n+2)$	4.5722	—	—	—	—	—	—
$S_{1.2}(n-1.-n)$	—	—	5.0962	5.2146	—	—	—
$S_{1.2}(n+1.-n-2)$	4.5722	4.2495_n	—	—	—	—	—
$S_{1.2}(n-1.-n-2)$	—	4.4986_n	—	$-\infty$	5.9858	6.890_n	—
$S_{0.3}(n.-n-3)$	—	—	$-\infty$	5.1219	6.6263	—	—

	$n=0$	1	2	3	4	5	6
$S_{0.0}(n+1 . -n+1)_{+\sigma}$	—	3.0874	—	—	—	—	—
$S_{0.0}(n-1 . -n-1)_{-\sigma}$	3.0584	4.1602_n	3.7054_n	3.8744_n	1.8182	3.373	2.716
$S_{0.0}(n+1 . -n-1)_{+\delta}$	3.4492	4.3702_n	—	—	—	—	—
$S_{0.0}(n-1 . -n+1)_{-\delta}$	3.4492	$-\infty$	3.7135	4.1313_n	—	—	—
$S_{1.0}(n-2 . -n-1)_{-\sigma}$	—	—	3.6774	—	$-\infty$	4.599	—
$S_{0.1}(n-1 . -n-2)_{-\sigma}$	—	4.5160_n	—	$-\infty$	4.8359_n	—	—

Radius Vector w^2.

	$n=0$	1	2	3	4	5
$S_{0.0}(n\,.-n)$	$-\infty$	2.665	4.417n	3.081n	2.507n	2.08n
$S_{1.0}(n+1\,.-n)$	2.104	3.125	3.677	2.733	2.151	1.54
$S_{1.0}(n-1\,.-n)$	2.104	3.100n	5.218n	4.880n	4.807	3.90
$S_{0.1}(n\,.-n+1)$	3.118	2.221n	2.334n	2.498	2.469	2.33
$S_{0.1}(n\,.-n-1)$	3.118	4.586	5.117	4.996n	4.092n	3.61n
$S_{2.0}(n+2\,.-n)$	2.830n	—	—	—	—	—
$S_{2.0}(n\,.-n)$	$-\infty$	4.160	4.530n	2.847	4.219n	3.32n
$S_{2.0}(n-2\,.-n)$	2.830n	2.314n	4.884n	5.372n	5.610	3.95
$S_{1.1}(n+1\,.-n+1)$	—	3.278	—	—	—	—
$S_{1.1}(n-1\,.-n+1)$	3.817n	$-\infty$	3.851n	5.162n	4.235n	—
$S_{1.1}(n+1\,.-n-1)$	3.817n	5.295n	4.227	—	—	—
$S_{1.1}(n-1\,.-n-1)$	3.410	5.205n	5.781	6.100n	4.404n	5.76
$S_{0.2}(n\,.-n+2)$	4.839	—	3.042n	—	—	—
$S_{0.2}(n\,.-n)$	$-\infty$	4.201	5.036	4.177	4.017	3.84
$S_{0.2}(n\,.-n-2)$	4.839	4.771n	5.808	4.304	4.928n	4.97n
$S_{3.0}(n-1\,.-n)$	—	—	5.254n	5.456	—	—
$S_{3.0}(n-3\,.-n)$	—	—	—	5.075n	—	$-\infty$
$S_{2.1}(n-2\,.-n+1)$	—	—	—	5.772n	5.729n	—
$S_{2.1}(n\,.-n-1)$	—	5.939	6.066	—	—	—
$S_{2.1}(n-2\,.-n-1)$	—	—	5.199	—	$-\infty$	—
$S_{1.2}(n-1\,.-n+2)$	5.228n	—	—	—	—	—
$S_{1.2}(n-1\,.-n)$	—	—	5.644n	6.142	—	—
$S_{1.2}(n+1\,.-n-2)$	5.228n	5.574n	—	—	—	—
$S_{1.2}(n-1\,.-n-2)$	—	5.914n	—	$-\infty$	—	8.10
$S_{0.3}(n\,.-n-3)$	—	—	$-\infty$	—	7.828n	—

	$n=0$	1	2	3	4	5
$S_{0.0}(n+1 \, . -n+1)_{+\sigma}$	—	3.321_n	—	—	—	—
$S_{0.0}(n-1 \, . -n-1)_{-\sigma}$	3.032_n	4.853	4.085_n	4.547	2.624_n	4.20_n
$S_{0.0}(n+1 \, . -n-1)_{+\delta}$	3.647_n	4.467	—	—	—	—
$S_{0.0}(n-1 \, . -n+1)_{-\delta}$	3.647_n	$-\infty$	3.925_n	4.901	—	—

Third Coordinate.

	$n=0$	1	2	3	4	5	6
$T_{0.0}(n\,.-n+1)_{+\pi'}$	1.6025*n*	1.3579	1.5804	0.9814	0.5106	0.093	9.708
$T_{0.0}(n\,.-n-1)_{-\pi'}$	1.6025	1.9777	2.2172	1.5832*n*	0.8681*n*	0.358*n*	9.919*n*
$T_{1.0}(n+1\,.-n+1)_{+\pi'}$	2.3567*n*	2.0943*n*	1.5542*n*	1.0700*n*	0.6096*n*	0.158*n*	9.699*n*
$T_{1.0}(n-1\,.-n+1)_{+\pi'}$	2.5429	$-\infty$	2.7615	2.8955*n*	2.0761*n*	1.603*n*	1.212*n*
$T_{1.0}(n+1\,.-n-1)_{-\pi'}$	2.5429*n*	2.8753	1.8778	1.2416	0.6830	0.140	9.568
$T_{1.0}(n-1\,.-n-1)_{-\pi'}$	2.3567	3.2866*n*	2.9277*n*	2.5727*n*	1.9018	2.105	1.548
$T_{0.1}(n\,.-n+2)_{+\pi'}$	2.8382*n*	2.6392	1.8395	1.1926	0.5276	9.580	9.415
$T_{0.1}(n\,.-n)_{+\pi'}$	$-\infty$	2.8153*n*	3.0046	2.2339	1.7950	1.429	1.093
$T_{0.1}(n\,.-n)_{-\pi'}$	$-\infty$	1.9224	1.8351	1.4081	1.0814	0.772	0.471
$T_{0.1}(n\,.-n-2)_{-\pi'}$	2.8382	2.9116	2.9096	2.1230*n*	2.3929*n*	1.842*n*	1.417*n*
$T_{2.0}(n\,.-n+1)_{+\pi'}$	—	2.6132	—	—	—	—	—
$T_{2.0}(n-2\,.-n+1)_{+\pi'}$	—	2.3791	3.5196*n*	2.1376*n*	3.5757*n*	—	—
$T_{2.0}(n\,.-n-1)_{-\pi'}$	—	3.5311	3.6444	—	—	—	—
$T_{2.0}(n-2\,.-n-1)_{-\pi'}$	—	—	3.7594*n*	3.6347	$-\infty$	3.290	—
$T_{1.1}(n-1\,.-n+2)_{+\pi'}$	3.4177	—	—	—	—	—	—
$T_{1.1}(n-1\,.-n)_{+\pi'}$	—	3.6721	3.2533	4.0198	—	—	—
$T_{1.1}(n-1\,.-n)_{-\pi'}$	—	3.0137*n*	2.2143*n*	3.0295	—	—	—
$T_{1.1}(n+1\,.-n-2)_{-\pi'}$	3.4177*n*	3.9573*n*	—	—	—	—	—
$T_{1.1}(n-1\,.-n-2)_{-\pi'}$	—	4.1615	4.0743*n*	$-\infty$	3.8724*n*	3.975	—
$T_{0.2}(n\,.-n+3)_{+\pi'}$	3.4831	—	—	—	—	—	—
$T_{0.2}(n\,.-n-1)_{+\pi'}$	—	3.2716*n*	3.8295*n*	—	—	—	—
$T_{0.2}(n\,.-n-3)_{-\pi'}$	3.4831*n*	3.8615	$-\infty$	3.8596	3.8875*n*	—	—
$T_{0.0}(n-1\,.-n-2)_{-\sigma-\pi'}$	—	2.5129*n*	2.2504	$-\infty$	—	—	—

Third Coordinate w.

	$n=0$	1	2	3	4	5
$T_{0.0}(n.-n+1)_{+\pi'}$	2.2007	1.8434n	2.2317n	1.6990n	1.283n	0.916n
$T_{0.0}(n.-n-1)_{-\pi'}$	2.2007n	2.4130n	2.4403	2.5355	1.749	1.284
$T_{1.0}(n+1.-n+1)_{+\pi'}$	2 8242	2.6740	2.2682	1.8621	1.468	1.076
$T_{1.0}(n-1.-n+1)_{+\pi'}$	3.0735n	$-\infty$	3.2781n	3 8482	2.929	2.474
$T_{1.0}(n+1.-n-1)_{-\pi'}$	3.0735	3.7832n	2 4472n	2.2443n	1.685n	1.196n
$T_{1.0}(n-1.-n-1)_{-\pi'}$	2.8242n	4.1262	3.2355	2.9294	2.824n	3.136n
$T_{0.1}(n.-n+2)_{+\pi'}$	3.7136	3.0463n	2.4907n	1.9552n	1.398n	0.699n
$T_{0.1}(n.-n)_{+\pi'}$	$-\infty$	3.2878	3.9294n	3.0461n	2.625n	2.289n
$T_{0.1}(n.-n)_{-\pi'}$	$-\infty$	2.7959n	2.2087n	2.0561n	1.822n	1.614n
$T_{0.1}(n.-n-2)_{-\pi'}$	3.7136n	3.5038n	3.2876n	2.9973	3.394	2.793
$T_{2.0}(n.-n+1)_{+\pi'}$	—	2.7152n	—	—	—	—
$T_{2.0}(n-2.-n+1)_{+\pi'}$	—	3.0580n	—	3.7025n	2.489	—
$T_{2.0}(n.-n-1)_{-\pi'}$	—	4.3455n	3.1603n	—	—	—
$T_{2.0}(n-2.-n-1)_{-\pi'}$	—	—	4.0275n	—	$-\infty$	—
$T_{1.1}(n-1.-n+2)_{+\pi'}$	4.2038n	—	—	—	—	—
$T_{1.1}(n-1.-n)_{+\pi'}$	—	—	2.6613n	3.6837	—	—
$T_{1.1}(n-1.-n)_{-\pi'}$	—	—	3.3532	3.8293	—	—
$T_{1.1}(n+1.-n-2)_{-\pi'}$	4 2038	3.7736n	—	—	—	—
$T_{1.1}(n-1.-n-2)_{-\pi'}$	—	4.5616	—	$-\infty$	—	5.302n
$T_{0.2}(n.-n+3)_{+\pi'}$	3.8309	—	—	—	—	—
$T_{0.2}(n.-n-1)_{+\pi'}$	—	3.8681	3.7884n	—	—	—
$T_{0.2}(n.-n-3)_{-\pi'}$	3.8309n	—	$-\infty$	—	5.217	—

Third Coordinate w^2.

	$n=0$	1	2	3	4	5
$T_{0.0}(n.-n+1)_{+\pi'}$	2.702_n	1.977	2 528	2.033	1.66	1.34
$T_{0.0}(n.-n-1)_{-\pi'}$	2.702	3.031_n	3.376	3.190_n	2.24_n	1.75_n
$T_{1.0}(n+1.-n+1)_{+\pi'}$	2.964_n	2.938_n	2.617_n	2.274_n	1.94_n	1.60_n
$T_{1.0}(n-1.-n+1)_{+\pi'}$	2.803	$-\infty$	3.016	4.548_n	3.41_n	2.96_n
$T_{1.0}(n+1.-n-1)_{-\pi'}$	2.803_n	4.474	3.181	2.796	2.23	1.79
$T_{1.0}(n-1.-n-1)_{-\pi'}$	2.964	4.926_n	4.416_n	3.543	3.18	3.82
$T_{0.1}(n.-n+2)_{+\pi'}$	4.516_n	3.167	2 815	2.372	1.91	1.37
$T_{0.1}(n.-n)_{+\pi'}$	$-\infty$	3.747_n	4.616	3.493	3.06	2.75
$T_{0.1}(n.-n)_{-\pi'}$	$-\infty$	3 002	2.747_n	2.128	2.09	1.95
$T_{0.1}(n.-n-2)_{-\pi'}$	4 516	4.502	3.851_n	3.312_n	4.05_n	3.34_n
$T_{2.0}(n.-n+1)_{+\pi'}$	—	4.109	—	—	—	—
$T_{2.0}(n-2.-n+1)_{+\pi'}$	—	3.434	—	2.336_n	4.81_n	—
$T_{2.0}(n.-n-1)_{-\pi'}$	—	4.549_n	4.695	—	—	—
$T_{2.0}(n-2.-n-1)_{-\pi'}$	—	—	4.862	—	$-\infty$	—
$T_{1.1}(n-1.-n+2)_{+\pi'}$	4.458_n	—	—	—	—	—
$T_{1.1}(n-1.-n)_{+\pi'}$	—	—	4.341_n	5.077	—	—
$T_{1.1}(n-1.-n)_{-\pi'}$	—	—	3.430	4.500	—	—
$T_{1.1}(n+1.-n-2)_{-\pi'}$	4.458	5.076_n	—	—	—	—
$T_{1.1}(n-1.-n-2)_{-\pi'}$	—	5.293	—	$-\infty$	—	6.43
$T_{0.2}(n.-n+3)_{+\pi'}$	4.577	—	—	—	—	—
$T_{0.2}(n.-n-1)_{+\pi'}$	—	4.337	4.934_n	—	—	—
$T_{0.2}(n.-n-3)_{-\pi'}$	4.577_n	—	$-\infty$	—	6.32_n	—

Terms of the fourth degree. Argument: (2 — 6)

	n		w
$R_{4.0}(n-4.-n)$	$n=6$	4.81766_n	5.6908
$R_{3.1}(n-3.-n-1)$	$n=5$	5.64896	6.1696_n
$R_{2.2}(n-2.-n-2)$	$n=4$	6.05144_n	6.7471
$R_{1.3}(n-1.-n-3)$	$n=3$	6.09992	6.6971_n
$R_{0.4}(n.-n-4)$	$n=2$	5.71948_n	6.0603
$R_{2.0}(n-3.-n-1)_{-\sigma}$	$n=5$	4.07151_n	5.0775
$R_{1.1}(n-2.-n-2)_{-\sigma}$	$n=4$	4.70183	4.8138_n
$R_{0.2}(n-1.-n-3)_{-\sigma}$	$n=3$	4.72222_n	5.0946
$S_{4.0}(n-4.-n)$	$n=6$	5.03260_n	5.7405
$S_{3.1}(n-3.-n-1)$	$n=5$	5.77677	6.4217_n
$S_{2.2}(n-2.-n-2)$	$n=4$	6.07795_n	6.6499
$S_{1.3}(n-1.-n-3)$	$n=3$	6.00521	6.5233_n
$S_{0.4}(n.-n-4)$	$n=2$	5.47381_n	5.8379
$S_{2.0}(n-3.-n-1)_{-\sigma}$	$n=5$	4.45522_n	5.2023
$S_{1.1}(n-2.-n-2)_{-\sigma}$	$n=4$	4.88055	5.5475_n
$S_{0.2}(n-1.-n-3)_{-\sigma}$	$n=3$	4.64616_n	5.1995

$l =$ (53)

[2.07446$_n$	2.6382 w	2.840$_n$ w^2]	$\eta\,\eta'$	$\sin \varDelta$
[3.1678$_n$	3.858 w	]	$\eta^3\eta'$	$\sin \varDelta$
[3.1284	3.873$_n$ w	]	$\eta^2\eta'^2$	$\sin 2\varDelta$
[3.3445$_n$	4.033 w	]	$\eta\,\eta'^3$	$\sin \varDelta$
[3.5764	4.202$_n$ w	]	$j^2\eta^2$	$\sin(\varDelta+\Sigma)$
[3.2356$_n$	3.937 w	]	$j^2\eta\,\eta'$	$\sin \Sigma$
[3.5214	4.227$_n$ w	]	$j^2\eta\,\eta'$	$\sin \varDelta$

$m =$ (53)

[2.07447$_n$	2.6382 w	2.840$_n$ w^2]	η'	$\sin \varDelta$
[3.15360$_n$	3.8584 w	4.228$_n$ w^2]	$\eta^2\eta'$	$\sin \varDelta$
[3.34447$_n$	4.0333 w	4.408$_n$ w^2]	η'^3	$\sin \varDelta$
[3.13604	3.8733$_n$ w	4.289 w^2]	$\eta\,\eta'^2$	$\sin 2\varDelta$
[3.23563$_n$	3.9366 w	4.325$_n$ w^2]	$j^2\,\eta'$	$\sin \Sigma$
[3 52141	4.2265$_n$ w	4.619 w^2]	$j^2\,\eta'$	$\sin \varDelta$
[3.57642	4.2020$_n$ w	4.527 w^2]	$j^2\eta$	$\sin(\varDelta+\Sigma)$

$n =$ (53)

[2.26102	2.7464$_n$ w	2.880 w^2]	η	
[2.74826	3 5903$_n$ w	4.078 w^2]	η^3	
[3.58321	4.2621$_n$ w	4.637 w^2]	$\eta\,\eta'^2$	
[3.58321$_n$	4.2621 w	4.638$_n$ w^2]	$j^2\eta$	
[2.07447$_n$	2.6382 w	2.840$_n$ w^2]	η'	$\cos \varDelta$
[3.55160$_n$	4.2797 w	4.686$_n$ w^2]	$\eta^2\eta'$	$\cos \varDelta$
[3.34447$_n$	4.0333 w	4.408$_n$ w^2]	η'^3	$\cos \varDelta$
[3.13604	3.8733$_n$ w	4.289 w^2]	$\eta\,\eta'^2$	$\cos 2\varDelta$
[3.23563$_n$	3.9366 w	4.325$_n$ w^2]	$j^2\,\eta'$	$\cos \Sigma$
[3.52141	4.2265$_n$ w	4.619 w^2]	$j^2\,\eta'$	$\cos \varDelta$
[3.57642	4.2020$_n$ w	4.527 w^2]	$j^2\eta$	$\cos(\varDelta+\Sigma)$

$F-\eta(H+G) =$ (37)

[4.42378	4.8206$_n$ w	4.803 w^2]	η'^3	$D^2\,\vartheta^2$
[4.68478$_n$	5.2241 w	5.448$_n$ w^2]	$\eta\,\eta'^2$	$D^3\,\vartheta^2$
[4.46409	5.1023$_n$ w	5.444 w^2]	$\eta^2\eta'$	$D^4\,\vartheta^2$
[3.76332$_n$	4.4778 w	4.899$_n$ w^2]	η^3	$D^5\,\vartheta^2$
[3.50318	4.1550$_n$ w	4.456 w^2]	$j^2\,\eta'$	$D^3S^{-1}\vartheta^2$
[3.21712$_n$	3.9493 w	4.357$_n$ w^2]	$j^2\eta$	$D^4S^{-1}\vartheta^2$

$-[(H)-(G')] =$ (37)

[3.90663	4.4046$_n$ W	4.498 W^2]	$\eta'^2 D^3 \vartheta^2$
[3.98696$_n$	4.5897 W	4.844$_n$ W^2]	$\eta\,\eta' D^4 \vartheta^2$
[3.46229	4.1456$_n$ W	4.500 W^2]	$\eta^2 D^5 \vartheta^2$
[2.43896	3.1513$_n$ W	3.513 W^2]	$j^2 D^4 S^{-1} \vartheta^2$
[4.1068$_n$	4.607 W	]	$\eta^4 D^5 \vartheta^2$
[3.9641$_n$	4.569 W	]	$\eta^4 D^{-5} \vartheta^{-2}$
[4.6133	5.290$_n$ W	]	$\eta^3 \eta' D^6 \vartheta^2$
[4.6396	4.926$_n$ W	]	$\eta^3 \eta' D^4 \vartheta^2$
[4.6066	5.102$_n$ W	]	$\eta^3 \eta' D^{-4} \vartheta^{-2}$
[5.2383$_n$	5.845 W	]	$\eta^2 \eta'^2 D^5 \vartheta^2$
[4.6101$_n$	3.958$_n$ W	]	$\eta^2 \eta'^2 D^3 \vartheta^2$
[4.7536$_n$	5.040 W	]	$\eta^2 \eta'^2 D^{-3} \vartheta^{-2}$
[5.3294	5.839$_n$ W	]	$\eta\,\eta'^3 D^4 \vartheta^2$
[3.7728	4.793 W	]	$\eta\,\eta'^3 D^2 \vartheta^2$
[4.3744	4.250 W	]	$\eta\,\eta'^3 D^{-2} \vartheta^{-2}$
[5.0097$_n$	5.406 W	]	$\eta'^4 D^3 \vartheta^2$
[3.9736	4.656$_n$ W	]	$\eta'^4 D^{-1} \vartheta^{-2}$
[3.8712	4.722$_n$ W	]	$j^2 \eta^2 D^4 S^{-1} \vartheta^2$
[3.3698	4.286$_n$ W	]	$j^2 \eta^2 D^{-4} S\, \vartheta^2$
[4.6455$_n$	5.416 W	]	$j^2 \eta^2 D^5 \vartheta^2$
[4.1872$_n$	4.986 W	]	$j^2 \eta\,\eta' D^3 S^{-1} \vartheta^2$
[4.1224$_n$	4.941 W	]	$j^2 \eta\,\eta' D^{-3} S\, \vartheta^{-2}$
[4.2008	4.942$_n$ W	]	$j^2 \eta'^2 D^{-2} S\, \vartheta^{-2}$
[3.8966	4.571$_n$ W	]	$j^2 \eta\,\eta' D^5 S^{-1} \vartheta^2$
[5.1031	5.843$_n$ W	]	$j^2 \eta\,\eta' D^4 \vartheta^2$
[4.0678$_n$	4.644 W	]	$j^2 \eta'^2 D^4 S^{-1} \vartheta^2$
[4.9420$_n$	5.637 W	]	$j^2 \eta'^2 D^3 \vartheta^2$
[3.4610$_n$		]	$j^4 D^4 S^{-1} \vartheta^2$
[2.4016		]	$j^4 D^{-3} S^2 \vartheta^{-2}$

$(A_1) =$ (47, 49)

[1.66706$_n$	2.0849 W	2.097$_n$ W^2]	
[2.71602$_n$	3.3184 W	3.608$_n$ W^2]	η^2
[2.81574$_n$	3.4071 W	3.683$_n$ W^2]	η'^2
[2.97669	3.6163$_n$ W	3.927 W^2]	$\eta\,\eta' D$
[2.81574	3.4071$_n$ W	3.683 W^2]	j^2
[3.2891$_n$	4.122 W	]	η^4
[4.0973	4.905$_n$ W	]	$\eta^3 \eta' D$
[3.9104$_n$	4.717 W	]	$\eta^2 \eta'^2 D^2$
[4.2827$_n$	5.054 W	]	$\eta^2 \eta'^2$

to be continued.

[4.4217	5.186_n	W	]	$\eta\ \eta'^3\ D$
[3.9955_n	4.723	W	]	η'^4
[4.1707_n	4.904	W	]	$j^2\ \eta^2\ D\ S$
[4.2605	5.007_n	W	]	$j^2\ \eta^2$
[3.9195	4.668_n	W	]	$j^2\ \eta\ \eta'\ S$
[3.9195	4.668_n	W	]	$j^2\ \eta\ \eta'\ S^{-1}$
[4.4245_n	5.191	W	]	$j^2\ \eta\ \eta'\ D$
[3.9195_n	4.668	W	]	$j^2\ \eta\ \eta'\ D^{-1}$
[3.6928_n	4.485	W	]	$j^2\ \eta'^2\ D\ S^{-1}$
[4.3406	5.085_n	W	]	$j^2\ \eta'^2$
[3.3402_n			]	j^4
[4.38427_n	3.5735	W	4.639 W^2]	$\eta'^3\ D^2\ \vartheta^2$
[4.36335	4.6608_n	W	4.898_n W^2]	$\eta\ \eta'^2\ D^3\ \vartheta^2$
[4.46699_n	4.5966	W	4.705_n W^2]	$\eta^2\ \eta'\ D^4\ \vartheta^2$
[3.76738	4.0572_n	W	4.144 W^2]	$\eta^3\ D^5\ \vartheta^2$
[3.62812_n	4.2643	W	4.604_n W^2]	$j^2\ \eta'\ D^3\ S^{-1}\ \vartheta^2$
[3.50696	4.1469_n	W	4.559 W^2]	$j^2\ \eta\ D^4\ S^{-1}\ \vartheta^2$

$(B_1) =$ (65)

[2.37550_n	2.9392	W	3.141_n W^2]	$\eta\ \eta'\ D$
[3.4689_n	4.160	W	]	$\eta^3\ \eta'\ D$
[3.6455_n	4.334	W	]	$\eta\ \eta'^3\ D$
[3.4294	4.174_n	W	]	$\eta^2\ \eta'^2\ D^2$
[3.6922	4.399_n	W	]	$j^2\ \eta\ \eta'\ D$
[3.2355_n	3.937	W	]	$j^2\ \eta\ \eta'\ D^{-1}$
[3.5365	4.238_n	W	]	$j^2\ \eta\ \eta'\ S^{-1}$
[3.8774	4.503_n	W	]	$j^2\ \eta^2\ D\ S$
[4.54872	4.5995_n	W	4.460 W^2]	$\eta'^3\ D^2\ \vartheta^2$
[4.86087_n	5.2825	W	5.401_n W^2]	$\eta\ \eta'^2\ D^3\ \vartheta^2$
[4.68593	5.1299_n	W	5.417 W^2]	$\eta^2\ \eta'\ D^4\ \vartheta^2$
[4.0266_n	4.5791	W	4.927_n W^2]	$\eta^3\ D^5\ \vartheta^2$
[3.79819	4.0236_n	W	3.855_n W^2]	$j^2\ \eta'\ D^3\ S^{-1}\ \vartheta^2$
[3.39130_n	3.8089	W	3.450 W^2]	$j^2\ \eta\ D^4\ S^{-1}\ \vartheta^2$

Inclination Perturbations $P' = e^{i\,\Pi'}$

$(G) =$ (70)

[2.95284	3.5844_n	W	3.915 W^2]	$\iota\ \eta^2\ D\ P'$
[2.63357_n	3.3345	W	3.723_n W^2]	$\iota\ \eta\ \eta'\ P'$
[2.63357	3.3345_n	W	3.723 W^2]	$\iota\ \eta\ \eta'\ P'^{-1}$
[1.49872			]	$\iota\ j^2\ P'\ S^{-1}$
[1.49872			]	$\iota\ j^2\ D^{-1}\ P'^{-1}$

$(H) =$ (70)

$[1.65896_n$	$2.1444_{}$ w	2.278_n w$^2]$	ι	$D\ P'$
$[3.00135_n$	3.6730 w	4.044_n w$^2]$	$\iota\ \eta^2$	$D\ P'$
$[2.90352$	3.6131_n w	4.009 w$^2]$	$\iota\ \eta\ \eta'$	$D^2\ P'$
$[2.63357$	3.3345_n w	3.723 w$^2]$	$\iota\ \eta\ \eta'$	P'
$[2.63357_n$	3.3345 w	3.723_n w$^2]$	$\iota\ \eta\ \eta'$	P'^{-1}
$[3.00135_n$	3.6730 w	4.044_n w$^2]$	$\iota\ \eta'^2$	$D\ P'$
$[2.29500$	3.0600_n w	3.506 w$^2]$	$\iota\ \eta'^2$	$D\ P'^{-1}$
$[1.96004$		$]$	$\iota\ j^2$	$D\ P'$
$[1.49872$		$]$	$\iota\ j^2$	$P'^{-1}\ S$
$[2.13793_n$	2.8636 w	3.257_n w$^2]$	$\iota\ \eta$	$D^4\ P'^{-1}\ \vartheta^2$
$[2.42400$	3.0758_n w	3.377 w$^2]$	$\iota\ \eta'$	$D^3\ P'^{-1}\ \vartheta^2$

$(C) =$ (83)

$[1.96000_n$	2.4456 w	2.580_n w$^2]$	$\iota\ \eta$	$D\ P'$
$[2.43896_n$	3.7443 w	4.481_n w$^2]$	$\iota\ \eta^2$	$D^4\ P'^{-1}\ \vartheta^2$
$[2.72501$	4.1866_n w	4.691 w$^2]$	$\iota\ \eta\ \eta'$	$D^3\ P'^{-1}\ \vartheta^2$
$[\ \ —$	3.9067 w	4.696_n w$^2]$	$\iota\ \eta'^2$	$D^2\ P'^{-1}\ \vartheta^2$

$m =$ (75)

$[1.65896_n$	2.1444 w	2.278_n w$^2]$	ι	$\cos(\varDelta + \Pi')$
$[2.02547_n$	2.9392 w	3.453_n w$^2]$	$\iota\ \eta^2$	$\cos(\varDelta + \Pi')$
$[2.90352$	3.6131_n w	4.009 w$^2]$	$\iota\ \eta\ \eta'$	$\cos(2\varDelta + \Pi')$
$[3.00135_n$	3.6730 w	4.044_n w$^2]$	$\iota\ \eta'^2$	$\cos(\varDelta + \Pi')$
$[2.29500$	3.0600_n w	3.506 w$^2]$	$\iota\ \eta'^2$	$\cos(\varDelta - \Pi')$
$[2.08900$		$]$	$\iota\ j^2$	$\cos(\varDelta + \Pi')$
$[1.79975$		$]$	$\iota\ j^2$	$\cos(\Sigma - \Pi')$

$n =$ (75)

$[1.65896$	2.1444_n w	2.278 w$^2]$	ι	$\sin(\varDelta + \Pi')$
$[3.27880$	3.9320_n w	4.285 w$^2]$	$\iota\ \eta^2$	$\sin(\varDelta + \Pi')$
$[2.90352_n$	3.6131 w	4.009_n w$^2]$	$\iota\ \eta\ \eta'$	$\sin(2\varDelta + \Pi')$
$[3.23563_n$	3.9366 w	4.325_n w$^2]$	$\iota\ \eta\ \eta'$	$\sin\Pi'$
$[3.00135$	3.6730_n w	4.044 w$^2]$	$\iota\ \eta'^2$	$\sin(\varDelta + \Pi')$
$[2.29500_n$	3.0600 w	3.506_n w$^2]$	$\iota\ \eta'^2$	$\sin(\varDelta - \Pi')$
$[2.08900_n$		$]$	$\iota\ j^2$	$\sin(\varDelta + \Pi')$
$[1.79975_n$		$]$	$\iota\ j^2$	$\sin(\Sigma - \Pi')$

$[(H) + (G')] =$ (70)

[2.13793n	2.8636 W	3.257n W^2]	$\iota\ \eta$	$D^4\ P'^{-1}\ \vartheta^2$
[2.42400	3.0758n W	3.377 W^2]	$\iota\ \eta'$	$D^3\ P'^{-1}\ \vartheta^2$

$(C_1) =$ (83)

[1.96000n	2.4456 W	2.580n W^2]	$\iota\ \eta$	$D\ P'$
[2.61504n	3.7884 W	4.496n W^2]	$\iota\ \eta^2$	$D^4\ P'^{-1}\ \vartheta^2$
[2.90113	4.2120n W	4.702 W^2]	$\iota\ \eta\ \eta'$	$D^3\ P'^{-1}\ \vartheta^2$
[—	3.9067 W	4.696n W^2]	$\iota\ \eta'^2$	$D^2\ P'^{-1}\ \vartheta^2$