John Carver's Book

It is required to find the side of rectangle ~~square~~ inscribed in a given Triangle, whose Area shall be a Maximum

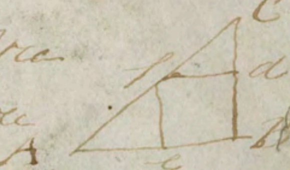
Let $fd = x$ Area

& $ef = dB = y$ Area

but $AB = a$

$BC = b$ a Mini Minimum

$xy =$ Maximum



But $AB - Be : ef :: AB : BC$

that is $a - x : y :: a : b$

Minimum

$y = \frac{ab - bx}{a}$ John Carver's Book

& $yx = \frac{abx - bx^2}{a}$ Penny

This in fluxions

i.e. $a\dot{x} - 2x\dot{x} = 0$ $y = \frac{ab - \frac{ba}{2}}{a} =$

$a = 2x$

$x = \frac{a}{2}$ $\frac{ab}{2a} = \frac{b}{2}$

Ans. J. Carver Sep 10 1818

Newtons Principia

Case 1st Propos. 35 Newtons Prin

$AC : AO$ or $SK :: CD \times Cc : Sy \times Dd$

Vin C : vin $c :: AC^{\frac{1}{2}} : SK^{\frac{1}{2}}$ or $AO^{\frac{1}{2}}$

vin c : vin $OKk :: SK^{\frac{1}{2}} : SC^{\frac{1}{2}}$

Therefore (compos^n

Vin C : vin $OKk :: AC^{\frac{1}{2}} : SC^{\frac{1}{2}}$ by

But $SC = \frac{CD^2}{SC}$ & $SC^{\frac{1}{2}} = \frac{CD}{AC^{\frac{1}{2}}}$

Vin C : vin $OKk : AC : CD$

$Cc : Kk :: AC : CD$

$Cc \times CD = Kk \times AC$

& $AC : SK :: Kk \times AC : Sy \times Dd$

$AC \times Sy \times Dd = SK \times Kk \times AC$

$Sy \times Dd = SK \times Kk$

Whence the rest of the Theor. is evident. Case 2nd

$TC : TD :: Cc : Dd$

$TD : TS :: CD : Sy$ by Sim. Tri^s

$TC : TS :: CD \times Cc : Sy \times Dd$ by Com

$TC : TS :: 2 : 1$ by Par^a

Wherefore $Bb \times Cc = \frac{1}{2} Sy \times Dd$

$Cc : Rh :: SK^{\frac{1}{2}} : \frac{1}{2} SC^{\frac{1}{2}}$

But SK is as $\frac{1}{2}$ $6D$ Parab

$\frac{1}{2} SC : \frac{1}{2} 6D :: 6D :: SK$ by the nature of

Jno Carver's Book

Sebr 7th 1818

$\frac{1}{2} SC^{\frac{1}{2}} = \frac{\frac{1}{2} 6D}{SK^{\frac{1}{2}}}$

8 - 10
10
4 . 16 -
5
14 -
7
15 - 15 -
4 . 10 -
4
6
5 10,
9 - 9
6
12
109 . 10

£
20
21
8
149
169 - 10
158 . 10
177 - 10
19
47 10

£ s D
20 . 3 . 4
£ 13 . 4 3/4
£ 7 7/8

$\frac{SK}{47} : SK^2 : SC - 6D^2$

$\frac{6D^2}{2} = SK \times SC$

$SC = \frac{6D^3}{SK}$

See Sanderson's Fluxions page 243

A Translation of the Quadrature of the Apollonian Parabola, shewing the use of the 4 principal Lemmas in the 1st Book of the Principia as also the use of the first and last Ratios in Geometrical Reasonings; Fig 58 —

Let AMm be a Parabola whose Parameter = t, Axis AP p, & regularly applied PM, mp

It is required the Area APM.

Let the Parallelograms APMQ, Apmq, and AMQ, MP meeting mp, mq in R & S respectively, then first $t \times AP = PM^2$ also $t \times Ap = pm^2$ Wherefore $t \times Pp = mp^2 - MP^2 =$ (by subtr.)

$= \overline{PS}^2 - \overline{PM}^2 = \overline{PS+PM} \times \overline{PS-PM} =$
$\overline{PS+PM} \times MS$; that is, the Rectangle
$t \times Pp$ is equal to the Rectangle
$\overline{PM+PS} \times MS$; Also the Solid $Ap \times$
$t \times Pp =$ the Solid $Ap \times \overline{PM+PS}$
$\times MS$, or the Solid $PM^2 \times Pp =$ to
the Solid $\overline{PM+PS} \times AS \times MS$. But
$PM \times Pp =$ to the Rectangle Pp
And $PM^2 \times Pp$ is equal to the
Rectangle PR drawn into the
Line PM. Likewise $\overline{PM+PS} \times A$
$\times MS$ is equal to Rectangle QS
drawn into the Line $\overline{PM+PS}$;
Therefore Rectangle PR is to the
Rectan. QS as PM+PS to PM
MP and MQ remaining the same
Let the Width of the Rectangles
PR & QS be diminished ad infini-
and the ultimate Ratios of the
evanescent Parallelograms shall
be 2PM to PM, as 2 to 1;

Therefore the ultimate Sum of
all the Parallelograms PR shall be
to the ultimate Sum of all the Par^ms
QS as 2 to 1, that is, the Internal
Space APM shall be double the
external Space AQM; & so the
internal Space APM shall be
$\frac{2}{3}$ of the whole Space PQ.
Q.E.I.

Concerning the First & Last
Ratios Page 244

However variable Magnitudes
are considered, as growing from
nothing, or vanishing into nothing;
Nevertheless it may happen, yea is,
oftentimes so, that their Ratios
neither begin, nor vanish, moreover
they may grow to a certain & determinate
Magnitude, but may become limited
or confined; Also, and both Mag-
nitudes of which their are Ratios
may go into nothing, or increase
to immensity, & their Ratio

are they, which are the first of begin-
-ing or the last of vanishing to be
considered in a different or separate
manner; & even the last ratios of
increased magnitudes and infin-
therefore they are Ratios first & last;
not ratios of the first & last
magnitudes, such are not given
in the nature of things; not
limited or confined Ratios, or
indeed, of being or existence; but
the limits of these Ratios to
which surely Ratios of magnitudes
without limit of increasing or decreas-
-ing are able to approach nearer
them for any one given diff^ce
and never pass over, nor reach too
unless at that time when the magnitudes
of which they are the Ratios even may
become infinite or of no value.

All these however seem at first
view difficult of conception, neverthe-
less a notable example amongst
among others occurring to the
Reasons, is able I think to be illustra-
-ted sufficiently. — — —

~~Wherefore~~ let the 2 Quantities A and
B be explained. of which let A be
$= 4x^2+3x$ and $B = 2x^2+x$ vari-
-able because of the variable quan-
x and infinite or of no value
even as their root x shall be
infinite or nothing; I say
now the last Ratio of vanishing
A to vanishing B even ipsam fore
quam has 3 to 1, & the the ultimate
ration of A infinite to infinite B
that is hereafter to be as much (even
for quam habet) 2 to 1. For when
A may be $= 4x^2+3x$, and $B = 2x^2+x$,

A shall be to B (by dividing by x) as $4x+3$, to $2x+1$: & this Ratio is greater than $4x+2$ to $2x+1$ or 2 to 1, and less than the Ratio $6x+3$ to $2x+1$ or 3 to 1. – Let x vanish and at the same time by the vanishing of $4x$ & $2x$ A shall be to B at the last as 3 to 1. Again when A is to B universally, as $4x+3$ to $2x+1$, or (by dividing by x) as $4+\frac{3}{x}$ to $2+\frac{1}{x}$; Let x be increased in infinitum, and by the vanishing of $\frac{3}{x}$ and $\frac{1}{x}$, A will be to B at last as 4 to 2 or 2 to 1; Therefore A will never be to B even in the Ratio of 3 to 1, or in the Ratio 2 to 1, but in the intermediate Ratio and these Ratios. viz. 3 to 1 and 2 to 1 are certain limits, and now determine of all the Ratios which the Quantity

A is able to have to the Quantity B – between which limits these indeterminate Ratios are restrained or detained within bounds, and to which they may come nearer than to any one given Diff^ence (See Newton under the end of the First Section)

Of ~~the~~ Curvature & the Angle of Contact

See Page 246

Definition –

If to some very little part of a Curve, or an Arc, and to the same Parts. Let the Arc be applied of the smallest circular or round, that with the former may agree as much as is able to be done; the Curvature of these are said to be the same. But if another may compass the place of the Other, the Curvature of the outermost is said to be less, and of the inmost greater &c. & universally the Quantity of Curvature in any one plane is reckoned always by the reciprocal ratio of the Diam^r^ of a Circle equally curved; from whence the ~~the~~ Curvature is called finite ~~when~~ or when the Diameter of Curvature that is the Dia^r^ of a Circle equally curved is finite.

Theorem 1 Fig 59

If thro some Curve the points A B, C continually approaching one another, & in the Plane B of the farthest uniting, Let the Circle D always pass over I say the ~~seam~~ Circle D will have at last the same Curvature as the Curve in the plane B

Demon^n^ –

(If thou disputest) Let another Circle have the same Curvature & let ~~them~~ accord as much as possible with the other Curve in the plane B. this Circle E shall even pass thro the 3 Points A. B. C, or it will not; Let it pass thro therefore, and the 2 Circles shall cut themselves in more than 2 Points contrary to what is demonstrated by Euclid in the 10 Propor^n^ of 3 Book of the Elements, If it cannot pass thro, then Circle D accords more with the Curve in the plane B then the Circle E of the same Curvature (that is) than the Circle more apt. 2, E. A. therefore the Circle D hath a determined Curvature.

Q. E. D

Corollary 1

If out of 3 Points A. B. C only A & B unite, the 3rd C remaining immovable, the Circle D shall surely the Curve in the plane B, but it shall not have the same curvature necessarily, unless the 3 point C shall cease to the other 2 A and B

Scholium

As there are Infinite number of Circles given, which will pass over 2 given points, & only one which will pass over 3; So there are given Circles an infinite number, which will touch the same Curve in the same plane, but only one that with hath the same Curvature, & shall congruee or accord with the Curve in the plane of Contact. Furthermore from the demonstrated Arguments it will easily appear that the smallest particle of all Curves, where the Curvature is finite, delights in all the same

affections or relations, as to Chords, Tangents, Sines (sagittas) &c &c which Arcs are of the same Curvature which are the best Circles, of them therefore some from the nature of Circles being derived ~~as it were~~ from their proper source especially those in Newton's Philosophy which shall occur will be of the greatest use, in general to illustrate & explain shall not trouble me. And there of this kind

1 Fig 60

If a Circle ABC shall touch a certain right line AD, & let the point B be moved, thro' the means of towards the immovable point A and let it ultimately join the same, & if thro' the immovable point A & the movable point B. the Chord AB is always understood to pass; I say the Angle BAD between the Chord AB and the Tangent AD, will be ultimately less than any given right lined ~~the~~ figure

For the this Angle B A D will be always equal to the Angle A C B in the Alternate Segment, which in the joining of A and B vanishes.

2

If from a point B the subtense be be applied in any manner whatever making any Angle finite B D A with the Tangent A D, I say, the Chord A B. the Tangent A D, & Arc A B will be ultimately in the ratio of equality. For place A C parallel to the subtense B D, then the Triangles C A B and A B D will be similar; from whence A B and A D and A D & D B shall be to one another as C A & C B and C B + B A, Let now the point B join with the Point A & C B & C A and C B + B A will be ultimately in the ratio of Equality whereas A B and A D and A D & D B

will be in a ratio of Equality, much more shall A B & A D and the Arc A B be in a ratio of equality.

3

Wherefore in every Computation concerning ultimate Ratios these Lines may be used one for the other

4

If the Chord B F G is drawn parallel to the Tangent A D, cutting the Chord A C in F, the sine verse A F will ultimately bisect the Arc B A G and the Chord B F G; For by the Parallels A D, B G. the point A is in the middle of the Arc B A G, and if A F & B D are also made parallel. B F or A D equal to the Arc A B; & by like reasoning F G shall be equal to the Arc A G.

5

The subtense BD, and its equal & parallel vers'd sine AF shall be as the square of the Arc BA, or its double BAG directly, & as the Chord AC inversely; if the position of the Chord AC be parallel to the subtense BD, and the Chord BG to the Tangent AD. For by similar Triangles CAB, ADB. CA is to AB as AB to BD; Therefore universally BD or AF is equal to the square of the Chord AB applied to the Chord AC; & the vanish Chord is equal to the vanishing Arc

6

And therefore in any curve-lined Figure, that is given an Angle to D and Curvature at A, & subtense BD vanishing, and vers'd sine AF shall be as the square of the Arc BA or the Arc BAG.

7

On which account If there shall be more Arcs, being small, & nearest the square of these shall be as their V- sine (sagittae) which the Chords bisect, and converge to a given point. For from the smallness & nearness of the Arcs, the curvature of all will be the same, the Tangents are all on the same right line as it were, and all the vers'd sines are, as if parallel

8

If from some points A and B of a Curve any right lines are drawn as BF, BF so inclined as that BF shall be parallel to the Tangent at A, and if AF is produced to C so as the length AC shall be equal to that which shall be ultimately by applying the square of the vanishing Arc AB, or the Chord AB or the ordinate BF to intersecting

AF, A Circle which touches the Curve in A and passes thro C shall have the same Curvature with the Curve in the point of Contact.

9

Or if the Line AD touch some Curve in the Point A that may cut the subtense BD in B, and if from the points A and B the Lines AC & BC are drawn, making the ABC, BAC respectively equal to the Angles ADB, ABD, these Lines AC. BC will ultimately concur in the Periphery of a Circle equal to the Curve, and agreeing ~~same~~ with that other Curve in A.

10

Lastly, if in some curvelined Figure the curvature at A shall not be given, if nevertheless the Line AC is terminated parallel to subtense

BD tends to the Curvature of a Circle of equally curved, then as if the Arc AB is compleated into a Circle equally curved ABC, the vanishing subtense BD shall be equall to the Square of nascent Arc AEB applied to that Line AC.

The following Theorem respects Arcs the smallest of Curves, whose curvatures are infinitely greater or infinitely less than Circles. See Scholium at the end of the Section of the Principia.

Theo. 2 Fig 61

If the Curves AB, AC and the Line AD mutually touch in the point A by whatever laws the Line DBC shall be ~~either~~ or to DB even universally or only in vanishing as AD^m, and CD as AD^n, making m greater than n; I say the Angle of Contact

(1) antequam ... quam ... ligem recta BC

B A D will be infinitly less than
C A D & therefore no curved Line
of the same genus with A C is able
to be drawn between the Curve
A B and its Tangent A D.

C Demonn

For Let B D be $= \frac{AD^m}{p}$, and $C D = \frac{AD^n}{q}$
making p and q finite & constant.
B D, will be to C D as $\frac{AD^m}{p}$ to $\frac{AD^n}{q}$, or as
AD^m to $AD^n \times \frac{p}{q}$ or as AD^{m-n} to $\frac{p}{q}$
Let now AD vanish & at the same
time AD^{m-n}, & B D will now be to C D
as the vanishing Quantity AD^{m-n} to
the finite quantity $\frac{p}{q}$, or as finite to
infinite; Therefore the vanishing sub-
-tense B D is infinitly less than vanishing
subtense C D, also on this account the
the Angle B A D than the Angle C A D
Q. E. D.

Demonstration of the 8th Corollary of
the 4th Proposition.

If two bodies revolve in similar Orbits
whose centripetal forces tending to their
Centers similarly placed; I say
these very forces in similar Places
are as the squares of the least
Arcs that are described together
applied to the Radii from these places
being drawn into centers of forces

Fig 62

Let S & s be the centers of forces
B and b similar places of orbits
A B C and a b c the least Arcs
described in the same time being the
Points B & b are in the middle. &
Let these Arcs be compleated
into equally curved Circles A B C E
& a b c e cutting the Radii B S and
b s produced if needful in E and e;
& because the Arcs A B C & a b c a

described in the same time, the centripetal force as in B will be to vim centripetam in b, as the vers'd sine of the Arc, ABC which besects the Chord, & passes thro' the center of force to the similar vers'd sine of the Arc abc: But then vers'd sines (sagittae) are to each other as $\frac{ABC^2}{BE}$ to $\frac{(abc)^2}{be}$. by that which we have written above of curvature. Ergo the centripetal force in B is to the centripetal force in b as $\frac{ABC^2}{BE}$ to $\frac{abc^2}{be}$. but in similar Figures all lineaments are proportional: Wherefore BE : be :: BS : bs: Let BS and bs be substituted for BE & be & the force centripetal in B will be to the centripetal force in b. as $\frac{ABC^2}{BS}$ to $\frac{abc^2}{bs}$. Q.E.D.

Cor.y 1

The Centripetal forces in similar places are as the squares of y^e velocities in those places directly & as the Radii drawn to the center of Forces inversely. For the velocities as the arcs describ'd in the same times

Coroll.y 2nd

Thus the centripetal forces in similar places are as the Radii drawn to the Centres of Forces directly, and as the squares of the Periodic Times inversely. For Let the Arcs ABC and abc be reser'd now to no more than what is described in least Time together even similar Particles of similar Figures, in the least describ'd Times T and t; and the velocity in B will be to velocity in b. as $\frac{ABC}{T}$ to $\frac{abc}{t}$, even as the Spaces describ'd directly & the Times inv

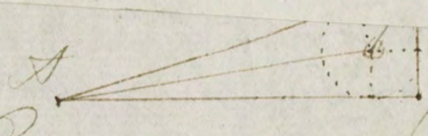
But the Arc ABC is to the similar Arc abc as the length BS to the similar length bs; Wherefore the velocity in B is to the velocity in b as $\frac{BS}{T}$ to $\frac{bs}{t}$, & the square of the vel^y in B is to the square of the vel^y in b as $\frac{BS^2}{T^2}$ to $\frac{bs^2}{t^2}$. But by the law Coroll^y the centripetal forces in B & b are as the squares of the vel^ies in these places directly And as the Radii from these places drawn unto the centers of Forces Inversely; Therefore the centripetal force in B is to the centripetal force in b as $\frac{BS}{T^2}$ to $\frac{bs}{t^2}$. But truly, as the similar parts of similar Areas SABC, sabc ~~to the~~ are to the whole Areas in the same Ratio, and the Times T and t will be in the same ratio to the whole periodic times: Wherefore the centripetal force in similar places B & b shall be as the Radii from the same places drawn unto the Centers of Forces directly; & as the square of the periodic Times Inversely.

The rest follow, in the same way, being demonstrated in a circle.

Demon: of the 9 Cor of the 4 Propos:—

If a Body revolve uniformly in a circular orb, the centripetal tending to the center running thro' any Arc AB in the time t. I say that the Arc AB will be a mean proportional between AE the Diam^r of the Circle, & the space of the right lined descent ended in the same time t; If the body revolving with all the circular is deprived of the right lined, ~~being turned~~, & shall be let fall towards the Center, urged by the same centripetal force, by which before it was forced in a circular Orb For by the given centripetal force the Descent (s) as the square of the Time t, that is, from the supposition of the uniform motion of the Body, as the square of the Arc AB, or as $\frac{AB^2}{AE}$; from whence if in some particular case the lengths s and $\frac{AB^2}{AE}$ appear to be equal, they will be always equal. Let the line AD touch the circle in A, and the subtensa BD is made parallel to the Diameter AE. & if the Arc AB is made infinitly

little, the subtensa BD will be $= \frac{AB^2}{AE}$, as appears from those theorems demonstrated of curvatures



Let x = Radius

y = ~~Surface~~ Area less Quadr

$\frac{y}{x}$ = Side $a - x$ = Side Great

$\frac{y}{x} : y :: a - x : ax - x^2$ Area Greater

$y + x^2 + ax - x^2$ = Area of the whole

$+ax$ $\frac{ax-y}{x}$ Side Great

$\frac{y}{x} + x = \frac{y+x^2}{x}$ = Perpen

$\frac{ax-y}{x}$ Area Greater Quadr

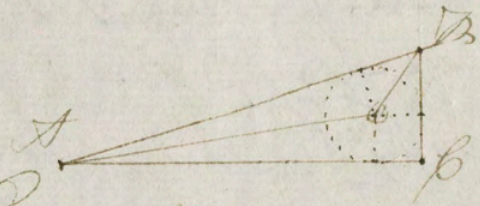
$\frac{ax-y}{x} + y + x^2 = ax + x^2$ = Area of the Whole

$\cancel{ax} = 2ax + 2x^2$

$p = 2a + 2x \,.\, a + 2x$

... by the given centripetal
the Descent (s) as the square
the Time t, that is, from the
supposition of the uniform
tion of the Body, as the squ
of the Arc BAB, or as A/A
from whence if in some pa
case the lengths s and A/A
appear to be equal, they will
always equal. Let the line
AD touch the circle in A
the subtensa BD is made p
lel to the Diameter AE. &
the Arc AB is made in

there



Let x = Radius

y = ~~Circle~~ Area less Quadr

$\frac{y}{x}$ = Side $a - x$ = Side Greater

$\frac{y}{x} : y :: a - x : ax - x^2$ Area Greater

$y + x^2 + ax - x^2$ = Area of the whole

$y + ax$ $\frac{ax - y}{x}$ Side Great

$\frac{y}{x} + x = \frac{y + x^2}{x}$ = Perp

$ax - y$ Area Greater Quadr

$\frac{ax - y}{x} + y + x^2 = ax + x^2$ = Area of the Whole

$\cancel{ax} = 2ax + 2x^2$

ta $p = 2a + 2x \quad a + 2x$

$y =$ less Area

$x =$ Radius then $\frac{y}{x} =$ side of less

$a - \frac{y}{x}$ side of Greater

$ax - y$ Area of Greater

$ax - y + y =$ Area of

$ax + x^2 =$ Area of whole triangle

$\frac{a}{2} \times x = \frac{ax}{2} =$ Area AOB

$\frac{y}{x} + x \times \frac{x}{2} = \frac{y + x^2}{2} =$ Area COB

$a - \frac{y}{x} + x \times x = \frac{ax - y + x^2}{2} = AOC$

$\frac{ax}{2} + \frac{y + x^2}{2} + \frac{ax - y + x^2}{2} = ax + x^2$

$\frac{y + x^2}{x} =$ Perpen^r

$\frac{y^2 + 2yx^2 + x^4}{x^2} =$ Squ^r Pd

$\frac{ax - y + x^2}{x} =$ Base

$a^2x^2 - axy + ax^3$
$- axy + ax^3 + y^2 - yx^2$
$- yx^2 + x^4$

$\frac{a^2x^2 - 2axy + y^2 - 2yx^2 + ax^3 + x^4}{x^2} = \square$ Base

$\square$ Base $+ \square$ Perpen $= \frac{a^2x^2 - 2axy + 2y^2 + ax^3 + 2x^4}{x^2}$

$= a^2$ or $a^2x^2 - 2axy + 2y^2 + ax^3 + 2x^4 = a^2x^2$

$2y^2 - 2axy = -ax^3 - 2x^4$

$y^2 - axy = -\frac{ax^3}{2} - x^4$

$y^2 - axy + \frac{a^2x^2}{4} = \frac{a^2x^2}{4} - \frac{ax^3}{2} - x^4$

$y - \frac{ax}{2} = \left(\frac{a^2x^2}{4} - \frac{ax^3}{2} - x^4\right)^{\frac{1}{2}}$

$y = \left(\frac{a^2x^2}{4} - \frac{ax^3}{2} - x^4\right)^{\frac{1}{2}} + \frac{ax}{2}$

Again $x^2 + \frac{y^2}{x^2} = BO^2$

$x^2 + \left(a - \frac{y}{x}\right)^2 = AO^2 = x^2 + a^2 - \frac{2ay}{x} + \frac{y^2}{x^2}$

$\left(\frac{x^4 + y^2}{x^2}\right)^{\frac{1}{2}} = BO$ & $\frac{x^4 + a^2x^2 - 2ayx + y^2}{x^2}$

$\left(\frac{x^4 + a^2x^2 - 2ayx + y^2}{x^2}\right)^{\frac{1}{2}} - \left(\frac{x^4 + y^2}{x^2}\right)^{\frac{1}{2}} = b$

$\left(\frac{x^4 + a^2x^2 - 2ayx + y^2}{x^2}\right)^{\frac{1}{2}} = b + \left(\frac{x^4 + y^2}{x^2}\right)^{\frac{1}{2}}$

$b^2x^2 + 2bx(x^4+y^2)^{\frac{1}{2}} = a^2x^2 - 2ayx$

or $2bx(x^4+y^2)^{\frac{1}{2}} = a^2-b^2x^2 - 2ayx$

call $a^2-b^2=m$

$2bx(x^4+y^2)^{\frac{1}{2}} = mx^2 - 2ayx$

$4b^2x^2(x^4+y^2) = m^2x^4 - 4amyx^3 + 4a^2y^2x^2$

Whence $4b^2x^4 + 4b^2x^2y^2 = m^2x^4 - 4amyx^3 + 4a^2y^2x^2$

or $4b^2x^2 + 4b^2y^2 = \frac{m^2}{4}x^2 - amyx + a^2y^2$

$(a^2-b^2)y^2 - amyx = b^2x^4 - \frac{m^2x^2}{4}$

$my^2 - amyx = b^2x^4 - \frac{m^2x^2}{4}$

$y^2 - ayx = \frac{b^2x^4}{m} - \frac{mx^2}{4}$

$y - \frac{ax}{2} = \left(\frac{a^2x^2}{4} + \frac{b^2x^4}{m} - \frac{mx^2}{4}\right)^{\frac{1}{2}} = \left(\frac{b^2x^4}{m} + \frac{b^2x^2}{4}\right.$

$y = \left(\frac{b^2x^4}{m} + \frac{b^2x^2}{4}\right)^{\frac{1}{2}} + \frac{ax}{2} = \left(\frac{a^2x^2}{4} - \frac{ax^3}{2} - x^4\right)^{\frac{1}{2}} + \frac{ax}{2}$

or $\frac{b^2x^4}{m} + \frac{b^2x^2}{4} = \frac{a^2x^2}{4} - \frac{ax^3}{2} - x^4$

$4b^2x^4 + mb^2x^2 = ma^2x^2 - 2max^3$

$4b^2x^2 + mb^2 = ma^2 - 2max$

$4b^2x^2 + 2max = ma^2 - b^2 \times m = m^2$

$x^2 + \frac{2max}{4b^2} = \frac{m^2}{4b^2}$

$x + \frac{ma}{4b^2} = \left(\frac{m^2}{4b^2} + \frac{m^2a^2}{16b^4}\right)^{\frac{1}{2}}$

$x = \left(\frac{4m^2b^2 + m^2a^2}{16b^4}\right)^{\frac{1}{2}} - \frac{ma}{4b^2} =$

$(4m^2b^2 + m^2a^2)^{\frac{1}{2}} \times \frac{1}{4b^2} - \frac{ma}{4b^2} =$

$(4b^2 + a^2)^{\frac{1}{2}} \times \frac{m}{4b^2} - \frac{ma}{4b^2}$

$a = 10$ b_2

$100 - 4 = 96 = m$

$\frac{48}{10}$ $\frac{24}{5}$

8944

447 2

$\frac{24}{5} \times 2{,}236 = 24 \times ,45,3,6$

245

4)19,664

5 –

$(x^2-b^2)^{\frac{1}{2}} = AC$

$\therefore (x^2-2xd+d^2-b^2)^{\frac{1}{2}} = BC$

$(x^2-b^2)^{\frac{1}{2}} + (x^2-2xd+d^2-b^2)^{\frac{1}{2}} = a$

$a-(x^2-b^2)^{\frac{1}{2}} = (x^2-2xd+d^2-b^2)^{\frac{1}{2}}$

$a^2-2a(x^2-b^2)^{\frac{1}{2}}+x^2-b^2 = x^2-2xd+d^2-b^2$

$\therefore a^2-2a(x^2-b^2) = d^2-2xd$

$a^2-d^2+2xd = 2a(x^2-b^2)^{\frac{1}{2}}$ Call $a^2-d^2=m$

$\therefore m+2xd = 2a(x^2-b^2)^{\frac{1}{2}}$

$m^2+4mxd+4x^2d^2 = 4a^2x^2-4a^2b^2$

$\therefore \overline{4a^2-4d^2}\,x^2-2mxd = m^2+4a^2b^2$

$\therefore 4mx^2-2mxd = m^2+4a^2b^2$

$x^2-\frac{dx}{2} = \frac{m}{4}+\frac{a^2b^2}{m}$

$x^2-\frac{dx}{2}+\frac{d^2}{4} = \frac{d^2+m}{4}+\frac{a^2b^2}{m}$

$x-\frac{d}{2} = \left(\frac{d^2+m}{4}+\frac{a^2b^2}{m}\right)^{\frac{1}{2}}$

$x = \left(\frac{d^2+m}{4}+\frac{a^2b^2}{m}\right)^{\frac{1}{2}}+\frac{d}{2}$

£
2011 " Feby 1st 1820
40 nearly "
1971

£
1972

Put x = Sum

$\frac{x+d}{2} = AO \quad \frac{x-d}{2} = BO$

$x : \frac{x+d}{2} :: a : \frac{ax+ad}{2x} = Am$

$a - \frac{ax+ad}{2x} = \frac{ax-ad}{2x} = Bm$

$\frac{a^2 \times \overline{x-d}^2}{4x^2} + = \frac{\overline{x^2-2dx+d^2} \times a^2}{4x^2}$

$\overline{\frac{x^4-2x^3d+x^2d^2-x^2a^2+2dxa^2-a^2d^2}{4x^2}}\Big|^{\frac{1}{2}} = r$

put $d^2-a^2d = m$

$r = \overline{\frac{x^4-2x^3d-x^2a^2+2dxa^2+m}{4x^2}}\Big|^{\frac{1}{2}} = r$

R

$\overline{\frac{ax+ad}{2x}+r}^2 + \overline{\frac{ax-ad}{2x}+r}^2 = a^2$

$\overline{\frac{ax+ad+2rx}{2x}}^2 + \overline{\frac{ax-ad+2rx}{2x}}^2 = a^2$

$ax+ad+2rx$

$ax+ad+2rx$

$a^2x^2+a^2xd+2arx^2$

$-a^2xd+ \quad 2arx^2 \quad +a^2d^2+2adrx$

$+2adrx+4r^2x^2$

$a^2x^2+2a^2xd+4arx^2+a^2d^2+4adrx+4r^2x^2$

$2a^2x^2+8arx^2+2a^2d^2+8r^2x^2 = a^2$

~~$2a^2+8ar+8r^2$ make~~

a

John McCaver

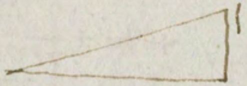
New Miles

1819 Ak

April 5 1819

Because $O\,r = C\,r$ Ergo [illegible]

Let $x =$ Area



$\cancel{A}B + BC + AC \times r = AC + BC$

$AB : AC : AC : \cancel{D}$ Cm

$AB \times Ah = AC^2$

a = [illegible]

Let x = Sum

d =

$\frac{x+d}{2} + \frac{x-d}{2}$ = [illegible]

$\frac{x+d}{2} : \frac{x-d}{2} : A[illegible] :: [illegible]$

$x : Am + mB = a :: : \frac{x+d}{2}$

$\frac{ax+ad}{2x} = Am$

$a - \frac{ax+ad}{2x} = \frac{ax-ad}{2x} = m[illegible]$

$\frac{x^2+2xd+d^2}{4} -$

$\frac{x+d}{2} : ax$

$4 - \frac{3+2}{3} = 2\frac{1}{3}$

12

$x = 6\frac{1}{3} \quad x+d = A6 \quad 6$

$x \; 2x+d : a :: x+d$

$\frac{ax+ad}{2x+d} = Am$

$a - \frac{ax+ad}{2x+d}$

$2ax + ad - ax - ad$

$\frac{ax}{2x+d} = mB$

$x^2 - \frac{a^2x^2}{4x^2+4xd+d^2} = Om$ [illegible]

$\frac{4x^4+4x^3d+d^2x^2-a^2x^2}{4x^2+4xd+d^2} = r^2$

$\left(\frac{4x^2+4xd+d^2-a^2}{2x+d}\right)^{\frac{1}{2}} x = r$

$\left(\frac{2ax+ad}{2x+d}\right)^2 + \frac{4ax+2ad}{2xa} \; [illegible]$

$CP = \frac{BQ \times AC}{AO}$

$CP^2 = \frac{BQ^2 \times AC^2}{AO^2}$

BQ^2